Computer Algebra Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

4-Trig-functions/4.4-Cotangent/113-4.4.7-d-trig-^m-a+b-c-cot-^n-^p

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [64]. This is test number [113].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (64)	0.00 (0)
Mathematica	100.00 (64)	0.00 (0)
Fricas	100.00 (64)	0.00 (0)
Maple	98.44 (63)	1.56 (1)
Giac	84.38 (54)	15.62 (10)
Mupad	60.94 (39)	39.06 (25)
Maxima	35.94 (23)	64.06 (41)
Sympy	17.19 (11)	82.81 (53)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

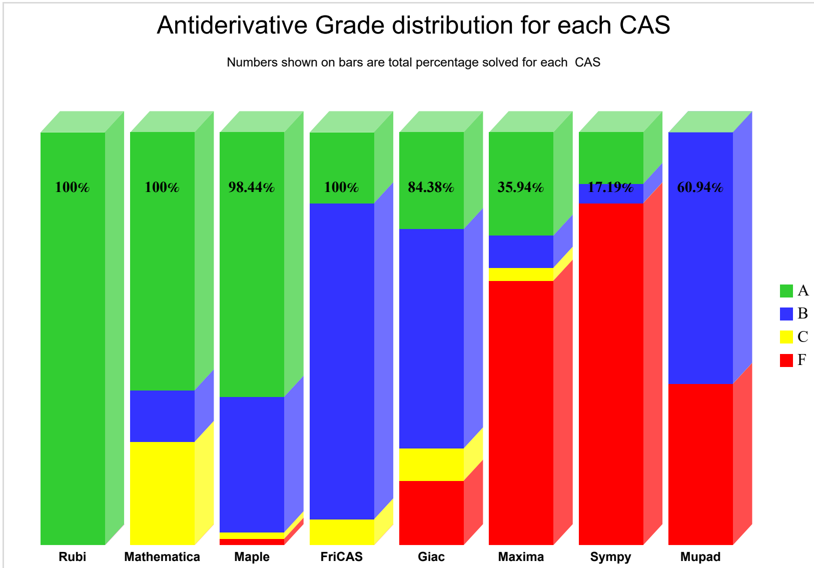
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

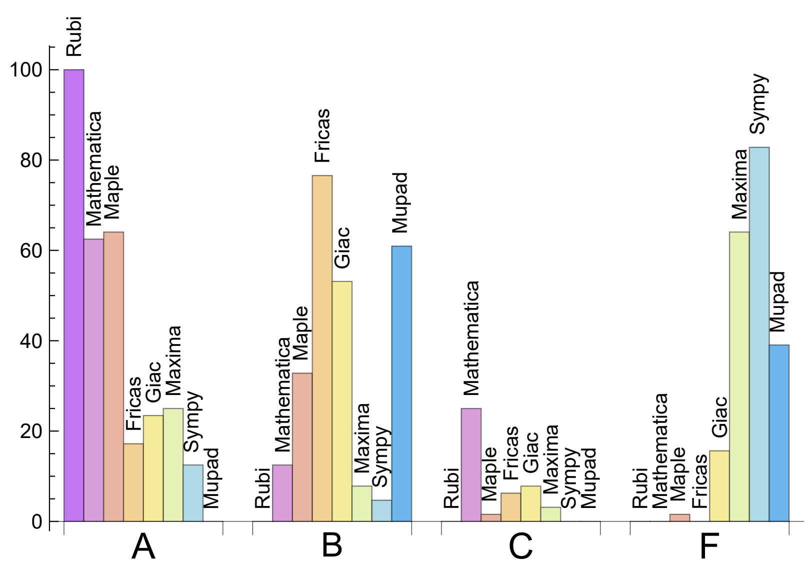
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Maple	64.062	32.812	1.562	1.562
Mathematica	62.500	12.500	25.000	0.000
Maxima	25.000	7.812	3.125	64.062
Giac	23.438	53.125	7.812	15.625
Fricas	17.188	76.562	6.250	0.000
Sympy	12.500	4.688	0.000	82.812
Mupad	0.000	60.938	0.000	39.062

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Fricas	0	0.00	0.00	0.00
Maple	1	100.00	0.00	0.00
Giac	10	0.00	10.00	90.00
Mupad	25	0.00	100.00	0.00
Maxima	41	56.10	0.00	43.90
Sympy	53	100.00	0.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Rubi	0.29
Fricas	0.34
Maxima	0.38
Maple	0.42
Giac	0.75
Mathematica	1.23
Sympy	6.49
Mupad	12.79

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Rubi	75.31	1.06	64.50	1.06
Maxima	142.13	4.02	52.00	1.21
Mathematica	158.78	1.85	72.00	1.03
Maple	161.00	2.05	84.00	1.23
Giac	246.44	2.91	164.50	2.30
Mupad	311.87	3.12	47.00	1.00
Fricas	454.30	5.52	337.00	5.31
Sympy	1072.82	8.82	88.00	1.45

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

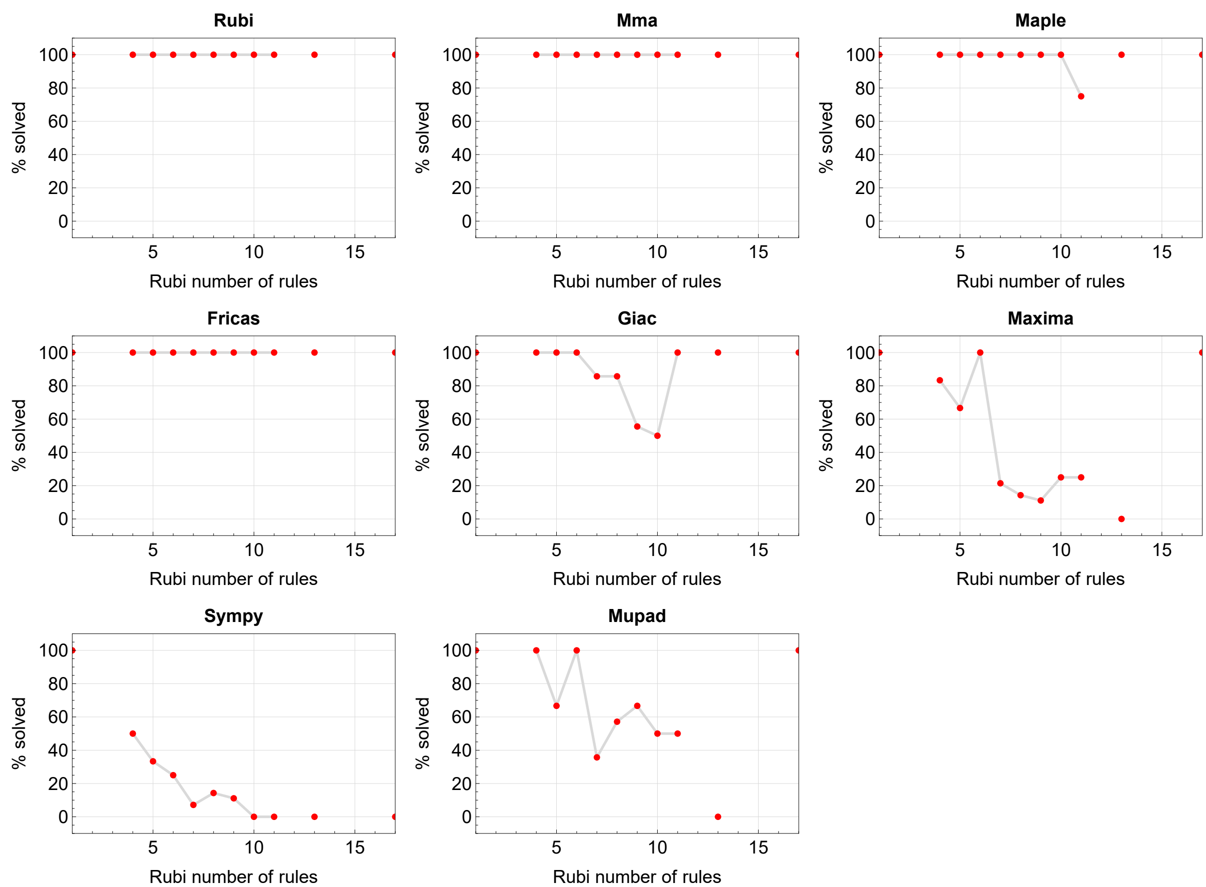


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

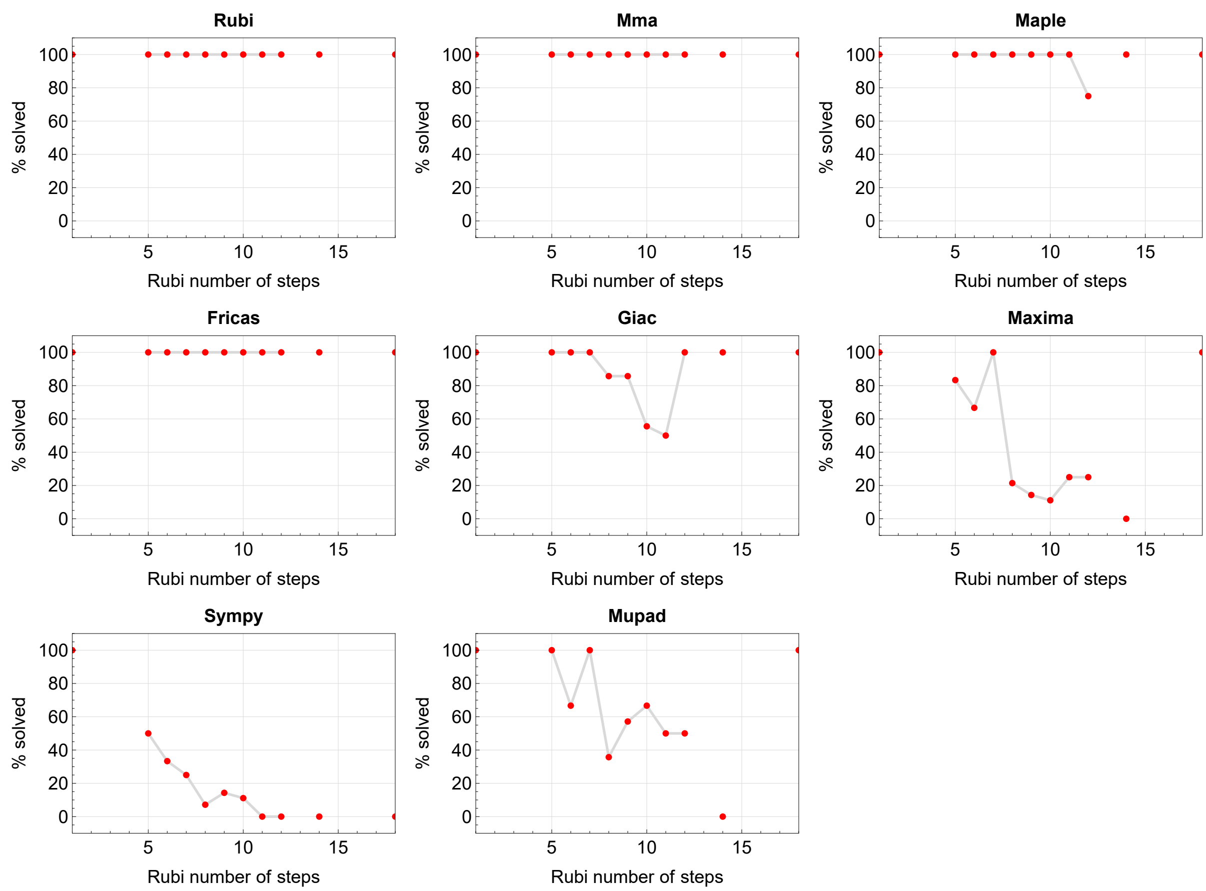


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

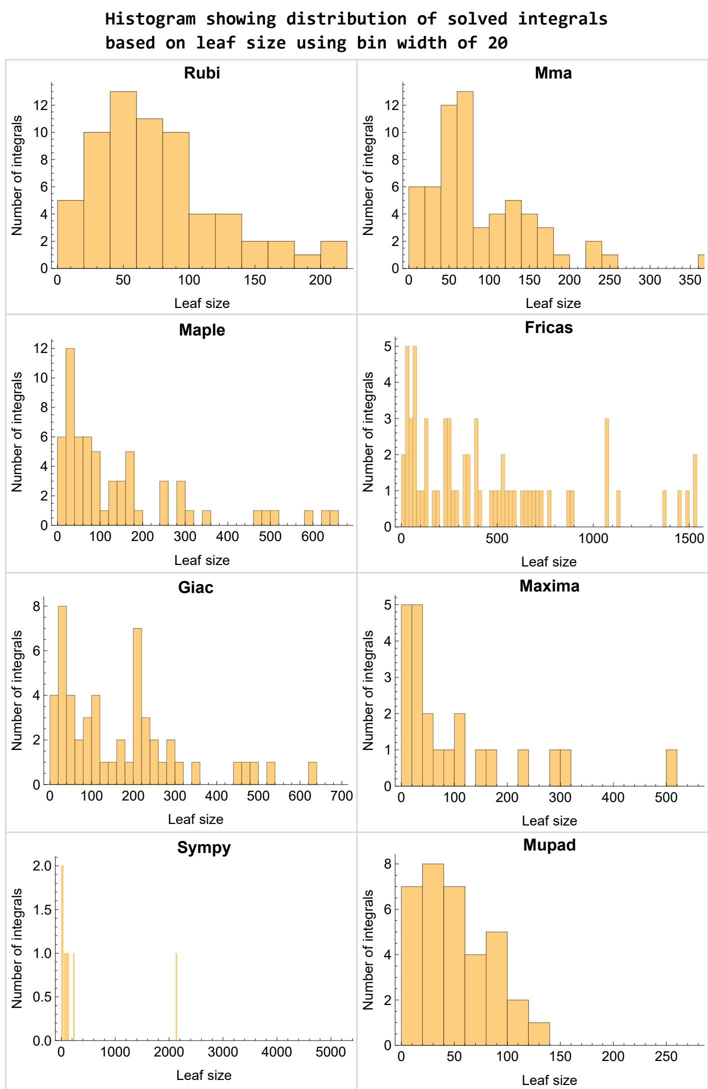


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

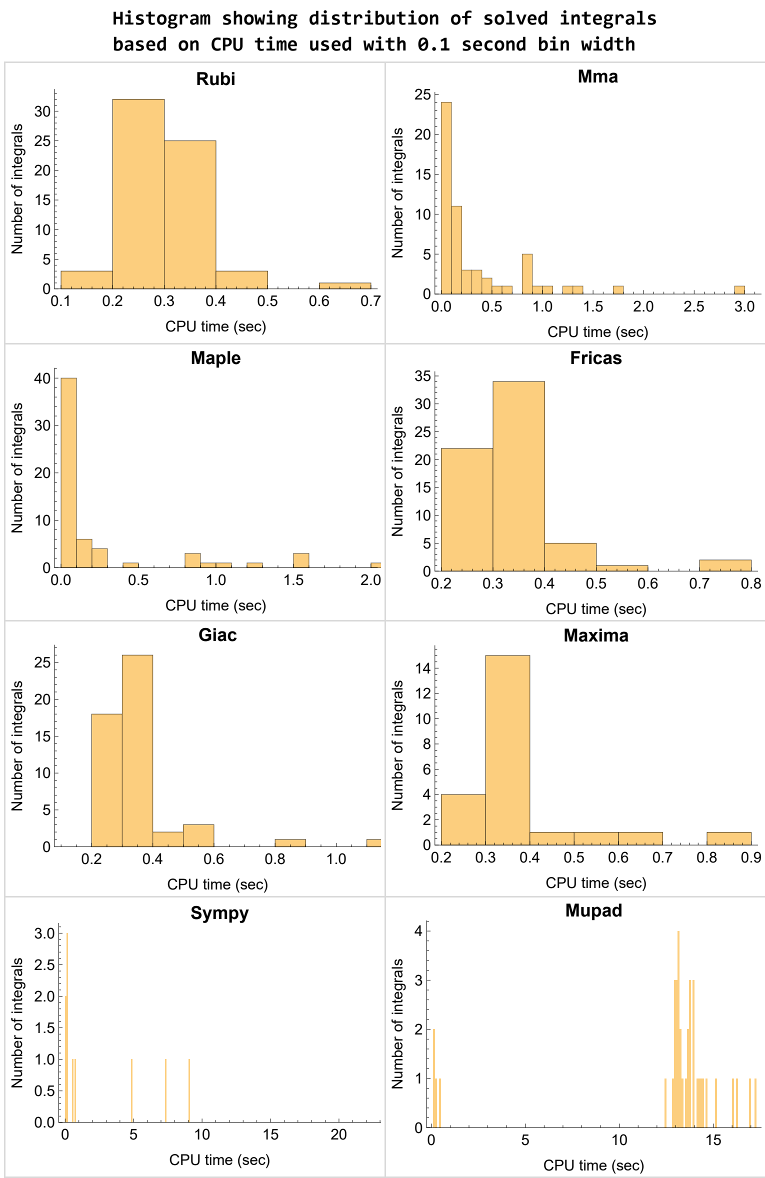


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

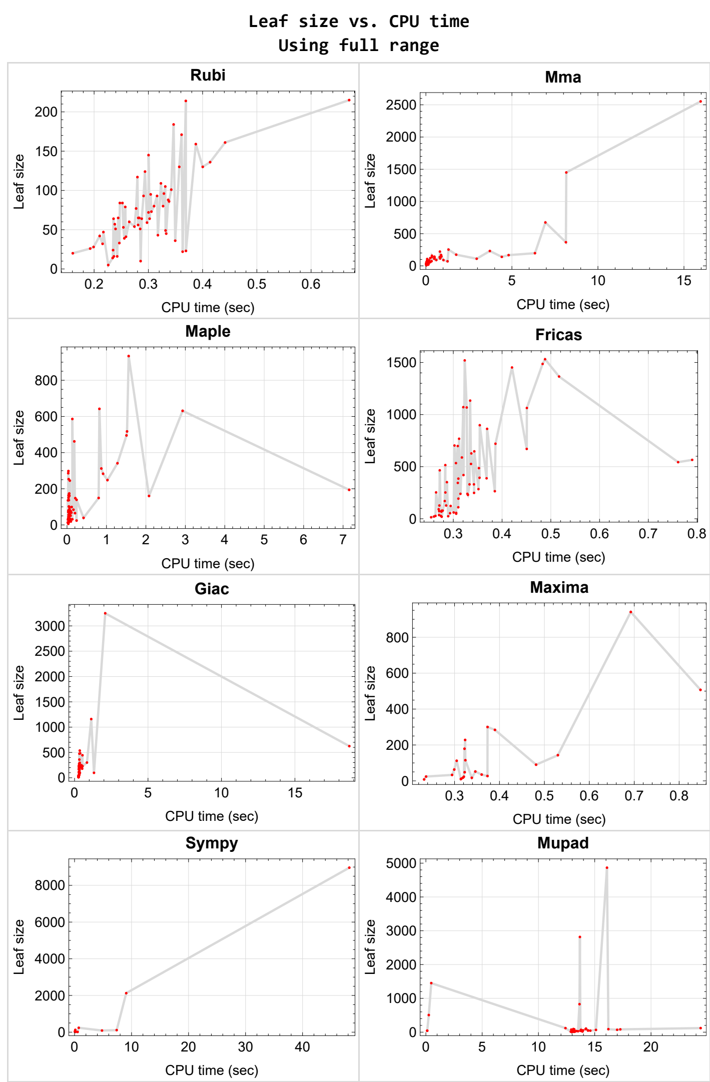


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{1\}$

Mathematica $\{25, 35, 36, 37, 40, 43, 48, 53, 55, 58\}$

Maple $\{24, 30, 52, 53, 58\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

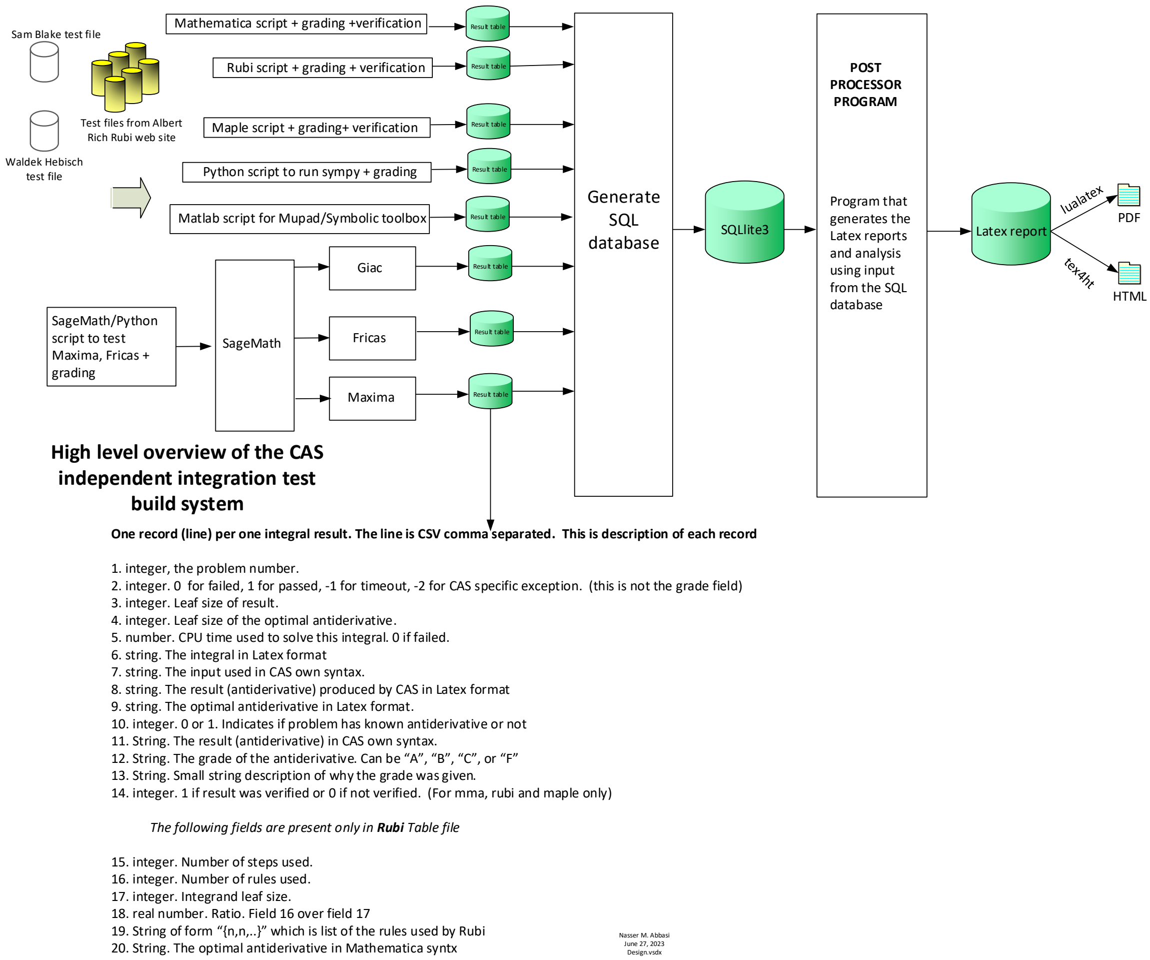
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 3, 4, 5, 6, 7, 10, 11, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 26, 27, 28, 29, 31, 32, 33, 39, 40, 41, 42, 43, 44, 46, 47, 49, 60, 61, 62, 63, 64 }

B grade { 8, 9, 12, 30, 34, 38, 45, 50 }

C grade { 2, 24, 25, 35, 36, 37, 48, 51, 52, 53, 54, 55, 56, 57, 58, 59 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 1, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 49, 50, 51, 54, 55, 56, 59, 60, 62 }

B grade { 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 47, 48, 52, 53, 58, 61, 63, 64 }

C grade { 2 }

F normal fail { 57 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 5, 14, 18, 21, 24, 25, 29, 30, 47, 48, 59 }

B grade { 2, 3, 4, 6, 7, 8, 9, 10, 15, 16, 17, 19, 20, 22, 23, 26, 27, 28, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 60, 61, 62, 63, 64 }

C grade { 1, 11, 12, 13 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.5 Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 10, 12, 13, 14, 15, 16, 17, 18, 59 }

B grade { 8, 9, 11, 40, 43 }

C grade { 39, 42 }

F normal fail { 21, 22, 24, 25, 27, 29, 30, 31, 32, 38, 41, 45, 47, 48, 52, 53, 57, 58, 60, 61, 62, 63, 64 }

F(-1) timedout fail { }

F(-2) exception fail { 19, 20, 23, 26, 28, 33, 34, 35, 36, 37, 44, 46, 49, 50, 51, 54, 55, 56 }

2.1.6 Giac

A grade { 1, 2, 5, 6, 7, 8, 9, 14, 15, 16, 17, 18, 59, 62, 63 }

B grade { 3, 4, 10, 20, 21, 23, 24, 25, 30, 34, 35, 36, 37, 38, 41, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 60, 61, 64 }

C grade { 11, 12, 13, 39, 40 }

F normal fail { }

F(-1) timedout fail { 42 }

F(-2) exception fail { 19, 22, 26, 27, 28, 29, 31, 32, 33 }

2.1.7 Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 21, 26, 28, 29, 34, 38, 39, 40, 42, 43, 44, 46, 47, 49, 51, 52, 54, 56, 57, 59 }

C grade { }

F normal fail { }

F(-1) timedout fail { 15, 22, 23, 24, 25, 27, 30, 31, 32, 33, 35, 36, 37, 41, 45, 48, 50, 53, 55, 58, 60, 61, 62, 63, 64 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 2, 3, 4, 10, 16, 51, 56, 59 }

B grade { 5, 6, 7 }

C grade { }

F normal fail { 1, 8, 9, 11, 12, 13, 14, 15, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 52, 53, 54, 55, 57, 58, 60, 61, 62, 63, 64 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	215	148	160	179	465	0	248	828
N.S.	1	0.92	0.64	0.69	0.77	2.00	0.00	1.06	3.55
time (sec)	N/A	0.663	0.905	2.079	0.322	0.272	0.000	0.473	13.647

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	34	29	23	48	22	40	20
N.S.	1	1.00	1.70	1.45	1.15	2.40	1.10	2.00	1.00
time (sec)	N/A	0.158	0.016	0.023	0.321	0.306	0.065	0.274	13.255

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	51	71	48	63	127	68	114	45
N.S.	1	1.09	1.51	1.02	1.34	2.70	1.45	2.43	0.96
time (sec)	N/A	0.233	1.259	0.054	0.299	0.271	0.096	0.320	0.121

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	79	111	77	112	253	126	229	76
N.S.	1	1.01	1.42	0.99	1.44	3.24	1.62	2.94	0.97
time (sec)	N/A	0.251	2.954	0.091	0.305	0.265	0.134	0.378	12.915

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	56	48	252	238	65	41
N.S.	1	1.00	1.00	1.14	0.98	5.14	4.86	1.33	0.84
time (sec)	N/A	0.324	0.070	0.076	0.323	0.284	0.740	0.305	0.118

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	117	90	99	115	534	2125	123	119
N.S.	1	1.21	0.93	1.02	1.19	5.51	21.91	1.27	1.23
time (sec)	N/A	0.270	1.021	0.118	0.324	0.305	9.066	0.316	12.401

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	184	138	148	228	1068	8964	206	4866
N.S.	1	1.23	0.92	0.99	1.52	7.12	59.76	1.37	32.44
time (sec)	N/A	0.334	0.352	0.210	0.324	0.328	48.202	0.398	16.089

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	24	51	19	300	91	0	32	18
N.S.	1	1.09	2.32	0.86	13.64	4.14	0.00	1.45	0.82
time (sec)	N/A	0.226	0.134	0.098	0.373	0.270	0.000	0.265	13.184

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	28	6	35	53	0	10	5
N.S.	1	1.00	5.60	1.20	7.00	10.60	0.00	2.00	1.00
time (sec)	N/A	0.209	0.033	0.033	0.360	0.293	0.000	0.273	12.971

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	14	12	13	10	21	14	28	12
N.S.	1	1.17	1.00	1.08	0.83	1.75	1.17	2.33	1.00
time (sec)	N/A	0.226	0.030	0.018	0.314	0.260	0.187	0.280	13.109

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	C	F	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	39	48	32	284	73	0	34	31
N.S.	1	1.11	1.37	0.91	8.11	2.09	0.00	0.97	0.89
time (sec)	N/A	0.246	0.115	0.039	0.390	0.271	0.000	0.268	13.017

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	C	F	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	16	30	15	17	19	0	11	14
N.S.	1	1.14	2.14	1.07	1.21	1.36	0.00	0.79	1.00
time (sec)	N/A	0.228	0.039	0.042	0.338	0.276	0.000	0.280	12.874

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	16	14	15	12	14	0	28	13
N.S.	1	1.14	1.00	1.07	0.86	1.00	0.00	2.00	0.93
time (sec)	N/A	0.222	0.030	0.032	0.315	0.255	0.000	0.265	13.203

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	36	19	29	24	27	0	25	17
N.S.	1	1.29	0.68	1.04	0.86	0.96	0.00	0.89	0.61
time (sec)	N/A	0.340	0.030	0.079	0.237	0.263	0.000	0.278	13.096

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	22	32	38	27	77	0	49	0
N.S.	1	0.71	1.03	1.23	0.87	2.48	0.00	1.58	0.00
time (sec)	N/A	0.352	0.152	0.044	0.373	0.276	0.000	0.291	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	8	27	12	11	10
N.S.	1	1.00	1.00	1.10	0.80	2.70	1.20	1.10	1.00
time (sec)	N/A	0.276	0.032	0.037	0.232	0.274	0.541	0.264	12.978

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	45	19	39	52	78	0	12	20
N.S.	1	1.25	0.53	1.08	1.44	2.17	0.00	0.33	0.56
time (sec)	N/A	0.330	0.027	0.419	0.346	0.278	0.000	0.272	13.185

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	23	19	24	18	35	0	25	34
N.S.	1	0.79	0.66	0.83	0.62	1.21	0.00	0.86	1.17
time (sec)	N/A	0.360	0.141	0.242	0.319	0.271	0.000	0.266	13.354

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	72	65	84	0	330	0	0	66
N.S.	1	1.09	0.98	1.27	0.00	5.00	0.00	0.00	1.00
time (sec)	N/A	0.296	0.203	0.163	0.000	0.342	0.000	0.000	15.109

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	53	48	71	0	248	0	95	53
N.S.	1	1.10	1.00	1.48	0.00	5.17	0.00	1.98	1.10
time (sec)	N/A	0.252	0.034	0.032	0.000	0.343	0.000	0.340	13.957

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	65	60	194	0	351	0	187	69
N.S.	1	1.08	1.00	3.23	0.00	5.85	0.00	3.12	1.15
time (sec)	N/A	0.274	0.034	7.154	0.000	0.287	0.000	0.325	13.781

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	93	140	174	0	768	0	0	0
N.S.	1	1.04	1.57	1.96	0.00	8.63	0.00	0.00	0.00
time (sec)	N/A	0.324	4.414	0.056	0.000	0.312	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	78	137	0	515	0	210	0
N.S.	1	1.00	1.20	2.11	0.00	7.92	0.00	3.23	0.00
time (sec)	N/A	0.239	0.173	0.046	0.000	0.284	0.000	0.498	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	51	51	44	312	0	193	0	239	0
N.S.	1	1.00	0.86	6.12	0.00	3.78	0.00	4.69	0.00
time (sec)	N/A	0.277	0.102	0.870	0.000	0.310	0.000	0.294	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	B	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	85	88	174	495	0	239	0	476	0
N.S.	1	1.04	2.05	5.82	0.00	2.81	0.00	5.60	0.00
time (sec)	N/A	0.327	1.763	1.508	0.000	0.329	0.000	0.329	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	95	91	150	0	486	0	0	120
N.S.	1	1.08	1.03	1.70	0.00	5.52	0.00	0.00	1.36
time (sec)	N/A	0.283	0.607	0.038	0.000	0.353	0.000	0.000	24.407

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	136	253	286	0	1134	0	0	0
N.S.	1	1.07	1.99	2.25	0.00	8.93	0.00	0.00	0.00
time (sec)	N/A	0.401	1.316	0.031	0.000	0.335	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	77	63	136	0	330	0	0	70
N.S.	1	1.12	0.91	1.97	0.00	4.78	0.00	0.00	1.01
time (sec)	N/A	0.262	0.188	0.024	0.000	0.333	0.000	0.000	16.991

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	80	75	341	0	565	0	0	506
N.S.	1	1.07	1.00	4.55	0.00	7.53	0.00	0.00	6.75
time (sec)	N/A	0.301	0.087	1.279	0.000	0.790	0.000	0.000	0.260

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	80	80	222	642	0	543	0	625	0
N.S.	1	1.00	2.78	8.02	0.00	6.79	0.00	7.81	0.00
time (sec)	N/A	0.313	0.815	0.821	0.000	0.761	0.000	18.701	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	169	462	0	1520	0	0	0
N.S.	1	1.00	0.99	2.70	0.00	8.89	0.00	0.00	0.00
time (sec)	N/A	0.359	0.885	0.185	0.000	0.323	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	124	143	298	0	1071	0	0	0
N.S.	1	0.98	1.13	2.37	0.00	8.50	0.00	0.00	0.00
time (sec)	N/A	0.290	0.499	0.035	0.000	0.321	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	84	107	170	0	703	0	0	0
N.S.	1	0.97	1.23	1.95	0.00	8.08	0.00	0.00	0.00
time (sec)	N/A	0.238	0.077	0.048	0.000	0.302	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	111	68	0	239	0	97	41
N.S.	1	1.00	2.36	1.45	0.00	5.09	0.00	2.06	0.87
time (sec)	N/A	0.212	0.482	0.111	0.000	0.315	0.000	1.323	13.726

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	B	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	85	84	231	102	0	526	0	300	0
N.S.	1	0.99	2.72	1.20	0.00	6.19	0.00	3.53	0.00
time (sec)	N/A	0.245	3.719	0.046	0.000	0.336	0.000	0.846	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	B	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	135	145	367	162	0	898	0	1160	0
N.S.	1	1.07	2.72	1.20	0.00	6.65	0.00	8.59	0.00
time (sec)	N/A	0.297	8.138	0.039	0.000	0.354	0.000	1.136	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	B	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	190	214	2553	253	0	1452	0	3249	0
N.S.	1	1.13	13.44	1.33	0.00	7.64	0.00	17.10	0.00
time (sec)	N/A	0.375	15.969	0.042	0.000	0.420	0.000	2.089	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	57	123	51	0	110	0	257	104
N.S.	1	1.06	2.28	0.94	0.00	2.04	0.00	4.76	1.93
time (sec)	N/A	0.225	0.505	0.090	0.000	0.310	0.000	0.330	14.224

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	B	F	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	62	34	507	68	0	170	88
N.S.	1	1.00	1.94	1.06	15.84	2.12	0.00	5.31	2.75
time (sec)	N/A	0.203	0.104	0.078	0.848	0.276	0.000	0.285	14.192

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	C	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	26	31	90	56	0	34	85
N.S.	1	1.00	0.93	1.11	3.21	2.00	0.00	1.21	3.04
time (sec)	N/A	0.194	0.056	0.081	0.481	0.306	0.000	0.270	13.177

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	64	121	48	0	170	0	179	0
N.S.	1	1.05	1.98	0.79	0.00	2.79	0.00	2.93	0.00
time (sec)	N/A	0.229	0.255	0.041	0.000	0.282	0.000	0.520	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	B	F	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	60	35	941	123	0	0	34
N.S.	1	1.00	1.43	0.83	22.40	2.93	0.00	0.00	0.81
time (sec)	N/A	0.209	0.090	0.030	0.692	0.294	0.000	0.000	13.918

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	48	21	143	60	0	45	20
N.S.	1	1.00	1.85	0.81	5.50	2.31	0.00	1.73	0.77
time (sec)	N/A	0.185	0.073	0.063	0.530	0.301	0.000	0.301	13.907

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	56	52	44	0	284	0	96	44
N.S.	1	1.08	1.00	0.85	0.00	5.46	0.00	1.85	0.85
time (sec)	N/A	0.266	0.191	0.037	0.000	0.352	0.000	0.364	14.609

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	158	80	0	588	0	229	0
N.S.	1	1.00	2.47	1.25	0.00	9.19	0.00	3.58	0.00
time (sec)	N/A	0.274	0.353	0.035	0.000	0.318	0.000	0.544	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	29	0	127	0	61	27
N.S.	1	1.00	1.00	0.88	0.00	3.85	0.00	1.85	0.82
time (sec)	N/A	0.242	0.027	0.078	0.000	0.285	0.000	0.328	13.513

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	65	60	149	0	419	0	203	93
N.S.	1	1.08	1.00	2.48	0.00	6.98	0.00	3.38	1.55
time (sec)	N/A	0.282	0.056	0.802	0.000	0.321	0.000	0.372	13.099

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	B	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	134	283	0	229	0	216	0
N.S.	1	1.00	2.48	5.24	0.00	4.24	0.00	4.00	0.00
time (sec)	N/A	0.269	0.852	0.914	0.000	0.330	0.000	0.337	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	64	59	68	0	385	0	109	52
N.S.	1	1.08	1.00	1.15	0.00	6.53	0.00	1.85	0.88
time (sec)	N/A	0.282	0.259	0.057	0.000	0.310	0.000	0.320	14.397

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F(-2)	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	137	99	0	388	0	159	0
N.S.	1	1.00	2.32	1.68	0.00	6.58	0.00	2.69	0.00
time (sec)	N/A	0.282	0.806	0.058	0.000	0.368	0.000	0.339	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	60	44	56	0	344	88	105	47
N.S.	1	1.09	0.80	1.02	0.00	6.25	1.60	1.91	0.85
time (sec)	N/A	0.264	0.052	0.038	0.000	0.309	4.816	0.305	14.409

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	84	109	75	517	0	863	0	295	1451
N.S.	1	1.30	0.89	6.15	0.00	10.27	0.00	3.51	17.27
time (sec)	N/A	0.314	0.065	1.522	0.000	0.369	0.000	0.354	0.479

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	B	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	92	101	674	631	0	393	0	359	0
N.S.	1	1.10	7.33	6.86	0.00	4.27	0.00	3.90	0.00
time (sec)	N/A	0.336	6.947	2.931	0.000	0.354	0.000	0.328	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	96	69	88	0	698	0	219	88
N.S.	1	1.17	0.84	1.07	0.00	8.51	0.00	2.67	1.07
time (sec)	N/A	0.315	0.110	0.062	0.000	0.309	0.000	0.294	16.211

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	B	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	94	105	197	161	0	720	0	281	0
N.S.	1	1.12	2.10	1.71	0.00	7.66	0.00	2.99	0.00
time (sec)	N/A	0.311	6.340	0.060	0.000	0.386	0.000	0.340	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	93	47	75	0	627	110	215	82
N.S.	1	1.19	0.60	0.96	0.00	8.04	1.41	2.76	1.05
time (sec)	N/A	0.281	0.056	0.046	0.000	0.337	7.393	0.320	17.273

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	118	159	78	0	0	1531	0	483	2817
N.S.	1	1.35	0.66	0.00	0.00	12.97	0.00	4.09	23.87
time (sec)	N/A	0.372	0.074	0.000	0.000	0.488	0.000	0.383	13.682

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	B	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	141	161	1450	934	0	647	0	537	0
N.S.	1	1.14	10.28	6.62	0.00	4.59	0.00	3.81	0.00
time (sec)	N/A	0.426	8.162	1.563	0.000	0.343	0.000	0.355	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	43	57	31	33	24	34	34	37
N.S.	1	1.16	1.54	0.84	0.89	0.65	0.92	0.92	1.00
time (sec)	N/A	0.310	0.047	0.135	0.294	0.289	0.124	0.264	13.795

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	86	86	139	0	1063	0	204	0
N.S.	1	0.96	0.96	1.54	0.00	11.81	0.00	2.27	0.00
time (sec)	N/A	0.323	0.187	0.242	0.000	0.451	0.000	0.303	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	130	167	245	0	1486	0	445	0
N.S.	1	1.03	1.33	1.94	0.00	11.79	0.00	3.53	0.00
time (sec)	N/A	0.398	4.810	0.071	0.000	0.483	0.000	0.539	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	65	0	264	0	58	0
N.S.	1	1.00	1.00	1.59	0.00	6.44	0.00	1.41	0.00
time (sec)	N/A	0.257	0.022	0.202	0.000	0.385	0.000	0.320	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	73	73	248	0	670	0	111	0
N.S.	1	0.99	0.99	3.35	0.00	9.05	0.00	1.50	0.00
time (sec)	N/A	0.294	0.332	1.025	0.000	0.451	0.000	0.290	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	130	114	586	0	1365	0	276	0
N.S.	1	1.11	0.97	5.01	0.00	11.67	0.00	2.36	0.00
time (sec)	N/A	0.360	0.831	0.135	0.000	0.517	0.000	0.313	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [61] had the largest ratio of [.866666999999999965]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	18	17	0.92	25	0.680
2	A	1	1	1.00	12	0.083
3	A	5	4	1.09	14	0.286
4	A	5	4	1.01	14	0.286
5	A	6	5	1.00	14	0.357
6	A	7	6	1.21	14	0.429
7	A	8	7	1.23	14	0.500
8	A	7	6	1.09	10	0.600
9	A	6	5	1.00	10	0.500
10	A	6	5	1.17	10	0.500
11	A	8	7	1.11	12	0.583
12	A	7	6	1.14	12	0.500
13	A	6	5	1.14	12	0.417
14	A	11	10	1.29	17	0.588
15	A	10	9	0.71	17	0.529
16	A	9	8	1.00	15	0.533
17	A	12	11	1.25	15	0.733
18	A	9	8	0.79	17	0.471
19	A	10	9	1.09	17	0.529
20	A	9	8	1.10	15	0.533
21	A	9	8	1.08	15	0.533
22	A	9	8	1.04	17	0.471

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	8	7	1.00	12	0.583
24	A	8	7	1.00	17	0.412
25	A	9	8	1.04	17	0.471
26	A	11	10	1.08	17	0.588
27	A	11	10	1.07	17	0.588
28	A	10	9	1.12	15	0.600
29	A	10	9	1.07	15	0.600
30	A	10	9	1.00	17	0.529
31	A	10	9	1.00	16	0.562
32	A	9	8	0.98	16	0.500
33	A	8	7	0.97	16	0.438
34	A	5	4	1.00	16	0.250
35	A	6	5	0.99	16	0.312
36	A	8	7	1.07	16	0.438
37	A	9	8	1.13	16	0.500
38	A	8	7	1.06	12	0.583
39	A	7	6	1.00	12	0.500
40	A	5	4	1.00	12	0.333
41	A	9	8	1.05	10	0.800
42	A	8	7	1.00	10	0.700
43	A	5	4	1.00	10	0.400
44	A	9	8	1.08	17	0.471
45	A	8	7	1.00	17	0.412
46	A	8	7	1.00	15	0.467
47	A	9	8	1.08	15	0.533
48	A	8	7	1.00	17	0.412
49	A	9	8	1.08	17	0.471
50	A	6	5	1.00	17	0.294
51	A	9	8	1.09	15	0.533
52	A	10	9	1.30	15	0.600
53	A	8	7	1.10	17	0.412
54	A	10	9	1.17	17	0.529
55	A	8	7	1.12	17	0.412
56	A	10	9	1.19	15	0.600

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
57	A	12	11	1.35	15	0.733
58	A	9	8	1.14	17	0.471
59	A	5	4	1.16	8	0.500
60	A	12	11	0.96	15	0.733
61	A	14	13	1.03	15	0.867
62	A	8	7	1.00	15	0.467
63	A	11	10	0.99	15	0.667
64	A	12	11	1.11	15	0.733

CHAPTER 3

Listing of integrals

3.1 $\int \frac{A+C\cot^2(c+dx)}{\sqrt{b\tan(c+dx)}}\,dx$

3.1.1 Optimal result

Integrand size = 25, antiderivative size = 233

$$\begin{aligned}\int \frac{A+C\cot^2(c+dx)}{\sqrt{b\tan(c+dx)}}\,dx &= -\frac{(A-C)\arctan\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}\sqrt{b}d} \\ &\quad+\frac{(A-C)\arctan\left(1+\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}\sqrt{b}d} \\ &\quad-\frac{(A-C)\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}\sqrt{b}d} \\ &\quad+\frac{(A-C)\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)+\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}\sqrt{b}d} \\ &\quad-\frac{2bC}{3d(b\tan(c+dx))^{3/2}}\end{aligned}$$

output

```
-1/2*(A-C)*arctan(1-2^(1/2)*(b*tan(d*x+c))^(1/2)/b^(1/2))/d*2^(1/2)/b^(1/2
)+1/2*(A-C)*arctan(1+2^(1/2)*(b*tan(d*x+c))^(1/2)/b^(1/2))/d*2^(1/2)/b^(1/
2)-1/4*(A-C)*ln(b^(1/2)-2^(1/2)*(b*tan(d*x+c))^(1/2)+b^(1/2)*tan(d*x+c))/d
*2^(1/2)/b^(1/2)+1/4*(A-C)*ln(b^(1/2)+2^(1/2)*(b*tan(d*x+c))^(1/2)+b^(1/2)
*tan(d*x+c))/d*2^(1/2)/b^(1/2)-2/3*b*C/d/(b*tan(d*x+c))^(3/2)
```

3.1.2 Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.64

$$\int \frac{A+C\cot^2(c+dx)}{\sqrt{b\tan(c+dx)}}\,dx$$
$$=\frac{-8C\cot(c+dx)-3\sqrt{2}(A-C)\left(2\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)-2\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)+\cdots\right.}{12d\sqrt{b\tan\cdots}}$$

input
```
Integrate[(A + C*Cot[c + d*x]^2)/Sqrt[b*Tan[c + d*x]],x]
```

output
```
(-8*C*Cot[c + d*x] - 3*Sqrt[2]*(A - C)*(2*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c +
d*x]]] - 2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]] + Log[1 - Sqrt[2]*Sqrt[T
an[c + d*x]] + Tan[c + d*x]] - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c
+ d*x]])*Sqrt[Tan[c + d*x]])/(12*d*Sqrt[b*Tan[c + d*x]])
```

3.1.3 Rubi [A] (warning: unable to verify)

Time = 0.66 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.92, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.680$, Rules used = {3042, 4156, 3042, 4112, 27, 2030, 3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+C\cot^2(c+dx)}{\sqrt{b\tan(c+dx)}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+C\tan\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{-b\cot\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 4156}$$

$$b^2\int \frac{A\tan^2(c+dx)+C}{(b\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$b^2\int \frac{A\tan(c+dx)^2+C}{(b\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{4112}$$

$$b^2\left(\frac{\int \frac{b(A-C)\tan(c+dx)}{(b\tan(c+dx))^{3/2}}dx}{b^2}-\frac{2C}{3bd(b\tan(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{27}$$

$$b^2\left(\frac{(A-C)\int \frac{\tan(c+dx)}{(b\tan(c+dx))^{3/2}}dx}{b}-\frac{2C}{3bd(b\tan(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{2030}$$

$$b^2\left(\frac{(A-C)\int \frac{1}{\sqrt{b\tan(c+dx)}}dx}{b^2}-\frac{2C}{3bd(b\tan(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(\frac{(A-C)\int \frac{1}{\sqrt{b\tan(c+dx)}}dx}{b^2}-\frac{2C}{3bd(b\tan(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3957}$$

$$b^2\left(\frac{(A-C)\int \frac{1}{\sqrt{b\tan(c+dx)}(\tan^2(c+dx)b^2+b^2)}d(b\tan(c+dx))}{bd}-\frac{2C}{3bd(b\tan(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{266}$$

$$b^2\left(\frac{2(A-C)\int \frac{1}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}}{bd}-\frac{2C}{3bd(b\tan(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{755}$$

$$b^2\left(\frac{2(A-C)\left(\frac{\int \frac{b-b^2\tan^2(c+dx)}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}}{2b}+\frac{\int \frac{b^2\tan^2(c+dx)+b}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}}{2b}\right)}{bd}-\frac{2C}{3bd(b\tan(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{1476}$$

3.1. $\int \frac{A+C\cot^2(c+dx)}{\sqrt{b\tan(c+dx)}}\,dx$

$$b^2\left(\frac{2(A-C)\left(\frac{\frac{1}{2}\int\frac{1}{b^2\tan^2(c+dx)-\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}+\frac{1}{2}\int\frac{1}{b^2\tan^2(c+dx)+\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2b}+\frac{\int\frac{b-b^2\tan^2}{b^4\tan^4(c+}\cdots\right.}{bd}\cdots\right.$$

$$\Big\downarrow \; 1082$$

$$b^2\left(\frac{2(A-C)\left(\frac{\frac{\int\frac{1}{-b^2\tan^2(c+dx)-1}d\left(1-\sqrt{2}\sqrt{b}\tan(c+dx)\right)}{\sqrt{2}\sqrt{b}}-\frac{\int\frac{1}{-b^2\tan^2(c+dx)-1}d\left(\sqrt{2}\sqrt{b}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{b}}}{2b}+\frac{\int\frac{b-b^2\tan^2(c+dx)}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}}{2b}\right)}{bd}\cdots\right.$$

$$\Big\downarrow \; 217$$

$$b^2\left(\frac{2(A-C)\left(\frac{\int\frac{b-b^2\tan^2(c+dx)}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}}{2b}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{b}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{b}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{b}\tan(c+dx)\right)}{\sqrt{2}\sqrt{b}}}{2b}\right)}{bd}-\frac{2C}{3bd(b\tan(c+dx))^3}\cdots\right.$$

$$\Big\downarrow \; 1479$$

$$b^2\left(\frac{2(A-C)\left(\frac{-\frac{\int-\frac{\sqrt{2}\sqrt{b}-2\sqrt{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{b}+\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{b^2\tan^2(c+dx)+\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}}{2b}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{b}\tan\cdots\right.}{\sqrt{2}\sqrt{\cdots}}\cdots\right.}{bd}\cdots\right.$$

$$\Big\downarrow \; 25$$

$$b^2\left(\frac{2(A-C)\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt{b}-2\sqrt{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{b}+\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{b^2\tan^2(c+dx)+\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}}{2b}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{b}\tan(c+\cdots\right.}{\sqrt{2}\sqrt{b}}\cdots\right.}{bd}\cdots\right.$$

$\downarrow$ 27

$$b^2\left(\frac{2(A-C)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{b}-2\sqrt{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}+\frac{\int \frac{\sqrt{b}+\sqrt{2}\sqrt{b\tan(c+dx)}}{b^2\tan^2(c+dx)+\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{b}}}{2b}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{b}\tan(c+\cdots\right.}{\sqrt{2}\sqrt{b}}\cdots}{\cdots}\right.}{bd}\cdots\right.$$

$\downarrow$ 1103

$$b^2\left(\frac{2(A-C)\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{b}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{b}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{b}\tan(c+dx)\right)}{\sqrt{2}\sqrt{b}}}{2b}+\frac{\frac{\log\left(\sqrt{2}b^{3/2}\tan(c+dx)+b^2\tan^2(c+dx)+b\right)}{2\sqrt{2}\sqrt{b}}-\frac{\log\left(-\sqrt{2}b^{3/2}\tan(c+dx)+b^2\cdots\right.}{2\sqrt{2}\sqrt{b}}}{2b}\cdots\right.}{bd}\cdots\right.$$

input
```
Int[(A + C*Cot[c + d*x]^2)/Sqrt[b*Tan[c + d*x]],x]
```

output
```
b^2*((2*(A - C)*((-(ArcTan[1 - Sqrt[2]*Sqrt[b]*Tan[c + d*x]]/(Sqrt[2]*Sqrt
[b])) + ArcTan[1 + Sqrt[2]*Sqrt[b]*Tan[c + d*x]]/(Sqrt[2]*Sqrt[b]))/(2*b)
+ (-1/2*Log[b - Sqrt[2]*b^(3/2)*Tan[c + d*x] + b^2*Tan[c + d*x]^2]/(Sqrt[2
]*Sqrt[b]) + Log[b + Sqrt[2]*b^(3/2)*Tan[c + d*x] + b^2*Tan[c + d*x]^2]/(2
*Sqrt[2]*Sqrt[b]))/(2*b)))/(b*d) - (2*C)/(3*b*d*(b*Tan[c + d*x])^(3/2)))
```

3.1.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.1. $\int \frac{A+C\cot^2(c+dx)}{\sqrt{b\tan(c+dx)}}\,dx$

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4112
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 + a^2*C)*((a + b*Tan[e + f*x])^(m
 + 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[
e + f*x])^(m + 1)*Simp[a*(A - C) - (A*b - b*C)*Tan[e + f*x], x], x], x] /;
FreeQ[{a, b, e, f, A, C}, x] && NeQ[A*b^2 + a^2*C, 0] && LtQ[m, -1] && NeQ[
a^2 + b^2, 0]
```

rule 4156
```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Cot[e + f*x])^(m - n*p
)*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

3.1.4 Maple [A] (verified)

Time = 2.08 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.69

method	result
derivativedivides	$\frac{2b\left(-\frac{C}{3(b\tan(dx+c))^{\frac{3}{2}}}+\frac{(A-C)\left(b^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{b\tan(dx+c)+\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^2}}{b\tan(dx+c)-\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{b\tan(dx+c)}}{\left(b^2\right)^{\frac{1}{4}}}+1\right)-2\cdots}{8b^2}\right.}{d}$
default	$\frac{2b\left(-\frac{C}{3(b\tan(dx+c))^{\frac{3}{2}}}+\frac{(A-C)\left(b^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{b\tan(dx+c)+\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^2}}{b\tan(dx+c)-\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{b\tan(dx+c)}}{\left(b^2\right)^{\frac{1}{4}}}+1\right)-2\cdots}{8b^2}\right.}{d}$
parts	$\frac{A\left(b^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{b\tan(dx+c)+\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^2}}{b\tan(dx+c)-\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{b\tan(dx+c)}}{\left(b^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{b\tan(dx\cdots}}{\left(b^2\right)^{\frac{1}{4}}}\right.\right.}{4db}$

input
```
int((A+C*cot(d*x+c)^2)/(b*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*b*(-1/3*C/(b*tan(d*x+c))^(3/2)+1/8*(A-C)*(b^2)^(1/4)/b^2*2^(1/2)*(ln((
b*tan(d*x+c)+(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2))/(b*tan(
d*x+c)-(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2)))+2*arctan(2^(
1/2)/(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(b^2)^(1/4)*(b*
tan(d*x+c))^(1/2)+1)))
```

3.1.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 465, normalized size of antiderivative = 2.00

$$\int \frac{A + C\cot^2(c+dx)}{\sqrt{b\tan(c+dx)}}\,dx =$$

$$-\frac{3\,bd\left(-\frac{A^4-4\,A^3C+6\,A^2C^2-4\,AC^3+C^4}{b^2d^4}\right)^{\frac{1}{4}}\log\left(bd\left(-\frac{A^4-4\,A^3C+6\,A^2C^2-4\,AC^3+C^4}{b^2d^4}\right)^{\frac{1}{4}}-\sqrt{b\tan\left(dx+c\right)}(A-C)\right)}{}$$

input
```
integrate((A+C*cot(d*x+c)^2)/(b*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-1/6*(3*b*d*(-(A^4 - 4*A^3*C + 6*A^2*C^2 - 4*A*C^3 + C^4)/(b^2*d^4))^(1/4)
*log(b*d*(-(A^4 - 4*A^3*C + 6*A^2*C^2 - 4*A*C^3 + C^4)/(b^2*d^4))^(1/4) -
sqrt(b*tan(d*x + c))*(A - C))*tan(d*x + c)^2 + 3*I*b*d*(-(A^4 - 4*A^3*C +
6*A^2*C^2 - 4*A*C^3 + C^4)/(b^2*d^4))^(1/4)*log(I*b*d*(-(A^4 - 4*A^3*C + 6
*A^2*C^2 - 4*A*C^3 + C^4)/(b^2*d^4))^(1/4) - sqrt(b*tan(d*x + c))*(A - C))
*tan(d*x + c)^2 - 3*I*b*d*(-(A^4 - 4*A^3*C + 6*A^2*C^2 - 4*A*C^3 + C^4)/(b
^2*d^4))^(1/4)*log(-I*b*d*(-(A^4 - 4*A^3*C + 6*A^2*C^2 - 4*A*C^3 + C^4)/(b
^2*d^4))^(1/4) - sqrt(b*tan(d*x + c))*(A - C))*tan(d*x + c)^2 - 3*b*d*(-(A
^4 - 4*A^3*C + 6*A^2*C^2 - 4*A*C^3 + C^4)/(b^2*d^4))^(1/4)*log(-b*d*(-(A^4
 - 4*A^3*C + 6*A^2*C^2 - 4*A*C^3 + C^4)/(b^2*d^4))^(1/4) - sqrt(b*tan(d*x
+ c))*(A - C))*tan(d*x + c)^2 + 4*sqrt(b*tan(d*x + c))*C)/(b*d*tan(d*x + c
)^2)
```

3.1.6 Sympy [F]

$$\int \frac{A + C\cot^2(c + dx)}{\sqrt{b\tan(c + dx)}}\,dx = \int \frac{A + C\cot^2(c + dx)}{\sqrt{b\tan(c + dx)}}\,dx$$

input
```
integrate((A+C*cot(d*x+c)**2)/(b*tan(d*x+c))**(1/2),x)
```

output
```
Integral((A + C*cot(c + d*x)**2)/sqrt(b*tan(c + d*x)), x)
```

3.1.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.77

$$\int \frac{A + C\cot^2(c + dx)}{\sqrt{b\tan(c + dx)}}\,dx = \frac{3\left(2\sqrt{2}\sqrt{b}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{b}+2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{b}}\right) + 2\sqrt{2}\sqrt{b}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{b}-2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{b}}\right) + \sqrt{2}\sqrt{b}\log\left(b\right.\right.}{}$$

input
```
integrate((A+C*cot(d*x+c)^2)/(b*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/12*(3*(2*sqrt(2)*sqrt(b)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(b) + 2*sqrt(b*
tan(d*x + c)))/sqrt(b)) + 2*sqrt(2)*sqrt(b)*arctan(-1/2*sqrt(2)*(sqrt(2)*s
qrt(b) - 2*sqrt(b*tan(d*x + c)))/sqrt(b)) + sqrt(2)*sqrt(b)*log(b*tan(d*x
+ c) + sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(b) + b) - sqrt(2)*sqrt(b)*log(b*t
an(d*x + c) - sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(b) + b))*(A - C) - 8*C*b^2
/(b*tan(d*x + c))^(3/2))/(b*d)
```

3.1.8 Giac [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.06

$$\int \frac{A+C\cot^2(c+dx)}{\sqrt{b\tan(c+dx)}}\,dx$$
$$= \frac{\sqrt{2}\left(A\sqrt{|b|}-C\sqrt{|b|}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}+2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{|b|}}\right)}{2\,bd}$$
$$+ \frac{\sqrt{2}\left(A\sqrt{|b|}-C\sqrt{|b|}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}-2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{|b|}}\right)}{2\,bd}$$
$$+ \frac{\sqrt{2}\left(A\sqrt{|b|}-C\sqrt{|b|}\right)\log\left(b\tan\left(dx+c\right)+\sqrt{2}\sqrt{b\tan\left(dx+c\right)}\sqrt{|b|}+|b|\right)}{4\,bd}$$
$$- \frac{\sqrt{2}\left(A\sqrt{|b|}-C\sqrt{|b|}\right)\log\left(b\tan\left(dx+c\right)-\sqrt{2}\sqrt{b\tan\left(dx+c\right)}\sqrt{|b|}+|b|\right)}{4\,bd}$$
$$- \frac{2\,C}{3\sqrt{b\tan\left(dx+c\right)}d\tan\left(dx+c\right)}$$

input
```
integrate((A+C*cot(d*x+c)^2)/(b*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*(A*sqrt(abs(b)) - C*sqrt(abs(b)))*arctan(1/2*sqrt(2)*(sqrt(2)*
sqrt(abs(b)) + 2*sqrt(b*tan(d*x + c)))/sqrt(abs(b)))/(b*d) + 1/2*sqrt(2)*(
A*sqrt(abs(b)) - C*sqrt(abs(b)))*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b))
 - 2*sqrt(b*tan(d*x + c)))/sqrt(abs(b)))/(b*d) + 1/4*sqrt(2)*(A*sqrt(abs(b
)) - C*sqrt(abs(b)))*log(b*tan(d*x + c) + sqrt(2)*sqrt(b*tan(d*x + c))*sqr
t(abs(b)) + abs(b))/(b*d) - 1/4*sqrt(2)*(A*sqrt(abs(b)) - C*sqrt(abs(b)))*
log(b*tan(d*x + c) - sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(abs(b)) + abs(b))/(
b*d) - 2/3*C/(sqrt(b*tan(d*x + c))*d*tan(d*x + c))
```

3.1.9 Mupad [B] (verification not implemented)

Time = 13.65 (sec) , antiderivative size = 828, normalized size of antiderivative = 3.55

$$\int \frac{A+C\cot^2(c+dx)}{\sqrt{b\tan(c+dx)}}\,dx = -\frac{2\,C\,b}{3\,d\,(b\tan\left(c+d\,x\right))^{3/2}}$$

$$+\frac{(-1)^{1/4}\,\mathrm{atan}\left(\frac{\frac{(-1)^{1/4}\,(A-C)\left(\sqrt{b\tan(c+d\,x)}\left(16\,A^2\,b^2\,d^3-32\,A\,C\,b^2\,d^3+16\,C^2\,b^2\,d^3\right)-\frac{(-1)^{1/4}\,(A-C)\left(32\,A\,b^3\,d^4-32\,C\,b^3\,d^4\right)}{2\,\sqrt{b}\,d}\right)1\mathrm{i}}{2\,\sqrt{b}\,d}+\frac{(-1)^{1/4}\,(A \cdots}{\frac{(-1)^{1/4}\,(A-C)\left(\sqrt{b\tan(c+d\,x)}\left(16\,A^2\,b^2\,d^3-32\,A\,C\,b^2\,d^3+16\,C^2\,b^2\,d^3\right)-\frac{(-1)^{1/4}\,(A-C)\left(32\,A\,b^3\,d^4-32\,C\,b^3\,d^4\right)}{2\,\sqrt{b}\,d}\right)}{2\,\sqrt{b}\,d}-\frac{(-1)^{1/4}\,(A \cdots}\right.}{\sqrt{b}\,d}$$

$$+\frac{(-1)^{1/4}\,\mathrm{atan}\left(\frac{\frac{(-1)^{1/4}\,(A-C)\left(\sqrt{b\tan(c+d\,x)}\left(16\,A^2\,b^2\,d^3-32\,A\,C\,b^2\,d^3+16\,C^2\,b^2\,d^3\right)-\frac{(-1)^{1/4}\,(A-C)\left(32\,A\,b^3\,d^4-32\,C\,b^3\,d^4\right)1\mathrm{i}}{2\,\sqrt{b}\,d}\right)}{2\,\sqrt{b}\,d}+\frac{(-1)^{1/4}\,(\cdots}{\frac{(-1)^{1/4}\,(A-C)\left(\sqrt{b\tan(c+d\,x)}\left(16\,A^2\,b^2\,d^3-32\,A\,C\,b^2\,d^3+16\,C^2\,b^2\,d^3\right)-\frac{(-1)^{1/4}\,(A-C)\left(32\,A\,b^3\,d^4-32\,C\,b^3\,d^4\right)1\mathrm{i}}{2\,\sqrt{b}\,d}\right)1\mathrm{i}}{2\,\sqrt{b}\,d}-\frac{(-1)^{1/4}\,(\cdots}\right.}{\sqrt{b}\,d}$$

input
```
int((A + C*cot(c + d*x)^2)/(b*tan(c + d*x))^(1/2),x)
```

output
```
((-1)^(1/4)*atan((((-1)^(1/4)*(A - C)*((b*tan(c + d*x))^(1/2)*(16*A^2*b^2*
d^3 + 16*C^2*b^2*d^3 - 32*A*C*b^2*d^3) - ((-1)^(1/4)*(A - C)*(32*A*b^3*d^4
 - 32*C*b^3*d^4))/(2*b^(1/2)*d))*1i)/(2*b^(1/2)*d) + ((-1)^(1/4)*(A - C)*(
(b*tan(c + d*x))^(1/2)*(16*A^2*b^2*d^3 + 16*C^2*b^2*d^3 - 32*A*C*b^2*d^3)
+ ((-1)^(1/4)*(A - C)*(32*A*b^3*d^4 - 32*C*b^3*d^4))/(2*b^(1/2)*d))*1i)/(2
*b^(1/2)*d))/(((-1)^(1/4)*(A - C)*((b*tan(c + d*x))^(1/2)*(16*A^2*b^2*d^3
+ 16*C^2*b^2*d^3 - 32*A*C*b^2*d^3) - ((-1)^(1/4)*(A - C)*(32*A*b^3*d^4 - 3
2*C*b^3*d^4))/(2*b^(1/2)*d)))/(2*b^(1/2)*d) - ((-1)^(1/4)*(A - C)*((b*tan(
c + d*x))^(1/2)*(16*A^2*b^2*d^3 + 16*C^2*b^2*d^3 - 32*A*C*b^2*d^3) + ((-1)
^(1/4)*(A - C)*(32*A*b^3*d^4 - 32*C*b^3*d^4))/(2*b^(1/2)*d)))/(2*b^(1/2)*d
)))*(A - C)*1i)/(b^(1/2)*d) - (2*C*b)/(3*d*(b*tan(c + d*x))^(3/2)) + ((-1)
^(1/4)*atan((((-1)^(1/4)*(A - C)*((b*tan(c + d*x))^(1/2)*(16*A^2*b^2*d^3 +
 16*C^2*b^2*d^3 - 32*A*C*b^2*d^3) - ((-1)^(1/4)*(A - C)*(32*A*b^3*d^4 - 32
*C*b^3*d^4)*1i)/(2*b^(1/2)*d)))/(2*b^(1/2)*d) + ((-1)^(1/4)*(A - C)*((b*ta
n(c + d*x))^(1/2)*(16*A^2*b^2*d^3 + 16*C^2*b^2*d^3 - 32*A*C*b^2*d^3) + ((-
1)^(1/4)*(A - C)*(32*A*b^3*d^4 - 32*C*b^3*d^4)*1i)/(2*b^(1/2)*d)))/(2*b^(1
/2)*d))/(((-1)^(1/4)*(A - C)*((b*tan(c + d*x))^(1/2)*(16*A^2*b^2*d^3 + 16*
C^2*b^2*d^3 - 32*A*C*b^2*d^3) - ((-1)^(1/4)*(A - C)*(32*A*b^3*d^4 - 32*C*b
^3*d^4)*1i)/(2*b^(1/2)*d))*1i)/(2*b^(1/2)*d) - ((-1)^(1/4)*(A - C)*((b*tan
(c + d*x))^(1/2)*(16*A^2*b^2*d^3 + 16*C^2*b^2*d^3 - 32*A*C*b^2*d^3) + (...
```

3.2 $\int (a+b\cot^2(c+dx))\,dx$

3.2.1 Optimal result

Integrand size = 12, antiderivative size = 20

$$\int (a+b\cot^2(c+dx))\,dx = ax-bx-\frac{b\cot(c+dx)}{d}$$

output
```
a*x-b*x-b*cot(d*x+c)/d
```

3.2.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.70

$$\int (a+b\cot^2(c+dx))\,dx = ax-\frac{b\cot(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan^2(c+dx)\right)}{d}$$

input
```
Integrate[a + b*Cot[c + d*x]^2,x]
```

output
```
a*x - (b*Cot[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[c + d*x]^2])/d
```

3.2.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\cot^2(c+dx)\right)\,dx$$

$$\Big\downarrow \ 2009$$

$$ax-\frac{b\cot(c+dx)}{d}-bx$$

input
```
Int[a + b*Cot[c + d*x]^2,x]
```

output
```
a*x - b*x - (b*Cot[c + d*x])/d
```

3.2.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.2.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.45

method	result	size
risch	$ax-bx-\frac{2ib}{d\left(e^{2i(dx+c)}-1\right)}$	29
norman	$\frac{(a-b)x\tan(dx+c)-\frac{b}{d}}{\tan(dx+c)}$	30
parallelrisch	$\frac{b(-\tan(dx+c)xd-1)}{d\tan(dx+c)}+ax$	30
default	$ax+\frac{b\left(-\cot(dx+c)+\frac{\pi}{2}-\operatorname{arccot}(\cot(dx+c))\right)}{d}$	31
parts	$ax+\frac{b\left(-\cot(dx+c)+\frac{\pi}{2}-\operatorname{arccot}(\cot(dx+c))\right)}{d}$	31
derivativedivides	$\frac{-b\cot(dx+c)+(-a+b)\left(\frac{\pi}{2}-\operatorname{arccot}(\cot(dx+c))\right)}{d}$	34

input
```
int(a+b*cot(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
a*x-b*x-2*I*b/d/(exp(2*I*(d*x+c))-1)
```

3.2.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(20) = 40$.

Time = 0.31 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.40

$$\int \left(a+b\cot^2(c+dx)\right)\,dx = \frac{(a-b)dx\sin\left(2\,dx+2\,c\right)-b\cos\left(2\,dx+2\,c\right)-b}{d\sin\left(2\,dx+2\,c\right)}$$

input
```
integrate(a+b*cot(d*x+c)^2,x, algorithm="fricas")
```

output
```
((a - b)*d*x*sin(2*d*x + 2*c) - b*cos(2*d*x + 2*c) - b)/(d*sin(2*d*x + 2*c
))
```

3.2.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \left(a+b\cot^2(c+dx)\right)\,dx = ax+b\left(\begin{cases} -x-\frac{\cot\left(c+dx\right)}{d} & \text{for}\ d\neq 0\\ x\cot^2\left(c\right) & \text{otherwise}\end{cases}\right)$$

input
```
integrate(a+b*cot(d*x+c)**2,x)
```

output
```
a*x + b*Piecewise((-x - cot(c + d*x)/d, Ne(d, 0)), (x*cot(c)**2, True))
```

3.2.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \left(a+b\cot^2(c+dx)\right)\,dx=ax-\frac{\left(dx+c+\frac{1}{\tan(dx+c)}\right)b}{d}$$

input
```
integrate(a+b*cot(d*x+c)^2,x, algorithm="maxima")
```

output
```
a*x - (d*x + c + 1/tan(d*x + c))*b/d
```

3.2.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.00

$$\int \left(a+b\cot^2(c+dx)\right)\,dx=ax-\frac{\left(2\,dx+2\,c+\frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)b}{2\,d}$$

input
```
integrate(a+b*cot(d*x+c)^2,x, algorithm="giac")
```

output
```
a*x - 1/2*(2*d*x + 2*c + 1/tan(1/2*d*x + 1/2*c) - tan(1/2*d*x + 1/2*c))*b/
d
```

3.2.9 Mupad [B] (verification not implemented)

Time = 13.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \left(a+b\cot^2(c+dx)\right)\,dx=x\,(a-b)-\frac{b\cot(c+d\,x)}{d}$$

input
```
int(a + b*cot(c + d*x)^2,x)
```

output
```
x*(a - b) - (b*cot(c + d*x))/d
```

3.3 $\int (a+b\cot^2(c+dx))^2\,dx$

3.3.1 Optimal result

Integrand size = 14, antiderivative size = 47

$$\int (a+b\cot^2(c+dx))^2\,dx=(a-b)^2x-\frac{(2a-b)b\cot(c+dx)}{d}-\frac{b^2\cot^3(c+dx)}{3d}$$

output
```
(a-b)^2*x-(2*a-b)*b*cot(d*x+c)/d-1/3*b^2*cot(d*x+c)^3/d
```

3.3.2 Mathematica [A] (verified)

Time = 1.26 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.51

$$\int (a+b\cot^2(c+dx))^2\,dx = -\frac{\cot(c+dx)\left(b(6a-3b+b\cot^2(c+dx))+3(a-b)^2\text{arctanh}\left(\sqrt{-\tan^2(c+dx)}\right)\sqrt{-\tan^2(c+dx)}\right)}{3d}$$

input
```
Integrate[(a + b*Cot[c + d*x]^2)^2,x]
```

output
```
-1/3*(Cot[c + d*x]*(b*(6*a - 3*b + b*Cot[c + d*x]^2) + 3*(a - b)^2*ArcTanh
[Sqrt[-Tan[c + d*x]^2]]*Sqrt[-Tan[c + d*x]^2]))/d
```

3.3.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 4144, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\cot^2(c+dx)\right)^2 dx$$

$$\Big\downarrow \ 3042$$

$$\int \left(a+b\tan\left(c+dx+\frac{\pi}{2}\right)^2\right)^2 dx$$

$$\Big\downarrow \ 4144$$

$$-\frac{\int \frac{\left(b\cot^2(c+dx)+a\right)^2}{\cot^2(c+dx)+1}d\cot(c+dx)}{d}$$

$$\Big\downarrow \ 300$$

$$-\frac{\int \left(\frac{(a-b)^2}{\cot^2(c+dx)+1}+b^2\cot^2(c+dx)+(2a-b)b\right)d\cot(c+dx)}{d}$$

$$\Big\downarrow \ 2009$$

$$-\frac{(a-b)^2\arctan(\cot(c+dx))+b(2a-b)\cot(c+dx)+\frac{1}{3}b^2\cot^3(c+dx)}{d}$$

input
```
Int[(a + b*Cot[c + d*x]^2)^2,x]
```

output
```
-(((a - b)^2*ArcTan[Cot[c + d*x]] + (2*a - b)*b*Cot[c + d*x] + (b^2*Cot[c
+ d*x]^3)/3)/d)
```

3.3.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

3.3.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

method	result	size
parallelrisch	$\frac{-b^2\cot(dx+c)^3+(-6ab+3b^2)\cot(dx+c)+3dx(a-b)^2}{3d}$	48
norman	$\frac{(a^2-2ab+b^2)x\tan(dx+c)^3-\frac{b^2}{3d}-\frac{b(2a-b)\tan(dx+c)^2}{d}}{\tan(dx+c)^3}$	61
derivativedivides	$\frac{-\frac{b^2\cot(dx+c)^3}{3}-2\cot(dx+c)ab+\cot(dx+c)b^2+(-a^2+2ab-b^2)\left(\frac{\pi}{2}-\operatorname{arccot}(\cot(dx+c))\right)}{d}$	68
default	$\frac{-\frac{b^2\cot(dx+c)^3}{3}-2\cot(dx+c)ab+\cot(dx+c)b^2+(-a^2+2ab-b^2)\left(\frac{\pi}{2}-\operatorname{arccot}(\cot(dx+c))\right)}{d}$	68
parts	$a^2x+\frac{b^2\left(-\frac{\cot(dx+c)^3}{3}+\cot(dx+c)-\frac{\pi}{2}+\operatorname{arccot}(\cot(dx+c))\right)}{d}+\frac{2ab\left(-\cot(dx+c)+\frac{\pi}{2}-\operatorname{arccot}(\cot(dx+c))\right)}{d}$	69
risch	$a^2x-2abx+b^2x+\frac{4ib\left(-3a\,e^{4i(dx+c)}+3b\,e^{4i(dx+c)}+6\,e^{2i(dx+c)}a-3\,e^{2i(dx+c)}b-3a+2b\right)}{3d\left(e^{2i(dx+c)}-1\right)^3}$	92

input
```
int((a+b*cot(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*(-b^2*cot(d*x+c)^3+(-6*a*b+3*b^2)*cot(d*x+c)+3*d*x*(a-b)^2)/d
```

3.3.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. 2(45) = 90.

Time = 0.27 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.70

$$\int \left(a+b\cot^2(c+dx)\right)^2\,dx$$
$$=\frac{2\,b^2\cos\left(2\,dx+2\,c\right)-2\,\left(3\,ab-2\,b^2\right)\cos\left(2\,dx+2\,c\right)^2+6\,ab-2\,b^2+3\,\left(\left(a^2-2\,ab+b^2\right)dx\cos\left(2\,dx+2\,c\right)\right.}{3\,\left(d\cos\left(2\,dx+2\,c\right)-d\right)\sin\left(2\,dx+2\,c\right)}$$

input
```
integrate((a+b*cot(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/3*(2*b^2*cos(2*d*x + 2*c) - 2*(3*a*b - 2*b^2)*cos(2*d*x + 2*c)^2 + 6*a*b
 - 2*b^2 + 3*((a^2 - 2*a*b + b^2)*d*x*cos(2*d*x + 2*c) - (a^2 - 2*a*b + b^
2)*d*x)*sin(2*d*x + 2*c))/((d*cos(2*d*x + 2*c) - d)*sin(2*d*x + 2*c))
```

3.3.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.45

$$\int \left(a+b\cot^2(c+dx)\right)^2\,dx$$
$$=\begin{cases} a^2x-2abx-\frac{2ab\cot(c+dx)}{d}+b^2x-\frac{b^2\cot^3(c+dx)}{3d}+\frac{b^2\cot(c+dx)}{d} & \text{for } d\neq 0\\ x\left(a+b\cot^2(c)\right)^2 & \text{otherwise}\end{cases}$$

input
```
integrate((a+b*cot(d*x+c)**2)**2,x)
```

output
```
Piecewise((a**2*x - 2*a*b*x - 2*a*b*cot(c + d*x)/d + b**2*x - b**2*cot(c +
 d*x)**3/(3*d) + b**2*cot(c + d*x)/d, Ne(d, 0)), (x*(a + b*cot(c)**2)**2,
True))
```

3.3.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.34

$$\int \left(a+b\cot^2(c+dx)\right)^2\,dx=a^2x-\frac{2\left(dx+c+\frac{1}{\tan(dx+c)}\right)ab}{d}+\frac{\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)b^2}{3\,d}$$

input
```
integrate((a+b*cot(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
a^2*x - 2*(d*x + c + 1/tan(d*x + c))*a*b/d + 1/3*(3*d*x + 3*c + (3*tan(d*x
 + c)^2 - 1)/tan(d*x + c)^3)*b^2/d
```

3.3.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(45) = 90.

Time = 0.32 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.43

$$\int \left(a+b\cot^2(c+dx)\right)^2\,dx$$
$$=\frac{b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+24\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-15\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+24\,(a^2-2\,ab+b^2)(dx+c)-\frac{24\,ab\,t}{}}{24\,d}$$

input
```
integrate((a+b*cot(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/24*(b^2*tan(1/2*d*x + 1/2*c)^3 + 24*a*b*tan(1/2*d*x + 1/2*c) - 15*b^2*ta
n(1/2*d*x + 1/2*c) + 24*(a^2 - 2*a*b + b^2)*(d*x + c) - (24*a*b*tan(1/2*d*
x + 1/2*c)^2 - 15*b^2*tan(1/2*d*x + 1/2*c)^2 + b^2)/tan(1/2*d*x + 1/2*c)^3
)/d
```

3.3.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96

$$\int \left(a + b\cot^2(c + dx)\right)^2 \, dx = x\,(a-b)^2 - \frac{b^2\cot(c+d\,x)^3}{3\,d} - \frac{b\cot(c+d\,x)\,(2\,a-b)}{d}$$

input
```
int((a + b*cot(c + d*x)^2)^2,x)
```

output
```
x*(a - b)^2 - (b^2*cot(c + d*x)^3)/(3*d) - (b*cot(c + d*x)*(2*a - b))/d
```

3.4 $\int (a+b\cot^2(c+dx))^3\,dx$

3.4.1 Optimal result

Integrand size = 14, antiderivative size = 78

$$\int (a+b\cot^2(c+dx))^3\,dx=(a-b)^3x-\frac{b(3a^2-3ab+b^2)\cot(c+dx)}{d}-\frac{(3a-b)b^2\cot^3(c+dx)}{3d}-\frac{b^3\cot^5(c+dx)}{5d}$$

output
```
(a-b)^3*x-b*(3*a^2-3*a*b+b^2)*cot(d*x+c)/d-1/3*(3*a-b)*b^2*cot(d*x+c)^3/d-
1/5*b^3*cot(d*x+c)^5/d
```

3.4.2 Mathematica [A] (verified)

Time = 2.95 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.42

$$\int (a+b\cot^2(c+dx))^3\,dx=-\frac{\cot^5(c+dx)\left(\frac{15(a-b)^3\operatorname{arctanh}\left(\sqrt{-\tan^2(c+dx)}\right)\tan^8(c+dx)}{(-\tan^2(c+dx))^{3/2}}+b(3b^2+5(3a-b)b\tan^2(c+dx)+15(3a^2-3ab$$
$$15d$$

input
```
Integrate[(a + b*Cot[c + d*x]^2)^3,x]
```

output
```
-1/15*(Cot[c + d*x]^5*((15*(a - b)^3*ArcTanh[Sqrt[-Tan[c + d*x]^2]]*Tan[c
+ d*x]^8)/(-Tan[c + d*x]^2)^(3/2) + b*(3*b^2 + 5*(3*a - b)*b*Tan[c + d*x]^
2 + 15*(3*a^2 - 3*a*b + b^2)*Tan[c + d*x]^4)))/d
```

3.4.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4144, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\cot^2(c+dx)\right)^3 \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\tan\left(c+dx+\frac{\pi}{2}\right)^2\right)^3 dx$$

$$\Big\downarrow \text{4144}$$

$$-\frac{\int \frac{\left(b\cot^2(c+dx)+a\right)^3}{\cot^2(c+dx)+1}d\cot(c+dx)}{d}$$

$$\Big\downarrow \text{300}$$

$$-\frac{\int \left(b^3\cot^4(c+dx)+(3a-b)b^2\cot^2(c+dx)+b\left(3a^2-3ba+b^2\right)+\frac{(a-b)^3}{\cot^2(c+dx)+1}\right)d\cot(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b\left(3a^2-3ab+b^2\right)\cot(c+dx)+(a-b)^3\arctan(\cot(c+dx))+\frac{1}{3}b^2(3a-b)\cot^3(c+dx)+\frac{1}{5}b^3\cot^5(c+dx)}{d}$$

input
```
Int[(a + b*Cot[c + d*x]^2)^3,x]
```

output
```
-(((a - b)^3*ArcTan[Cot[c + d*x]] + b*(3*a^2 - 3*a*b + b^2)*Cot[c + d*x] +
 ((3*a - b)*b^2*Cot[c + d*x]^3)/3 + (b^3*Cot[c + d*x]^5)/5)/d)
```

3.4.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

3.4.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.99

method	result
parallelrisch	$\frac{-3b^3\cot(dx+c)^5+5(-3a\,b^2+b^3)\cot(dx+c)^3+15(-3a^2b+3a\,b^2-b^3)\cot(dx+c)+15dx(a-b)^3}{15d}$
norman	$\frac{(a^3-3a^2b+3a\,b^2-b^3)x\tan(dx+c)^5-\frac{b^3}{5d}-\frac{b\left(3a^2-3ab+b^2\right)\tan(dx+c)^4}{d}-\frac{b^2(3a-b)\tan(dx+c)^2}{3d}}{\tan(dx+c)^5}$
derivativedivides	$\frac{-\frac{b^3\cot(dx+c)^5}{5}-a\,b^2\cot(dx+c)^3+\frac{b^3\cot(dx+c)^3}{3}-3a^2b\cot(dx+c)+3\cot(dx+c)a\,b^2-\cot(dx+c)b^3+(-a^3+3a^2b-3a\,b^2+}{d}$
default	$\frac{-\frac{b^3\cot(dx+c)^5}{5}-a\,b^2\cot(dx+c)^3+\frac{b^3\cot(dx+c)^3}{3}-3a^2b\cot(dx+c)+3\cot(dx+c)a\,b^2-\cot(dx+c)b^3+(-a^3+3a^2b-3a\,b^2+}{d}$
parts	$a^3x+\frac{b^3\left(-\frac{\cot(dx+c)^5}{5}+\frac{\cot(dx+c)^3}{3}-\cot(dx+c)+\frac{\pi}{2}-\operatorname{arccot}(\cot(dx+c))\right)}{d}+\frac{3a\,b^2\left(-\frac{\cot(dx+c)^3}{3}+\cot(dx+c)-\frac{\pi}{2}+a\right.}{d}$
risch	$a^3x-3a^2bx+3a\,b^2x-b^3x-\frac{2ib\left(45a^2e^{8i(dx+c)}-90ab\,e^{8i(dx+c)}+45b^2e^{8i(dx+c)}-180a^2e^{6i(dx+c)}+270ab\,e^{6}\right.}{}$

input
```
int((a+b*cot(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/15*(-3*b^3*cot(d*x+c)^5+5*(-3*a*b^2+b^3)*cot(d*x+c)^3+15*(-3*a^2*b+3*a*b
^2-b^3)*cot(d*x+c)+15*d*x*(a-b)^3)/d
```

3.4.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. 2(74) = 148.

Time = 0.26 (sec) , antiderivative size = 253, normalized size of antiderivative = 3.24

$$\int (a + b\cot^2(c + dx))^3 \, dx =$$
$$-\frac{(45\,a^2b - 60\,ab^2 + 23\,b^3)\cos(2\,dx + 2\,c)^3 + 45\,a^2b - 30\,ab^2 + 13\,b^3 - (45\,a^2b - 30\,ab^2 + b^3)\cos(2\,dx +}{}$$

input
```
integrate((a+b*cot(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
-1/15*((45*a^2*b - 60*a*b^2 + 23*b^3)*cos(2*d*x + 2*c)^3 + 45*a^2*b - 30*a
*b^2 + 13*b^3 - (45*a^2*b - 30*a*b^2 + b^3)*cos(2*d*x + 2*c)^2 - (45*a^2*b
 - 60*a*b^2 + 11*b^3)*cos(2*d*x + 2*c) - 15*((a^3 - 3*a^2*b + 3*a*b^2 - b^
3)*d*x*cos(2*d*x + 2*c)^2 - 2*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*x*cos(2*d*
x + 2*c) + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*x)*sin(2*d*x + 2*c))/((d*cos(
2*d*x + 2*c)^2 - 2*d*cos(2*d*x + 2*c) + d)*sin(2*d*x + 2*c))
```

3.4.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.62

$$\int (a + b\cot^2(c + dx))^3 \, dx$$
$$= \begin{cases} a^3x - 3a^2bx - \frac{3a^2b\cot(c+dx)}{d} + 3ab^2x - \frac{ab^2\cot^3(c+dx)}{d} + \frac{3ab^2\cot(c+dx)}{d} - b^3x - \frac{b^3\cot^5(c+dx)}{5d} + \frac{b^3\cot^3(c+dx)}{3d} - b \\ x(a + b\cot^2(c))^3 \end{cases}$$

input
```
integrate((a+b*cot(d*x+c)**2)**3,x)
```

output
```
Piecewise((a**3*x - 3*a**2*b*x - 3*a**2*b*cot(c + d*x)/d + 3*a*b**2*x - a*
b**2*cot(c + d*x)**3/d + 3*a*b**2*cot(c + d*x)/d - b**3*x - b**3*cot(c + d
*x)**5/(5*d) + b**3*cot(c + d*x)**3/(3*d) - b**3*cot(c + d*x)/d, Ne(d, 0))
, (x*(a + b*cot(c)**2)**3, True))
```

3.4.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.44

$$\int \left(a + b\cot^2(c + dx)\right)^3 \, dx = a^3x - \frac{3\left(dx + c + \frac{1}{\tan(dx+c)}\right)a^2b}{d} + \frac{\left(3\,dx + 3\,c + \frac{3\,\tan(dx+c)^2 - 1}{\tan(dx+c)^3}\right)ab^2}{d} - \frac{\left(15\,dx + 15\,c + \frac{15\,\tan(dx+c)^4 - 5\,\tan(dx+c)^2 + 3}{\tan(dx+c)^5}\right)b^3}{15\,d}$$

input
```
integrate((a+b*cot(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
a^3*x - 3*(d*x + c + 1/tan(d*x + c))*a^2*b/d + (3*d*x + 3*c + (3*tan(d*x +
 c)^2 - 1)/tan(d*x + c)^3)*a*b^2/d - 1/15*(15*d*x + 15*c + (15*tan(d*x + c
)^4 - 5*tan(d*x + c)^2 + 3)/tan(d*x + c)^5)*b^3/d
```

3.4.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(74) = 148$.

Time = 0.38 (sec) , antiderivative size = 229, normalized size of antiderivative = 2.94

$$\int \left(a + b\cot^2(c + dx)\right)^3 \, dx$$
$$= \frac{3\,b^3\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^5 + 60\,ab^2\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^3 - 35\,b^3\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^3 + 720\,a^2b\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 900}{}$$

input
```
integrate((a+b*cot(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/480*(3*b^3*tan(1/2*d*x + 1/2*c)^5 + 60*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 35
*b^3*tan(1/2*d*x + 1/2*c)^3 + 720*a^2*b*tan(1/2*d*x + 1/2*c) - 900*a*b^2*t
an(1/2*d*x + 1/2*c) + 330*b^3*tan(1/2*d*x + 1/2*c) + 480*(a^3 - 3*a^2*b +
3*a*b^2 - b^3)*(d*x + c) - (720*a^2*b*tan(1/2*d*x + 1/2*c)^4 - 900*a*b^2*t
an(1/2*d*x + 1/2*c)^4 + 330*b^3*tan(1/2*d*x + 1/2*c)^4 + 60*a*b^2*tan(1/2*
d*x + 1/2*c)^2 - 35*b^3*tan(1/2*d*x + 1/2*c)^2 + 3*b^3)/tan(1/2*d*x + 1/2*
c)^5)/d
```

3.4.9 Mupad [B] (verification not implemented)

Time = 12.92 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.97

$$\int (a+b\cot^2(c+dx))^3\,dx = x\,(a-b)^3 - \frac{b^3\cot(c+d\,x)^5}{5\,d} - \frac{\cot(c+d\,x)^3\,(3\,a\,b^2-b^3)}{3\,d} - \frac{b\cot(c+d\,x)\,(3\,a^2-3\,a\,b+b^2)}{d}$$

input
```
int((a + b*cot(c + d*x)^2)^3,x)
```

output
```
x*(a - b)^3 - (b^3*cot(c + d*x)^5)/(5*d) - (cot(c + d*x)^3*(3*a*b^2 - b^3)
)/(3*d) - (b*cot(c + d*x)*(3*a^2 - 3*a*b + b^2))/d
```

3.5 $\int \frac{1}{a+b\cot^2(c+dx)}\,dx$

3.5.1 Optimal result

Integrand size = 14, antiderivative size = 49

$$\int \frac{1}{a+b\cot^2(c+dx)}\,dx=\frac{x}{a-b}+\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\cot(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)d}$$

output
```
x/(a-b)+arctan(cot(d*x+c)*b^(1/2)/a^(1/2))*b^(1/2)/(a-b)/d/a^(1/2)
```

3.5.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+b\cot^2(c+dx)}\,dx=\frac{\arctan(\tan(c+dx))-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{b}}\right)}{\sqrt{a}}}{ad-bd}$$

input
```
Integrate[(a + b*Cot[c + d*x]^2)^(-1),x]
```

output
```
(ArcTan[Tan[c + d*x]] - (Sqrt[b]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[b]])/S
qrt[a])/(a*d - b*d)
```

3.5.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 4143, 3042, 4158, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\cot^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a+b\tan\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{4143}$$

$$\frac{x}{a-b}-\frac{b\int \frac{\csc^2(c+dx)}{b\cot^2(c+dx)+a}dx}{a-b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x}{a-b}-\frac{b\int \frac{\sec\left(c+dx+\frac{\pi}{2}\right)^2}{b\tan\left(c+dx+\frac{\pi}{2}\right)^2+a}dx}{a-b}$$

$$\Big\downarrow \text{4158}$$

$$\frac{b\int \frac{1}{b\cot^2(c+dx)+a}d\cot(c+dx)}{d(a-b)}+\frac{x}{a-b}$$

$$\Big\downarrow \text{218}$$

$$\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\cot(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}d(a-b)}+\frac{x}{a-b}$$

input
```
Int[(a + b*Cot[c + d*x]^2)^(-1),x]
```

output
```
x/(a - b) + (Sqrt[b]*ArcTan[(Sqrt[b]*Cot[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a -
 b)*d)
```

3.5.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4143
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> Simp[x/(a -
 b), x] - Simp[b/(a - b)   Int[Sec[e + f*x]^2/(a + b*Tan[e + f*x]^2), x], x
] /; FreeQ[{a, b, e, f}, x] && NeQ[a, b]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

3.5.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.14

method	result	size
derivativedivides	$\frac{-\frac{\frac{\pi}{2}-\operatorname{arccot}(\cot(dx+c))}{a-b}+\frac{b\arctan\left(\frac{b\cot(dx+c)}{\sqrt{ab}}\right)}{(a-b)\sqrt{ab}}}{d}$	56
default	$\frac{-\frac{\frac{\pi}{2}-\operatorname{arccot}(\cot(dx+c))}{a-b}+\frac{b\arctan\left(\frac{b\cot(dx+c)}{\sqrt{ab}}\right)}{(a-b)\sqrt{ab}}}{d}$	56
risch	$\frac{x}{a-b}+\frac{\sqrt{-ab}\,\ln\left(e^{2i(dx+c)}-\frac{2i\sqrt{-ab}+a+b}{a-b}\right)}{2a(a-b)d}-\frac{\sqrt{-ab}\,\ln\left(e^{2i(dx+c)}+\frac{2i\sqrt{-ab}-a-b}{a-b}\right)}{2a(a-b)d}$	120

input
```
int(1/(a+b*cot(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/(a-b)*(1/2*Pi-arccot(cot(d*x+c)))+b/(a-b)/(a*b)^(1/2)*arctan(b*cot
(d*x+c)/(a*b)^(1/2)))
```

3.5.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 252, normalized size of antiderivative = 5.14

$$\int \frac{1}{a+b\cot^2(c+dx)}\,dx$$
$$= \left[\frac{4\,dx-\sqrt{-\frac{b}{a}}\log\left(\frac{(a^2+6\,ab+b^2)\cos(2\,dx+2\,c)^2+4\,(a^2-ab-(a^2+ab)\cos(2\,dx+2\,c))\sqrt{-\frac{b}{a}}\sin(2\,dx+2\,c)+a^2-6\,ab+b^2-2\,(a^2-b^2)\cos(2}{(a^2-2\,ab+b^2)\cos(2\,dx+2\,c)^2+a^2+2\,ab+b^2-2\,(a^2-b^2)\cos(2\,dx+2\,c)}\right.}{4\,(a-b)d}\right.$$

input
```
integrate(1/(a+b*cot(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/4*(4*d*x - sqrt(-b/a)*log(((a^2 + 6*a*b + b^2)*cos(2*d*x + 2*c)^2 + 4*(
a^2 - a*b - (a^2 + a*b)*cos(2*d*x + 2*c))*sqrt(-b/a)*sin(2*d*x + 2*c) + a^
2 - 6*a*b + b^2 - 2*(a^2 - b^2)*cos(2*d*x + 2*c))/((a^2 - 2*a*b + b^2)*cos
(2*d*x + 2*c)^2 + a^2 + 2*a*b + b^2 - 2*(a^2 - b^2)*cos(2*d*x + 2*c))))/((
a - b)*d), 1/2*(2*d*x + sqrt(b/a)*arctan(1/2*((a + b)*cos(2*d*x + 2*c) - a
 + b)*sqrt(b/a)/(b*sin(2*d*x + 2*c))))/((a - b)*d)]
```

3.5.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. 2(37) = 74.

Time = 0.74 (sec) , antiderivative size = 238, normalized size of antiderivative = 4.86

$$\int \frac{1}{a+b\cot^2(c+dx)}\,dx$$
$$= \begin{cases} \frac{\tilde{\infty}x}{\cot^2(c)} & \text{for } a=0 \wedge b=0 \wedge d=0 \\ \frac{x}{a} & \text{for } b=0 \\ \frac{-x+\frac{1}{d\cot(c+dx)}}{b} & \text{for } a=0 \\ \frac{dx\cot^2(c+dx)}{2bd\cot^2(c+dx)+2bd}+\frac{dx}{2bd\cot^2(c+dx)+2bd}-\frac{\cot(c+dx)}{2bd\cot^2(c+dx)+2bd} & \text{for } a=b \\ \frac{x}{a+b\cot^2(c)} & \text{for } d=0 \\ \frac{2dx\sqrt{-\frac{a}{b}}}{2ad\sqrt{-\frac{a}{b}}-2bd\sqrt{-\frac{a}{b}}}+\frac{\log\left(-\sqrt{-\frac{a}{b}}+\cot(c+dx)\right)}{2ad\sqrt{-\frac{a}{b}}-2bd\sqrt{-\frac{a}{b}}}-\frac{\log\left(\sqrt{-\frac{a}{b}}+\cot(c+dx)\right)}{2ad\sqrt{-\frac{a}{b}}-2bd\sqrt{-\frac{a}{b}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b*cot(d*x+c)**2),x)
```

output
```
Piecewise((zoo*x/cot(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (x/a, Eq(b, 0
)), ((-x + 1/(d*cot(c + d*x)))/b, Eq(a, 0)), (d*x*cot(c + d*x)**2/(2*b*d*c
ot(c + d*x)**2 + 2*b*d) + d*x/(2*b*d*cot(c + d*x)**2 + 2*b*d) - cot(c + d*
x)/(2*b*d*cot(c + d*x)**2 + 2*b*d), Eq(a, b)), (x/(a + b*cot(c)**2), Eq(d,
 0)), (2*d*x*sqrt(-a/b)/(2*a*d*sqrt(-a/b) - 2*b*d*sqrt(-a/b)) + log(-sqrt(
-a/b) + cot(c + d*x))/(2*a*d*sqrt(-a/b) - 2*b*d*sqrt(-a/b)) - log(sqrt(-a/
b) + cot(c + d*x))/(2*a*d*sqrt(-a/b) - 2*b*d*sqrt(-a/b)), True))
```

3.5.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int \frac{1}{a+b\cot^2(c+dx)}\,dx = -\frac{\frac{b\arctan\left(\frac{a\tan(dx+c)}{\sqrt{ab}}\right)}{\sqrt{ab}(a-b)} - \frac{dx+c}{a-b}}{d}$$

input
```
integrate(1/(a+b*cot(d*x+c)^2),x, algorithm="maxima")
```

output
```
-(b*arctan(a*tan(d*x + c)/sqrt(a*b))/(sqrt(a*b)*(a - b)) - (d*x + c)/(a -
b))/d
```

3.5.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.33

$$\int \frac{1}{a+b\cot^2(c+dx)}\,dx = -\frac{\frac{\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan(dx+c)}{\sqrt{ab}}\right)\right)b}{\sqrt{ab}(a-b)} - \frac{dx+c}{a-b}}{d}$$

input
```
integrate(1/(a+b*cot(d*x+c)^2),x, algorithm="giac")
```

output
```
-((pi*floor((d*x + c)/pi + 1/2)*sgn(a) + arctan(a*tan(d*x + c)/sqrt(a*b)))
*b/(sqrt(a*b)*(a - b)) - (d*x + c)/(a - b))/d
```

3.5.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \frac{1}{a+b\cot^2(c+dx)}\,dx = \frac{x}{a-b} + \frac{b\,\mathrm{atan}\left(\frac{b\cot(c+d\,x)}{\sqrt{a\,b}}\right)}{d\,\sqrt{a\,b}\,(a-b)}$$

input
```
int(1/(a + b*cot(c + d*x)^2),x)
```

output
```
x/(a - b) + (b*atan((b*cot(c + d*x))/(a*b)^(1/2)))/(d*(a*b)^(1/2)*(a - b))
```

3.6 $\int \frac{1}{\left(a+b\cot^2(c+dx)\right)^2}\,dx$

3.6.1 Optimal result

Integrand size = 14, antiderivative size = 97

$$\int \frac{1}{\left(a+b\cot^2(c+dx)\right)^2}\,dx = \frac{x}{(a-b)^2} + \frac{(3a-b)\sqrt{b}\arctan\left(\frac{\sqrt{b}\cot(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^2d} + \frac{b\cot(c+dx)}{2a(a-b)d\left(a+b\cot^2(c+dx)\right)}$$

output
```
x/(a-b)^2+1/2*b*cot(d*x+c)/a/(a-b)/d/(a+b*cot(d*x+c)^2)+1/2*(3*a-b)*arctan
(cot(d*x+c)*b^(1/2)/a^(1/2))*b^(1/2)/a^(3/2)/(a-b)^2/d
```

3.6.2 Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.93

$$\int \frac{1}{\left(a+b\cot^2(c+dx)\right)^2}\,dx = \frac{-2\arctan(\cot(c+dx)) + \frac{(3a-b)\sqrt{b}\arctan\left(\frac{\sqrt{b}\cot(c+dx)}{\sqrt{a}}\right)}{a^{3/2}} + \frac{(a-b)b\cot(c+dx)}{a(a+b\cot^2(c+dx))}}{2(a-b)^2d}$$

input
```
Integrate[(a + b*Cot[c + d*x]^2)^(-2),x]
```

output
```
(-2*ArcTan[Cot[c + d*x]] + ((3*a - b)*Sqrt[b]*ArcTan[(Sqrt[b]*Cot[c + d*x]
)/Sqrt[a]])/a^(3/2) + ((a - b)*b*Cot[c + d*x])/(a*(a + b*Cot[c + d*x]^2)))
/(2*(a - b)^2*d)
```

3.6.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.21, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4144, 316, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\cot^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\left(a+b\tan\left(c+dx+\frac{\pi}{2}\right)^2\right)^2}dx$$

$$\Big\downarrow \; 4144$$

$$-\frac{\int \frac{1}{(\cot^2(c+dx)+1)(b\cot^2(c+dx)+a)^2}d\cot(c+dx)}{d}$$

$$\Big\downarrow \; 316$$

$$-\frac{\frac{\int \frac{-b\cot^2(c+dx)+2a-b}{\left(\cot^2(c+dx)+1\right)\left(b\cot^2(c+dx)+a\right)}d\cot(c+dx)}{2a(a-b)}-\frac{b\cot(c+dx)}{2a(a-b)(a+b\cot^2(c+dx))}}{d}$$

$$\Big\downarrow \; 397$$

$$-\frac{\frac{\frac{2a\int \frac{1}{\cot^2(c+dx)+1}d\cot(c+dx)}{a-b}-\frac{b(3a-b)\int \frac{1}{b\cot^2(c+dx)+a}d\cot(c+dx)}{a-b}}{2a(a-b)}-\frac{b\cot(c+dx)}{2a(a-b)(a+b\cot^2(c+dx))}}{d}$$

$$\Big\downarrow \; 216$$

$$-\frac{\frac{\frac{2a\arctan(\cot(c+dx))}{a-b}-\frac{b(3a-b)\int \frac{1}{b\cot^2(c+dx)+a}d\cot(c+dx)}{a-b}}{2a(a-b)}-\frac{b\cot(c+dx)}{2a(a-b)(a+b\cot^2(c+dx))}}{d}$$

$$\Big\downarrow \; 218$$

$$-\frac{\frac{\frac{2a\arctan(\cot(c+dx))}{a-b}-\frac{\sqrt{b}(3a-b)\arctan\left(\frac{\sqrt{b}\cot(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)}}{2a(a-b)}-\frac{b\cot(c+dx)}{2a(a-b)(a+b\cot^2(c+dx))}}{d}$$

input
```
Int[(a + b*Cot[c + d*x]^2)^(-2),x]
```

output
```
-((((2*a*ArcTan[Cot[c + d*x]])/(a - b) - ((3*a - b)*Sqrt[b]*ArcTan[(Sqrt[b
]*Cot[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)))/(2*a*(a - b)) - (b*Cot[c + d*
x])/(2*a*(a - b)*(a + b*Cot[c + d*x]^2)))/d)
```

3.6.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

3.6.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.02

method	result
derivativedivides	$\frac{-\frac{\frac{\pi}{2}-\operatorname{arccot}(\cot(dx+c))}{(a-b)^2}+\frac{b\left(\frac{(a-b)\cot(dx+c)}{2a\left(a+b\cot(dx+c)^2\right)}+\frac{(3a-b)\arctan\left(\frac{b\cot(dx+c)}{\sqrt{ab}}\right)}{2a\sqrt{ab}}\right)}{(a-b)^2}}{d}$
default	$\frac{-\frac{\frac{\pi}{2}-\operatorname{arccot}(\cot(dx+c))}{(a-b)^2}+\frac{b\left(\frac{(a-b)\cot(dx+c)}{2a\left(a+b\cot(dx+c)^2\right)}+\frac{(3a-b)\arctan\left(\frac{b\cot(dx+c)}{\sqrt{ab}}\right)}{2a\sqrt{ab}}\right)}{(a-b)^2}}{d}$
risch	$\frac{x}{a^2-2ab+b^2}-\frac{ib\left(e^{2i(dx+c)}a+e^{2i(dx+c)}b-a+b\right)}{da(-a+b)^2\left(-a\,e^{4i(dx+c)}+b\,e^{4i(dx+c)}+2\,e^{2i(dx+c)}a+2\,e^{2i(dx+c)}b-a+b\right)}+\frac{3\sqrt{-ab}\,\ln\left(e^{2i(dx+c)}-\frac{2i\sqrt{}}{}\right.}{4a(a-b)^2d}$

input
```
int(1/(a+b*cot(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/(a-b)^2*(1/2*Pi-arccot(cot(d*x+c)))+1/(a-b)^2*b*(1/2*(a-b)/a*cot(d
*x+c)/(a+b*cot(d*x+c)^2)+1/2*(3*a-b)/a/(a*b)^(1/2)*arctan(b*cot(d*x+c)/(a*
b)^(1/2))))
```

3.6.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. 2(85) = 170.

Time = 0.31 (sec) , antiderivative size = 534, normalized size of antiderivative = 5.51

$$\int \frac{1}{\left(a+b\cot^2(c+dx)\right)^2}\,dx$$
$$= \left[\frac{8\,(a^2-ab)dx\cos\left(2\,dx+2\,c\right)-8\,(a^2+ab)dx+\left(3\,a^2+2\,ab-b^2-\left(3\,a^2-4\,ab+b^2\right)\cos\left(2\,dx+2\,c\right)\right)\sqrt{}}{8\left(\left(a^4-3\,a^3b+3\,a^2b^2-\right.\right.}\right.$$

input
```
integrate(1/(a+b*cot(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[1/8*(8*(a^2 - a*b)*d*x*cos(2*d*x + 2*c) - 8*(a^2 + a*b)*d*x + (3*a^2 + 2*
a*b - b^2 - (3*a^2 - 4*a*b + b^2)*cos(2*d*x + 2*c))*sqrt(-b/a)*log(((a^2 +
 6*a*b + b^2)*cos(2*d*x + 2*c)^2 + 4*(a^2 - a*b - (a^2 + a*b)*cos(2*d*x +
2*c))*sqrt(-b/a)*sin(2*d*x + 2*c) + a^2 - 6*a*b + b^2 - 2*(a^2 - b^2)*cos(
2*d*x + 2*c))/((a^2 - 2*a*b + b^2)*cos(2*d*x + 2*c)^2 + a^2 + 2*a*b + b^2
- 2*(a^2 - b^2)*cos(2*d*x + 2*c))) - 4*(a*b - b^2)*sin(2*d*x + 2*c))/((a^4
 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d*cos(2*d*x + 2*c) - (a^4 - a^3*b - a^2*b^
2 + a*b^3)*d), 1/4*(4*(a^2 - a*b)*d*x*cos(2*d*x + 2*c) - 4*(a^2 + a*b)*d*x
 - (3*a^2 + 2*a*b - b^2 - (3*a^2 - 4*a*b + b^2)*cos(2*d*x + 2*c))*sqrt(b/a
)*arctan(1/2*((a + b)*cos(2*d*x + 2*c) - a + b)*sqrt(b/a)/(b*sin(2*d*x + 2
*c))) - 2*(a*b - b^2)*sin(2*d*x + 2*c))/((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^
3)*d*cos(2*d*x + 2*c) - (a^4 - a^3*b - a^2*b^2 + a*b^3)*d)]
```

3.6.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2125 vs. $2(78) = 156$.

Time = 9.07 (sec) , antiderivative size = 2125, normalized size of antiderivative = 21.91

$$\int \frac{1}{\left(a + b\cot^2(c + dx)\right)^2}\, dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*cot(d*x+c)**2)**2,x)
```

output
```
Piecewise((zoo*x/cot(c)**4, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (x/a**2, Eq(b
, 0)), ((x - 1/(d*cot(c + d*x)) + 1/(3*d*cot(c + d*x)**3))/b**2, Eq(a, 0))
, (3*d*x*cot(c + d*x)**4/(8*b**2*d*cot(c + d*x)**4 + 16*b**2*d*cot(c + d*x
)**2 + 8*b**2*d) + 6*d*x*cot(c + d*x)**2/(8*b**2*d*cot(c + d*x)**4 + 16*b*
*2*d*cot(c + d*x)**2 + 8*b**2*d) + 3*d*x/(8*b**2*d*cot(c + d*x)**4 + 16*b*
*2*d*cot(c + d*x)**2 + 8*b**2*d) - 3*cot(c + d*x)**3/(8*b**2*d*cot(c + d*x
)**4 + 16*b**2*d*cot(c + d*x)**2 + 8*b**2*d) - 5*cot(c + d*x)/(8*b**2*d*co
t(c + d*x)**4 + 16*b**2*d*cot(c + d*x)**2 + 8*b**2*d), Eq(a, b)), (x/(a +
b*cot(c)**2)**2, Eq(d, 0)), (4*a**2*d*x*sqrt(-a/b)/(4*a**4*d*sqrt(-a/b) +
4*a**3*b*d*sqrt(-a/b)*cot(c + d*x)**2 - 8*a**3*b*d*sqrt(-a/b) - 8*a**2*b**
2*d*sqrt(-a/b)*cot(c + d*x)**2 + 4*a**2*b**2*d*sqrt(-a/b) + 4*a*b**3*d*sqr
t(-a/b)*cot(c + d*x)**2) + 3*a**2*log(-sqrt(-a/b) + cot(c + d*x))/(4*a**4*
d*sqrt(-a/b) + 4*a**3*b*d*sqrt(-a/b)*cot(c + d*x)**2 - 8*a**3*b*d*sqrt(-a/
b) - 8*a**2*b**2*d*sqrt(-a/b)*cot(c + d*x)**2 + 4*a**2*b**2*d*sqrt(-a/b) +
 4*a*b**3*d*sqrt(-a/b)*cot(c + d*x)**2) - 3*a**2*log(sqrt(-a/b) + cot(c +
d*x))/(4*a**4*d*sqrt(-a/b) + 4*a**3*b*d*sqrt(-a/b)*cot(c + d*x)**2 - 8*a**
3*b*d*sqrt(-a/b) - 8*a**2*b**2*d*sqrt(-a/b)*cot(c + d*x)**2 + 4*a**2*b**2*
d*sqrt(-a/b) + 4*a*b**3*d*sqrt(-a/b)*cot(c + d*x)**2) + 4*a*b*d*x*sqrt(-a/
b)*cot(c + d*x)**2/(4*a**4*d*sqrt(-a/b) + 4*a**3*b*d*sqrt(-a/b)*cot(c + d*
x)**2 - 8*a**3*b*d*sqrt(-a/b) - 8*a**2*b**2*d*sqrt(-a/b)*cot(c + d*x)**...
```

3.6.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.19

$$\int \frac{1}{\left(a+b\cot^2(c+dx)\right)^2}\,dx=\frac{\frac{b\tan(dx+c)}{a^2b-ab^2+(a^3-a^2b)\tan(dx+c)^2}-\frac{(3\,ab-b^2)\arctan\left(\frac{a\tan(dx+c)}{\sqrt{ab}}\right)}{(a^3-2\,a^2b+ab^2)\sqrt{ab}}+\frac{2\,(dx+c)}{a^2-2\,ab+b^2}}{2\,d}$$

input
```
integrate(1/(a+b*cot(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/2*(b*tan(d*x + c)/(a^2*b - a*b^2 + (a^3 - a^2*b)*tan(d*x + c)^2) - (3*a*
b - b^2)*arctan(a*tan(d*x + c)/sqrt(a*b))/((a^3 - 2*a^2*b + a*b^2)*sqrt(a*
b)) + 2*(d*x + c)/(a^2 - 2*a*b + b^2))/d
```

3.6.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.27

$$\int \frac{1}{(a+b\cot^2(c+dx))^2}\,dx = -\frac{\frac{\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan(dx+c)}{\sqrt{ab}}\right)\right)(3\,ab-b^2)}{(a^3-2\,a^2b+ab^2)\sqrt{ab}}-\frac{2\,(dx+c)}{a^2-2\,ab+b^2}-\frac{b\tan(dx+c)}{\left(a\tan(dx+c)^2+b\right)(a^2-ab)}}{2\,d}$$

input
```
integrate(1/(a+b*cot(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
-1/2*((pi*floor((d*x + c)/pi + 1/2)*sgn(a) + arctan(a*tan(d*x + c)/sqrt(a*
b)))*(3*a*b - b^2)/((a^3 - 2*a^2*b + a*b^2)*sqrt(a*b)) - 2*(d*x + c)/(a^2
- 2*a*b + b^2) - b*tan(d*x + c)/((a*tan(d*x + c)^2 + b)*(a^2 - a*b)))/d
```

3.6.9 Mupad [B] (verification not implemented)

Time = 12.40 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.23

$$\int \frac{1}{(a+b\cot^2(c+dx))^2}\,dx = \frac{\frac{a\,x}{(a-b)^2}+\frac{b\,x\cot(c+d\,x)^2}{(a-b)^2}+\frac{b\cot(c+d\,x)}{2\,a\,d\,(a-b)}}{b\cot\left(c+d\,x\right)^2+a} + \frac{\operatorname{atan}\left(\frac{b\cot(c+d\,x)}{\sqrt{a\,b}}\right)\,\left(3\,a\,b-b^2\right)}{\sqrt{a\,b}\,\left(2\,a^3\,d-a\,b\,\left(4\,a\,d-2\,b\,d\right)\right)}$$

input
```
int(1/(a + b*cot(c + d*x)^2)^2,x)
```

output
```
((a*x)/(a - b)^2 + (b*x*cot(c + d*x)^2)/(a - b)^2 + (b*cot(c + d*x))/(2*a*
d*(a - b)))/(a + b*cot(c + d*x)^2) + (atan((b*cot(c + d*x))/(a*b)^(1/2))*(
3*a*b - b^2))/((a*b)^(1/2)*(2*a^3*d - a*b*(4*a*d - 2*b*d)))
```

3.7 $\int \frac{1}{(a+b\cot^2(c+dx))^3}\,dx$

3.7.1 Optimal result

Integrand size = 14, antiderivative size = 150

$$\int \frac{1}{(a+b\cot^2(c+dx))^3}\,dx = \frac{x}{(a-b)^3}+\frac{\sqrt{b}(15a^2-10ab+3b^2)\arctan\left(\frac{\sqrt{b}\cot(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a-b)^3d} + \frac{b\cot(c+dx)}{4a(a-b)d\,(a+b\cot^2(c+dx))^2} + \frac{(7a-3b)b\cot(c+dx)}{8a^2(a-b)^2d\,(a+b\cot^2(c+dx))}$$

output
```
x/(a-b)^3+1/4*b*cot(d*x+c)/a/(a-b)/d/(a+b*cot(d*x+c)^2)^2+1/8*(7*a-3*b)*b*
cot(d*x+c)/a^2/(a-b)^2/d/(a+b*cot(d*x+c)^2)+1/8*(15*a^2-10*a*b+3*b^2)*arct
an(cot(d*x+c)*b^(1/2)/a^(1/2))*b^(1/2)/a^(5/2)/(a-b)^3/d
```

3.7.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.92

$$\int \frac{1}{(a+b\cot^2(c+dx))^3}\,dx = \frac{-8\arctan(\cot(c+dx))+\frac{\sqrt{b}(15a^2-10ab+3b^2)\arctan\left(\frac{\sqrt{b}\cot(c+dx)}{\sqrt{a}}\right)}{a^{5/2}}+\frac{2(a-b)^2b\cot(c+dx)}{a(a+b\cot^2(c+dx))^2}+\frac{(7a-3b)(a-b)b\cot(c+dx)}{a^2(a+b\cot^2(c+dx))}}{8(a-b)^3d}$$

input
```
Integrate[(a + b*Cot[c + d*x]^2)^(-3),x]
```

output
```
(-8*ArcTan[Cot[c + d*x]] + (Sqrt[b]*(15*a^2 - 10*a*b + 3*b^2)*ArcTan[(Sqrt
[b]*Cot[c + d*x])/Sqrt[a]])/a^(5/2) + (2*(a - b)^2*b*Cot[c + d*x])/(a*(a +
 b*Cot[c + d*x]^2)^2) + ((7*a - 3*b)*(a - b)*b*Cot[c + d*x])/(a^2*(a + b*C
ot[c + d*x]^2)))/(8*(a - b)^3*d)
```

3.7.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.23, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 4144, 316, 402, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\cot^2(c+dx)\right)^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\left(a+b\tan\left(c+dx+\frac{\pi}{2}\right)^2\right)^3}dx$$

$$\Big\downarrow \text{ 4144}$$

$$-\frac{\int \frac{1}{(\cot^2(c+dx)+1)(b\cot^2(c+dx)+a)^3}d\cot(c+dx)}{d}$$

$$\Big\downarrow \text{ 316}$$

$$-\frac{\frac{\int \frac{-3b\cot^2(c+dx)+4a-3b}{\left(\cot^2(c+dx)+1\right)\left(b\cot^2(c+dx)+a\right)^2}d\cot(c+dx)}{4a(a-b)}-\frac{b\cot(c+dx)}{4a(a-b)(a+b\cot^2(c+dx))^2}}{d}$$

$$\Big\downarrow \text{ 402}$$

$$-\frac{\frac{\frac{\int \frac{8a^2-7ba+3b^2-(7a-3b)b\cot^2(c+dx)}{\left(\cot^2(c+dx)+1\right)\left(b\cot^2(c+dx)+a\right)}d\cot(c+dx)}{2a(a-b)}-\frac{b(7a-3b)\cot(c+dx)}{2a(a-b)\left(a+b\cot^2(c+dx)\right)}}{4a(a-b)}-\frac{b\cot(c+dx)}{4a(a-b)(a+b\cot^2(c+dx))^2}}{d}$$

$$\Big\downarrow \text{ 397}$$

$$-\frac{\frac{\frac{\frac{8a^2\int \frac{1}{\cot^2(c+dx)+1}d\cot(c+dx)}{a-b}-\frac{b\left(15a^2-10ab+3b^2\right)\int \frac{1}{b\cot^2(c+dx)+a}d\cot(c+dx)}{a-b}}{2a(a-b)}-\frac{b(7a-3b)\cot(c+dx)}{2a(a-b)\left(a+b\cot^2(c+dx)\right)}}{4a(a-b)}-\frac{b\cot(c+dx)}{4a(a-b)(a+b\cot^2(c+dx))^2}}{d}$$

$$\Big\downarrow \text{216}$$

$$-\frac{\frac{\frac{\frac{8a^2\arctan(\cot(c+dx))}{a-b}-\frac{b\left(15a^2-10ab+3b^2\right)\int \frac{1}{b\cot^2(c+dx)+a}d\cot(c+dx)}{a-b}}{2a(a-b)}-\frac{b(7a-3b)\cot(c+dx)}{2a(a-b)\left(a+b\cot^2(c+dx)\right)}}{4a(a-b)}-\frac{b\cot(c+dx)}{4a(a-b)(a+b\cot^2(c+dx))^2}}{d}$$

$$\Big\downarrow \text{218}$$

$$-\frac{\frac{\frac{\frac{8a^2\arctan(\cot(c+dx))}{a-b}-\frac{\sqrt{b}\left(15a^2-10ab+3b^2\right)\arctan\left(\frac{\sqrt{b}\cot(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)}}{2a(a-b)}-\frac{b(7a-3b)\cot(c+dx)}{2a(a-b)\left(a+b\cot^2(c+dx)\right)}}{4a(a-b)}-\frac{b\cot(c+dx)}{4a(a-b)(a+b\cot^2(c+dx))^2}}{d}$$

input
```
Int[(a + b*Cot[c + d*x]^2)^(-3),x]
```

output
```
-((-1/4*(b*Cot[c + d*x])/(a*(a - b)*(a + b*Cot[c + d*x]^2)^2) + (((8*a^2*A
rcTan[Cot[c + d*x]])/(a - b) - (Sqrt[b]*(15*a^2 - 10*a*b + 3*b^2)*ArcTan[(
Sqrt[b]*Cot[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)))/(2*a*(a - b)) - ((7*a -
 3*b)*b*Cot[c + d*x])/(2*a*(a - b)*(a + b*Cot[c + d*x]^2)))/(4*a*(a - b)))
/d)
```

3.7.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

3.7. $\int \frac{1}{(a+b\cot^2(c+dx))^3}\,dx$

3.7.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.99

method	result
derivativedivides	$\frac{-\frac{\frac{\pi}{2}-\operatorname{arccot}(\cot(dx+c))}{(a-b)^3}+\frac{b\left(\frac{\frac{b(7a^2-10ab+3b^2)\cot(dx+c)^3}{8a^2}+\frac{(9a^2-14ab+5b^2)\cot(dx+c)}{8a}}{(a+b\cot(dx+c)^2)^2}+\frac{(15a^2-10ab+3b^2)\arctan\left(\frac{b\cot(dx+c}{\sqrt{ab}}\right.}{8a^2\sqrt{ab}}\right.}{(a-b)^3}}{d}$
default	$\frac{-\frac{\frac{\pi}{2}-\operatorname{arccot}(\cot(dx+c))}{(a-b)^3}+\frac{b\left(\frac{\frac{b(7a^2-10ab+3b^2)\cot(dx+c)^3}{8a^2}+\frac{(9a^2-14ab+5b^2)\cot(dx+c)}{8a}}{(a+b\cot(dx+c)^2)^2}+\frac{(15a^2-10ab+3b^2)\arctan\left(\frac{b\cot(dx+c}{\sqrt{ab}}\right.}{8a^2\sqrt{ab}}\right.}{(a-b)^3}}{d}$
risch	$\frac{x}{a^3-3a^2b+3ab^2-b^3}-\frac{ib\left(9a^3e^{6i(dx+c)}+a^2b\,e^{6i(dx+c)}-13ab^2e^{6i(dx+c)}+3b^3e^{6i(dx+c)}-27a^3e^{4i(dx+c)}-9a^2b\,e^{4i(dx+c)}\right.}{4\left(-a\,e^{4i(dx+c)}+b\,e^{4i(dx+c)}+\right.}$

input
```
int(1/(a+b*cot(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/(a-b)^3*(1/2*Pi-arccot(cot(d*x+c)))+b/(a-b)^3*((1/8*b*(7*a^2-10*a*
b+3*b^2)/a^2*cot(d*x+c)^3+1/8*(9*a^2-14*a*b+5*b^2)/a*cot(d*x+c))/(a+b*cot(
d*x+c)^2)^2+1/8*(15*a^2-10*a*b+3*b^2)/a^2/(a*b)^(1/2)*arctan(b*cot(d*x+c)/
(a*b)^(1/2))))
```

3.7.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 475 vs. $2(136) = 272$.

Time = 0.33 (sec) , antiderivative size = 1068, normalized size of antiderivative = 7.12

$$\int \frac{1}{(a+b\cot^2(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*cot(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
[1/32*(32*(a^4 - 2*a^3*b + a^2*b^2)*d*x*cos(2*d*x + 2*c)^2 - 64*(a^4 - a^2
*b^2)*d*x*cos(2*d*x + 2*c) + 32*(a^4 + 2*a^3*b + a^2*b^2)*d*x - (15*a^4 +
20*a^3*b - 2*a^2*b^2 - 4*a*b^3 + 3*b^4 + (15*a^4 - 40*a^3*b + 38*a^2*b^2 -
 16*a*b^3 + 3*b^4)*cos(2*d*x + 2*c)^2 - 2*(15*a^4 - 10*a^3*b - 12*a^2*b^2
+ 10*a*b^3 - 3*b^4)*cos(2*d*x + 2*c))*sqrt(-b/a)*log(((a^2 + 6*a*b + b^2)*
cos(2*d*x + 2*c)^2 + 4*(a^2 - a*b - (a^2 + a*b)*cos(2*d*x + 2*c))*sqrt(-b/
a)*sin(2*d*x + 2*c) + a^2 - 6*a*b + b^2 - 2*(a^2 - b^2)*cos(2*d*x + 2*c))/
((a^2 - 2*a*b + b^2)*cos(2*d*x + 2*c)^2 + a^2 + 2*a*b + b^2 - 2*(a^2 - b^2
)*cos(2*d*x + 2*c))) + 4*(9*a^3*b - 7*a^2*b^2 - 5*a*b^3 + 3*b^4 - 3*(3*a^3
*b - 7*a^2*b^2 + 5*a*b^3 - b^4)*cos(2*d*x + 2*c))*sin(2*d*x + 2*c))/((a^7
- 5*a^6*b + 10*a^5*b^2 - 10*a^4*b^3 + 5*a^3*b^4 - a^2*b^5)*d*cos(2*d*x + 2
*c)^2 - 2*(a^7 - 3*a^6*b + 2*a^5*b^2 + 2*a^4*b^3 - 3*a^3*b^4 + a^2*b^5)*d*
cos(2*d*x + 2*c) + (a^7 - a^6*b - 2*a^5*b^2 + 2*a^4*b^3 + a^3*b^4 - a^2*b^
5)*d), 1/16*(16*(a^4 - 2*a^3*b + a^2*b^2)*d*x*cos(2*d*x + 2*c)^2 - 32*(a^4
 - a^2*b^2)*d*x*cos(2*d*x + 2*c) + 16*(a^4 + 2*a^3*b + a^2*b^2)*d*x + (15*
a^4 + 20*a^3*b - 2*a^2*b^2 - 4*a*b^3 + 3*b^4 + (15*a^4 - 40*a^3*b + 38*a^2
*b^2 - 16*a*b^3 + 3*b^4)*cos(2*d*x + 2*c)^2 - 2*(15*a^4 - 10*a^3*b - 12*a^
2*b^2 + 10*a*b^3 - 3*b^4)*cos(2*d*x + 2*c))*sqrt(b/a)*arctan(1/2*((a + b)*
cos(2*d*x + 2*c) - a + b)*sqrt(b/a)/(b*sin(2*d*x + 2*c))) + 2*(9*a^3*b - 7
*a^2*b^2 - 5*a*b^3 + 3*b^4 - 3*(3*a^3*b - 7*a^2*b^2 + 5*a*b^3 - b^4)*co...
```

3.7.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8964 vs. $2(133) = 266$.

Time = 48.20 (sec) , antiderivative size = 8964, normalized size of antiderivative = 59.76

$$\int \frac{1}{(a+b\cot^2(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*cot(d*x+c)**2)**3,x)
```

output
```
Piecewise((zoo*x/cot(c)**6, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (x/a**3, Eq(b
, 0)), ((-x + 1/(d*cot(c + d*x)) - 1/(3*d*cot(c + d*x)**3) + 1/(5*d*cot(c
+ d*x)**5))/b**3, Eq(a, 0)), (15*d*x*cot(c + d*x)**6/(48*b**3*d*cot(c + d*
x)**6 + 144*b**3*d*cot(c + d*x)**4 + 144*b**3*d*cot(c + d*x)**2 + 48*b**3*
d) + 45*d*x*cot(c + d*x)**4/(48*b**3*d*cot(c + d*x)**6 + 144*b**3*d*cot(c
+ d*x)**4 + 144*b**3*d*cot(c + d*x)**2 + 48*b**3*d) + 45*d*x*cot(c + d*x)*
*2/(48*b**3*d*cot(c + d*x)**6 + 144*b**3*d*cot(c + d*x)**4 + 144*b**3*d*co
t(c + d*x)**2 + 48*b**3*d) + 15*d*x/(48*b**3*d*cot(c + d*x)**6 + 144*b**3*
d*cot(c + d*x)**4 + 144*b**3*d*cot(c + d*x)**2 + 48*b**3*d) - 15*cot(c + d
*x)**5/(48*b**3*d*cot(c + d*x)**6 + 144*b**3*d*cot(c + d*x)**4 + 144*b**3*
d*cot(c + d*x)**2 + 48*b**3*d) - 40*cot(c + d*x)**3/(48*b**3*d*cot(c + d*x
)**6 + 144*b**3*d*cot(c + d*x)**4 + 144*b**3*d*cot(c + d*x)**2 + 48*b**3*d
) - 33*cot(c + d*x)/(48*b**3*d*cot(c + d*x)**6 + 144*b**3*d*cot(c + d*x)**
4 + 144*b**3*d*cot(c + d*x)**2 + 48*b**3*d), Eq(a, b)), (x/(a + b*cot(c)**
2)**3, Eq(d, 0)), (16*a**4*d*x*sqrt(-a/b)/(16*a**7*d*sqrt(-a/b) + 32*a**6*
b*d*sqrt(-a/b)*cot(c + d*x)**2 - 48*a**6*b*d*sqrt(-a/b) + 16*a**5*b**2*d*s
qrt(-a/b)*cot(c + d*x)**4 - 96*a**5*b**2*d*sqrt(-a/b)*cot(c + d*x)**2 + 48
*a**5*b**2*d*sqrt(-a/b) - 48*a**4*b**3*d*sqrt(-a/b)*cot(c + d*x)**4 + 96*a
**4*b**3*d*sqrt(-a/b)*cot(c + d*x)**2 - 16*a**4*b**3*d*sqrt(-a/b) + 48*a**
3*b**4*d*sqrt(-a/b)*cot(c + d*x)**4 - 32*a**3*b**4*d*sqrt(-a/b)*cot(c +...
```

3.7.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.52

$$\int \frac{1}{\left(a+b\cot^2(c+dx)\right)^3}\,dx =$$

$$-\frac{\frac{(15\,a^2b-10\,ab^2+3\,b^3)\arctan\left(\frac{a\tan(dx+c)}{\sqrt{ab}}\right)}{(a^5-3\,a^4b+3\,a^3b^2-a^2b^3)\sqrt{ab}}-\frac{(9\,a^2b-5\,ab^2)\tan(dx+c)^3+(7\,ab^2-3\,b^3)\tan(dx+c)}{a^4b^2-2\,a^3b^3+a^2b^4+(a^6-2\,a^5b+a^4b^2)\tan(dx+c)^4+2\,(a^5b-2\,a^4b^2+a^3b^3)\tan(dx+c)^2}-\frac{\cdots}{a^3\cdots}}{8\,d}$$

input
```
integrate(1/(a+b*cot(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
-1/8*((15*a^2*b - 10*a*b^2 + 3*b^3)*arctan(a*tan(d*x + c)/sqrt(a*b))/((a^5
 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*sqrt(a*b)) - ((9*a^2*b - 5*a*b^2)*tan(d*
x + c)^3 + (7*a*b^2 - 3*b^3)*tan(d*x + c))/(a^4*b^2 - 2*a^3*b^3 + a^2*b^4
+ (a^6 - 2*a^5*b + a^4*b^2)*tan(d*x + c)^4 + 2*(a^5*b - 2*a^4*b^2 + a^3*b^
3)*tan(d*x + c)^2) - 8*(d*x + c)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3))/d
```

3.7.8 Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.37

$$\int \frac{1}{(a+b\cot^2(c+dx))^3}\,dx =$$

$$-\frac{\frac{(15\,a^2b-10\,ab^2+3\,b^3)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan(dx+c)}{\sqrt{ab}}\right)\right)}{(a^5-3\,a^4b+3\,a^3b^2-a^2b^3)\sqrt{ab}}-\frac{8\,(dx+c)}{a^3-3\,a^2b+3\,ab^2-b^3}-\frac{9\,a^2b\tan(dx+c)^3-5\,ab^2\tan(dx+c)^3+7\,ab^2\,\mathrm{ta}\cdots}{(a^4-2\,a^3b+a^2b^2)\left(a\tan(dx+\cdots\right.}}{8\,d}$$

input
```
integrate(1/(a+b*cot(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
-1/8*((15*a^2*b - 10*a*b^2 + 3*b^3)*(pi*floor((d*x + c)/pi + 1/2)*sgn(a) +
 arctan(a*tan(d*x + c)/sqrt(a*b)))/((a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*
sqrt(a*b)) - 8*(d*x + c)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - (9*a^2*b*tan(d*
x + c)^3 - 5*a*b^2*tan(d*x + c)^3 + 7*a*b^2*tan(d*x + c) - 3*b^3*tan(d*x +
 c))/((a^4 - 2*a^3*b + a^2*b^2)*(a*tan(d*x + c)^2 + b)^2))/d
```

3.7.9 Mupad [B] (verification not implemented)

Time = 16.09 (sec) , antiderivative size = 4866, normalized size of antiderivative = 32.44

$$\int \frac{1}{(a+b\cot^2(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(1/(a + b*cot(c + d*x)^2)^3,x)
```

output
```
((cot(c + d*x)^3*(7*a*b^2 - 3*b^3))/(8*a^2*(a^2 - 2*a*b + b^2)) + (cot(c +
 d*x)*(9*a*b - 5*b^2))/(8*a*(a^2 - 2*a*b + b^2)))/(a^2*d + b^2*d*cot(c + d
*x)^4 + 2*a*b*d*cot(c + d*x)^2) + (2*atan((((((96*a^2*b^10*d^2 - 800*a^3*b
^9*d^2 + 3040*a^4*b^8*d^2 - 6816*a^5*b^7*d^2 + 9760*a^6*b^6*d^2 - 9056*a^7
*b^5*d^2 + 5280*a^8*b^4*d^2 - 1760*a^9*b^3*d^2 + 256*a^10*b^2*d^2)/(64*(a^
10*d^3 - 6*a^9*b*d^3 + a^4*b^6*d^3 - 6*a^5*b^5*d^3 + 15*a^6*b^4*d^3 - 20*a
^7*b^3*d^3 + 15*a^8*b^2*d^3)) - (cot(c + d*x)*(256*a^4*b^9*d^2 - 1280*a^5*
b^8*d^2 + 2304*a^6*b^7*d^2 - 1280*a^7*b^6*d^2 - 1280*a^8*b^5*d^2 + 2304*a^
9*b^4*d^2 - 1280*a^10*b^3*d^2 + 256*a^11*b^2*d^2)*1i)/(32*(2*a^3*d - 2*b^3
*d + 6*a*b^2*d - 6*a^2*b*d)*(a^8*d^2 - 4*a^7*b*d^2 + a^4*b^4*d^2 - 4*a^5*b
^3*d^2 + 6*a^6*b^2*d^2)))*1i)/(2*a^3*d - 2*b^3*d + 6*a*b^2*d - 6*a^2*b*d)
- (cot(c + d*x)*(9*b^7 - 60*a*b^6 + 190*a^2*b^5 - 300*a^3*b^4 + 289*a^4*b^
3))/(32*(a^8*d^2 - 4*a^7*b*d^2 + a^4*b^4*d^2 - 4*a^5*b^3*d^2 + 6*a^6*b^2*d
^2)))/(2*a^3*d - 2*b^3*d + 6*a*b^2*d - 6*a^2*b*d) - ((((96*a^2*b^10*d^2 -
800*a^3*b^9*d^2 + 3040*a^4*b^8*d^2 - 6816*a^5*b^7*d^2 + 9760*a^6*b^6*d^2 -
 9056*a^7*b^5*d^2 + 5280*a^8*b^4*d^2 - 1760*a^9*b^3*d^2 + 256*a^10*b^2*d^2
)/(64*(a^10*d^3 - 6*a^9*b*d^3 + a^4*b^6*d^3 - 6*a^5*b^5*d^3 + 15*a^6*b^4*d
^3 - 20*a^7*b^3*d^3 + 15*a^8*b^2*d^3)) + (cot(c + d*x)*(256*a^4*b^9*d^2 -
1280*a^5*b^8*d^2 + 2304*a^6*b^7*d^2 - 1280*a^7*b^6*d^2 - 1280*a^8*b^5*d^2
+ 2304*a^9*b^4*d^2 - 1280*a^10*b^3*d^2 + 256*a^11*b^2*d^2)*1i)/(32*(2*a...
```

3.7. $\int \frac{1}{\left(a+b\cot^2(c+dx)\right)^3}\,dx$

3.8 $\int (1+\cot^2(x))^{3/2} \, dx$

3.8.1 Optimal result

Integrand size = 10, antiderivative size = 22

$$\int (1+\cot^2(x))^{3/2} \, dx = -\frac{1}{2}\operatorname{arcsinh}(\cot(x)) - \frac{1}{2}\cot(x)\sqrt{\csc^2(x)}$$

output
```
-1/2*arcsinh(cot(x))-1/2*cot(x)*(csc(x)^2)^(1/2)
```

3.8.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 51 vs. 2(22) = 44.

Time = 0.13 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.32

$$\int (1+\cot^2(x))^{3/2} \, dx = \frac{1}{8}\sqrt{\csc^2(x)}\left(-\csc^2\left(\frac{x}{2}\right) - 4\log\left(\cos\left(\frac{x}{2}\right)\right) + 4\log\left(\sin\left(\frac{x}{2}\right)\right) + \sec^2\left(\frac{x}{2}\right)\right)\sin(x)$$

input
```
Integrate[(1 + Cot[x]^2)^(3/2),x]
```

output
```
(Sqrt[Csc[x]^2]*(-Csc[x/2]^2 - 4*Log[Cos[x/2]] + 4*Log[Sin[x/2]] + Sec[x/2
]^2)*Sin[x])/8
```

3.8.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 4140, 3042, 4610, 211, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\cot^2(x)+1\right)^{3/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(\tan\left(x+\frac{\pi}{2}\right)^2+1\right)^{3/2} dx$$

$$\Big\downarrow \text{4140}$$

$$\int \csc^2(x)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(\sec\left(x+\frac{\pi}{2}\right)^2\right)^{3/2} dx$$

$$\Big\downarrow \text{4610}$$

$$-\int \sqrt{\cot^2(x)+1}d\cot(x)$$

$$\Big\downarrow \text{211}$$

$$-\frac{1}{2}\int \frac{1}{\sqrt{\cot^2(x)+1}}d\cot(x)-\frac{1}{2}\sqrt{\cot^2(x)+1}\cot(x)$$

$$\Big\downarrow \text{222}$$

$$-\frac{1}{2}\text{arcsinh}(\cot(x))-\frac{1}{2}\cot(x)\sqrt{\cot^2(x)+1}$$

input
```
Int[(1 + Cot[x]^2)^(3/2),x]
```

output
```
-1/2*ArcSinh[Cot[x]] - (Cot[x]*Sqrt[1 + Cot[x]^2])/2
```

3.8.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

3.8.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result
derivativedivides	$-\frac{\cot(x)\sqrt{\cot(x)^2+1}}{2}-\frac{\operatorname{arcsinh}(\cot(x))}{2}$
default	$-\frac{\cot(x)\sqrt{\cot(x)^2+1}}{2}-\frac{\operatorname{arcsinh}(\cot(x))}{2}$
risch	$-\frac{i\sqrt{-\frac{e^{2ix}}{\left(e^{2ix}-1\right)^2}}\,\left(e^{2ix}+1\right)}{e^{2ix}-1}-\sqrt{-\frac{e^{2ix}}{\left(e^{2ix}-1\right)^2}}\,\ln\left(e^{ix}+1\right)\sin\left(x\right)+\sqrt{-\frac{e^{2ix}}{\left(e^{2ix}-1\right)^2}}\,\ln\left(e^{ix}-1\right)\sin\left(x\right.$

input
```
int((cot(x)^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*cot(x)*(cot(x)^2+1)^(1/2)-1/2*arcsinh(cot(x))
```

3.8.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. 2(16) = 32.

Time = 0.27 (sec) , antiderivative size = 91, normalized size of antiderivative = 4.14

$$\int \left(1+\cot^2(x)\right)^{3/2}\,dx = -\frac{2\sqrt{2}\sqrt{-\frac{1}{\cos(2x)-1}}(\cos(2x)+1)+\log\left(\frac{1}{2}\sqrt{2}\sqrt{-\frac{1}{\cos(2x)-1}}\sin(2x)+1\right)\sin(2x)-\log\left(-\frac{1}{2}\sqrt{2}\sqrt{-\frac{1}{\cos(2x)-1}}\right.}{4\sin(2x)}$$

input
```
integrate((1+cot(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/4*(2*sqrt(2)*sqrt(-1/(cos(2*x) - 1))*(cos(2*x) + 1) + log(1/2*sqrt(2)*s
qrt(-1/(cos(2*x) - 1))*sin(2*x) + 1)*sin(2*x) - log(-1/2*sqrt(2)*sqrt(-1/(
cos(2*x) - 1))*sin(2*x) + 1)*sin(2*x))/sin(2*x)
```

3.8.6 Sympy [F]

$$\int \left(1+\cot^2(x)\right)^{3/2}\,dx = \int \left(\cot^2(x)+1\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((1+cot(x)**2)**(3/2),x)
```

output
```
Integral((cot(x)**2 + 1)**(3/2), x)
```

3.8.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 300 vs. 2(16) = 32.

Time = 0.37 (sec) , antiderivative size = 300, normalized size of antiderivative = 13.64

$$\int \left(1+\cot^2(x)\right)^{3/2}\,dx = -\frac{4(\cos(3x)+\cos(x))\cos(4x)-4(2\cos(2x)-1)\cos(3x)-8\cos(2x)\cos(x)+(2(2\cos(2x)-1)\cos}{}$$

input
```
integrate((1+cot(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/4*(4*(cos(3*x) + cos(x))*cos(4*x) - 4*(2*cos(2*x) - 1)*cos(3*x) - 8*cos
(2*x)*cos(x) + (2*(2*cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - 4*cos(2*x)^2 -
sin(4*x)^2 + 4*sin(4*x)*sin(2*x) - 4*sin(2*x)^2 + 4*cos(2*x) - 1)*log(cos(
x)^2 + sin(x)^2 + 2*cos(x) + 1) - (2*(2*cos(2*x) - 1)*cos(4*x) - cos(4*x)^
2 - 4*cos(2*x)^2 - sin(4*x)^2 + 4*sin(4*x)*sin(2*x) - 4*sin(2*x)^2 + 4*cos
(2*x) - 1)*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) + 4*(sin(3*x) + sin(x))
*sin(4*x) - 8*sin(3*x)*sin(2*x) - 8*sin(2*x)*sin(x) + 4*cos(x))/(2*(2*cos(
2*x) - 1)*cos(4*x) - cos(4*x)^2 - 4*cos(2*x)^2 - sin(4*x)^2 + 4*sin(4*x)*s
in(2*x) - 4*sin(2*x)^2 + 4*cos(2*x) - 1)
```

3.8.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.45

$$\int \left(1+\cot^2(x)\right)^{3/2} dx = \frac{1}{4}\left(\frac{2\cos(x)}{\cos(x)^2-1} - \log(\cos(x)+1) + \log(-\cos(x)+1)\right)\operatorname{sgn}(\sin(x))$$

input
```
integrate((1+cot(x)^2)^(3/2),x, algorithm="giac")
```

output
```
1/4*(2*cos(x)/(cos(x)^2 - 1) - log(cos(x) + 1) + log(-cos(x) + 1))*sgn(sin
(x))
```

3.8.9 Mupad [B] (verification not implemented)

Time = 13.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \left(1+\cot^2(x)\right)^{3/2} dx = -\frac{\operatorname{asinh}(\cot(x))}{2} - \frac{\cot(x)\sqrt{\cot(x)^2+1}}{2}$$

input
```
int((cot(x)^2 + 1)^(3/2),x)
```

output
```
- asinh(cot(x))/2 - (cot(x)*(cot(x)^2 + 1)^(1/2))/2
```

3.9 $\int \sqrt{1+\cot^2(x)}\,dx$

3.9.1 Optimal result

Integrand size = 10, antiderivative size = 5

$$\int \sqrt{1+\cot^2(x)}\,dx = -\operatorname{arcsinh}(\cot(x))$$

output
```
-arcsinh(cot(x))
```

3.9.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 28 vs. 2(5) = 10.

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 5.60

$$\int \sqrt{1+\cot^2(x)}\,dx = \sqrt{\csc^2(x)}\left(-\log\left(\cos\left(\frac{x}{2}\right)\right)+\log\left(\sin\left(\frac{x}{2}\right)\right)\right)\sin(x)$$

input
```
Integrate[Sqrt[1 + Cot[x]^2],x]
```

output
```
Sqrt[Csc[x]^2]*(-Log[Cos[x/2]] + Log[Sin[x/2]])*Sin[x]
```

3.9.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 4140, 3042, 4610, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cot^2(x)+1}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\tan\left(x+\frac{\pi}{2}\right)^2+1}dx$$

$$\Big\downarrow \text{4140}$$

$$\int \sqrt{\csc^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sec\left(x+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{4610}$$

$$-\int \frac{1}{\sqrt{\cot^2(x)+1}}d\cot(x)$$

$$\Big\downarrow \text{222}$$

$$-\text{arcsinh}(\cot(x))$$

input
```
Int[Sqrt[1 + Cot[x]^2],x]
```

output
```
-ArcSinh[Cot[x]]
```

3.9.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

3.9.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.20

method	result	size
derivativedivides	$-\operatorname{arcsinh}(\cot(x))$	6
default	$-\operatorname{arcsinh}(\cot(x))$	6
risch	$-2\sqrt{-\frac{e^{2ix}}{(e^{2ix}-1)^2}}\,\ln(e^{ix}+1)\sin(x)+2\sqrt{-\frac{e^{2ix}}{(e^{2ix}-1)^2}}\,\ln(e^{ix}-1)\sin(x)$	62

input
```
int((cot(x)^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-arcsinh(cot(x))
```

3.9.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(5) = 10$.

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 10.60

$$\int \sqrt{1+\cot^2(x)}\,dx = -\frac{1}{2}\log\left(\frac{1}{2}\sqrt{2}\sqrt{-\frac{1}{\cos(2x)-1}}\sin(2x)+1\right) + \frac{1}{2}\log\left(-\frac{1}{2}\sqrt{2}\sqrt{-\frac{1}{\cos(2x)-1}}\sin(2x)+1\right)$$

input
```
integrate((1+cot(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*log(1/2*sqrt(2)*sqrt(-1/(cos(2*x) - 1))*sin(2*x) + 1) + 1/2*log(-1/2*
sqrt(2)*sqrt(-1/(cos(2*x) - 1))*sin(2*x) + 1)
```

3.9.6 Sympy [F]

$$\int \sqrt{1+\cot^2(x)}\,dx = \int \sqrt{\cot^2(x)+1}\,dx$$

input
```
integrate((1+cot(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(cot(x)**2 + 1), x)
```

3.9.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(5) = 10$.

Time = 0.36 (sec) , antiderivative size = 35, normalized size of antiderivative = 7.00

$$\int \sqrt{1+\cot^2(x)}\,dx = -\frac{1}{2}\log\left(\cos(x)^2+\sin(x)^2+2\cos(x)+1\right) + \frac{1}{2}\log\left(\cos(x)^2+\sin(x)^2-2\cos(x)+1\right)$$

input
```
integrate((1+cot(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/2*log(cos(x)^2 + sin(x)^2
 - 2*cos(x) + 1)
```

3.9.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 2.00

$$\int \sqrt{1+\cot^2(x)}\,dx = \log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right|\right)\operatorname{sgn}(\sin{(x)})$$

input
```
integrate((1+cot(x)^2)^(1/2),x, algorithm="giac")
```

output
```
log(abs(tan(1/2*x)))*sgn(sin(x))
```

3.9.9 Mupad [B] (verification not implemented)

Time = 12.97 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \sqrt{1+\cot^2(x)}\,dx = -\operatorname{asinh}(\cot(x))$$

input
```
int((cot(x)^2 + 1)^(1/2),x)
```

output
```
-asinh(cot(x))
```

3.10 $\int \frac{1}{\sqrt{1+\cot^2(x)}}\,dx$

3.10.1 Optimal result

Integrand size = 10, antiderivative size = 12

$$\int \frac{1}{\sqrt{1+\cot^2(x)}}\,dx = -\frac{\cot(x)}{\sqrt{\csc^2(x)}}$$

output
```
-cot(x)/(csc(x)^2)^(1/2)
```

3.10.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+\cot^2(x)}}\,dx = -\frac{\cot(x)}{\sqrt{\csc^2(x)}}$$

input
```
Integrate[1/Sqrt[1 + Cot[x]^2],x]
```

output
```
-(Cot[x]/Sqrt[Csc[x]^2])
```

3.10.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4140, 3042, 4610, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\cot^2(x)+1}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\tan\left(x+\frac{\pi}{2}\right)^2+1}}dx$$

$$\Big\downarrow \text{4140}$$

$$\int \frac{1}{\sqrt{\csc^2(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\sec\left(x+\frac{\pi}{2}\right)^2}}dx$$

$$\Big\downarrow \text{4610}$$

$$-\int \frac{1}{\left(\cot^2(x)+1\right)^{3/2}}d\cot(x)$$

$$\Big\downarrow \text{208}$$

$$-\frac{\cot(x)}{\sqrt{\cot^2(x)+1}}$$

input
```
Int[1/Sqrt[1 + Cot[x]^2],x]
```

output
```
-(Cot[x]/Sqrt[1 + Cot[x]^2])
```

3.10.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

3.10.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$-\frac{\cot(x)}{\sqrt{\cot(x)^2+1}}$	13
default	$-\frac{\cot(x)}{\sqrt{\cot(x)^2+1}}$	13
risch	$-\frac{i e^{2ix}}{2\sqrt{-\frac{e^{2ix}}{(e^{2ix}-1)^2}}\,(e^{2ix}-1)}-\frac{i}{2(e^{2ix}-1)\sqrt{-\frac{e^{2ix}}{(e^{2ix}-1)^2}}}$	67

input
```
int(1/(cot(x)^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-cot(x)/(cot(x)^2+1)^(1/2)
```

3.10.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(10) = 20$.

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.75

$$\int \frac{1}{\sqrt{1+\cot^2(x)}}\,dx = -\frac{1}{2}\,\sqrt{2}\sqrt{-\frac{1}{\cos\left(2\,x\right)-1}}\sin\left(2\,x\right)$$

input
```
integrate(1/(1+cot(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(2)*sqrt(-1/(cos(2*x) - 1))*sin(2*x)
```

3.10.6 Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{\sqrt{1+\cot^2(x)}}\,dx = -\frac{\cot\left(x\right)}{\sqrt{\cot^2\left(x\right)+1}}$$

input
```
integrate(1/(1+cot(x)**2)**(1/2),x)
```

output
```
-cot(x)/sqrt(cot(x)**2 + 1)
```

3.10.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{\sqrt{1+\cot^2(x)}}\,dx = -\frac{1}{\sqrt{\tan\left(x\right)^2+1}}$$

input
```
integrate(1/(1+cot(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/sqrt(tan(x)^2 + 1)
```

3.10.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(10) = 20$.

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.33

$$\int \frac{1}{\sqrt{1+\cot^2(x)}}\,dx = \frac{2}{\left(\frac{\cos(x)-1}{\cos(x)+1}-1\right)\operatorname{sgn}(\sin(x))} + 2\operatorname{sgn}(\sin(x))$$

input
```
integrate(1/(1+cot(x)^2)^(1/2),x, algorithm="giac")
```

output
```
2/(((cos(x) - 1)/(cos(x) + 1) - 1)*sgn(sin(x))) + 2*sgn(sin(x))
```

3.10.9 Mupad [B] (verification not implemented)

Time = 13.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+\cot^2(x)}}\,dx = -\frac{\sin(2x)}{2\sqrt{\sin(x)^2}}$$

input
```
int(1/(cot(x)^2 + 1)^(1/2),x)
```

output
```
-sin(2*x)/(2*(sin(x)^2)^(1/2))
```

3.11 $\int (-1-\cot^2(x))^{3/2}\,dx$

3.11.1 Optimal result

Integrand size = 12, antiderivative size = 35

$$\int (-1-\cot^2(x))^{3/2}\,dx = -\frac{1}{2}\arctan\left(\frac{\cot(x)}{\sqrt{-\csc^2(x)}}\right)+\frac{1}{2}\cot(x)\sqrt{-\csc^2(x)}$$

output
```
-1/2*arctan(cot(x)/(-csc(x)^2)^(1/2))+1/2*cot(x)*(-csc(x)^2)^(1/2)
```

3.11.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.37

$$\int (-1-\cot^2(x))^{3/2}\,dx = -\frac{\csc\left(\frac{x}{2}\right)\left(\cot(x)\csc(x)+\log\left(\cos\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)\right)\right)\sec\left(\frac{x}{2}\right)}{4\sqrt{-\csc^2(x)}}$$

input
```
Integrate[(-1 - Cot[x]^2)^(3/2),x]
```

output
```
-1/4*(Csc[x/2]*(Cot[x]*Csc[x] + Log[Cos[x/2]] - Log[Sin[x/2]])*Sec[x/2])/S
qrt[-Csc[x]^2]
```

3.11.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {3042, 4140, 3042, 4610, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(-\cot^2(x)-1\right)^{3/2}dx$$

$$\Big\downarrow \; 3042$$

$$\int \left(-\tan\left(x+\frac{\pi}{2}\right)^2-1\right)^{3/2}dx$$

$$\Big\downarrow \; 4140$$

$$\int \left(-\csc^2(x)\right)^{3/2}dx$$

$$\Big\downarrow \; 3042$$

$$\int \left(-\sec\left(x+\frac{\pi}{2}\right)^2\right)^{3/2}dx$$

$$\Big\downarrow \; 4610$$

$$\int \sqrt{-\cot^2(x)-1}d\cot(x)$$

$$\Big\downarrow \; 211$$

$$\frac{1}{2}\cot(x)\sqrt{-\cot^2(x)-1}-\frac{1}{2}\int \frac{1}{\sqrt{-\cot^2(x)-1}}d\cot(x)$$

$$\Big\downarrow \; 224$$

$$\frac{1}{2}\cot(x)\sqrt{-\cot^2(x)-1}-\frac{1}{2}\int \frac{1}{\frac{\cot^2(x)}{-\cot^2(x)-1}+1}d\frac{\cot(x)}{\sqrt{-\cot^2(x)-1}}$$

$$\Big\downarrow \; 216$$

$$\frac{1}{2}\cot(x)\sqrt{-\cot^2(x)-1}-\frac{1}{2}\arctan\left(\frac{\cot(x)}{\sqrt{-\cot^2(x)-1}}\right)$$

input
```
Int[(-1 - Cot[x]^2)^(3/2),x]
```

output
```
-1/2*ArcTan[Cot[x]/Sqrt[-1 - Cot[x]^2]] + (Cot[x]*Sqrt[-1 - Cot[x]^2])/2
```

3.11.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

3.11.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

method	result	siz
derivativedivides	$\frac{\cot(x)\sqrt{-1-\cot(x)^2}}{2}-\frac{\arctan\left(\frac{\cot(x)}{\sqrt{-1-\cot(x)^2}}\right)}{2}$	32
default	$\frac{\cot(x)\sqrt{-1-\cot(x)^2}}{2}-\frac{\arctan\left(\frac{\cot(x)}{\sqrt{-1-\cot(x)^2}}\right)}{2}$	32
risch	$\frac{i\sqrt{\frac{e^{2ix}}{\left(e^{2ix}-1\right)^2}}\left(e^{2ix}+1\right)}{e^{2ix}-1}-\sqrt{\frac{e^{2ix}}{\left(e^{2ix}-1\right)^2}}\ln\left(e^{ix}-1\right)\sin\left(x\right)+\sqrt{\frac{e^{2ix}}{\left(e^{2ix}-1\right)^2}}\ln\left(e^{ix}+1\right)\sin\left(x\right)$	95

input
```
int((-1-cot(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*cot(x)*(-1-cot(x)^2)^(1/2)-1/2*arctan(cot(x)/(-1-cot(x)^2)^(1/2))
```

3.11.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.09

$$\int (-1 -\cot^2(x))^{3/2}\,dx = \frac{\left(-i\,e^{(4i\,x)}+2i\,e^{(2i\,x)}-i\right)\log\left(e^{(i\,x)}+1\right)+\left(i\,e^{(4i\,x)}-2i\,e^{(2i\,x)}+i\right)\log\left(e^{(i\,x)}-1\right)+2i\,e^{(3i}}{2\left(e^{(4i\,x)}-2\,e^{(2i\,x)}+1\right)}$$

input
```
integrate((-1-cot(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/2*((-I*e^(4*I*x) + 2*I*e^(2*I*x) - I)*log(e^(I*x) + 1) + (I*e^(4*I*x) -
2*I*e^(2*I*x) + I)*log(e^(I*x) - 1) + 2*I*e^(3*I*x) + 2*I*e^(I*x))/(e^(4*I
*x) - 2*e^(2*I*x) + 1)
```

3.11.6 Sympy [F]

$$\int \left(-1-\cot^2(x)\right)^{3/2} dx = \int \left(-\cot^2(x)-1\right)^{\frac{3}{2}} dx$$

input
```
integrate((-1-cot(x)**2)**(3/2),x)
```

output
```
Integral((-cot(x)**2 - 1)**(3/2), x)
```

3.11.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. 2(27) = 54.

Time = 0.39 (sec) , antiderivative size = 284, normalized size of antiderivative = 8.11

$$\int (-1 -\cot^2(x))^{3/2} dx = \frac{(2(2\cos(2x)-1)\cos(4x)-\cos(4x)^2-4\cos(2x)^2-\sin(4x)^2+4\sin(4x)\sin(2x)}{}$$

input
```
integrate((-1-cot(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/2*((2*(2*cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - 4*cos(2*x)^2 - sin(4*x)^2
 + 4*sin(4*x)*sin(2*x) - 4*sin(2*x)^2 + 4*cos(2*x) - 1)*arctan2(sin(x), co
s(x) + 1) - (2*(2*cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - 4*cos(2*x)^2 - sin
(4*x)^2 + 4*sin(4*x)*sin(2*x) - 4*sin(2*x)^2 + 4*cos(2*x) - 1)*arctan2(sin
(x), cos(x) - 1) + 2*(sin(3*x) + sin(x))*cos(4*x) - 2*(cos(3*x) + cos(x))*
sin(4*x) - 2*(2*cos(2*x) - 1)*sin(3*x) + 4*cos(3*x)*sin(2*x) + 4*cos(x)*si
n(2*x) - 4*cos(2*x)*sin(x) + 2*sin(x))/(2*(2*cos(2*x) - 1)*cos(4*x) - cos(
4*x)^2 - 4*cos(2*x)^2 - sin(4*x)^2 + 4*sin(4*x)*sin(2*x) - 4*sin(2*x)^2 +
4*cos(2*x) - 1)
```

3.11.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\int \left(-1-\cot^2(x)\right)^{3/2} dx = -\frac{1}{4}\left(\frac{2i\cos(x)}{\cos(x)^2-1} - i\log(\cos(x)+1) + i\log(-\cos(x)+1)\right)\operatorname{sgn}(\sin(x))$$

input
```
integrate((-1-cot(x)^2)^(3/2),x, algorithm="giac")
```

output
```
-1/4*(2*I*cos(x)/(cos(x)^2 - 1) - I*log(cos(x) + 1) + I*log(-cos(x) + 1))*
sgn(sin(x))
```

3.11.9 Mupad [B] (verification not implemented)

Time = 13.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \left(-1-\cot^2(x)\right)^{3/2} dx = \frac{\cot(x)\sqrt{-\cot(x)^2-1}}{2} - \frac{\operatorname{atan}\left(\frac{\cot(x)}{\sqrt{-\cot(x)^2-1}}\right)}{2}$$

input
```
int((- cot(x)^2 - 1)^(3/2),x)
```

output
```
(cot(x)*(- cot(x)^2 - 1)^(1/2))/2 - atan(cot(x)/(- cot(x)^2 - 1)^(1/2))/2
```

3.12 $\int \sqrt{-1-\cot^2(x)}\,dx$

3.12.1 Optimal result

Integrand size = 12, antiderivative size = 14

$$\int \sqrt{-1-\cot^2(x)}\,dx = \arctan\left(\frac{\cot(x)}{\sqrt{-\csc^2(x)}}\right)$$

output
```
arctan(cot(x)/(-csc(x)^2)^(1/2))
```

3.12.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 30 vs. 2(14) = 28.

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.14

$$\int \sqrt{-1-\cot^2(x)}\,dx = \frac{\csc(x)\left(\log\left(\cos\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)\right)\right)}{\sqrt{-\csc^2(x)}}$$

input
```
Integrate[Sqrt[-1 - Cot[x]^2],x]
```

output
```
(Csc[x]*(Log[Cos[x/2]] - Log[Sin[x/2]]))/Sqrt[-Csc[x]^2]
```

3.12.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 4140, 3042, 4610, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{-\cot^2(x)-1}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{-\tan\left(x+\frac{\pi}{2}\right)^2-1}dx$$

$$\Big\downarrow \text{4140}$$

$$\int \sqrt{-\csc^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{-\sec\left(x+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{4610}$$

$$\int \frac{1}{\sqrt{-\cot^2(x)-1}}d\cot(x)$$

$$\Big\downarrow \text{224}$$

$$\int \frac{1}{\frac{\cot^2(x)}{-\cot^2(x)-1}+1}d\frac{\cot(x)}{\sqrt{-\cot^2(x)-1}}$$

$$\Big\downarrow \text{216}$$

$$\arctan\left(\frac{\cot(x)}{\sqrt{-\cot^2(x)-1}}\right)$$

input
```
Int[Sqrt[-1 - Cot[x]^2],x]
```

output
```
ArcTan[Cot[x]/Sqrt[-1 - Cot[x]^2]]
```

3.12.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

3.12.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

method	result	size
derivativedivides	$\arctan\left(\frac{\cot(x)}{\sqrt{-1-\cot(x)^{2}}}\right)$	15
default	$\arctan\left(\frac{\cot(x)}{\sqrt{-1-\cot(x)^{2}}}\right)$	15
risch	$-2\sqrt{\frac{e^{2ix}}{(e^{2ix}-1)^{2}}}\,\ln\left(e^{ix}+1\right)\sin\left(x\right)+2\sqrt{\frac{e^{2ix}}{(e^{2ix}-1)^{2}}}\,\ln\left(e^{ix}-1\right)\sin\left(x\right)$	60

input
```
int((-1-cot(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arctan(cot(x)/(-1-cot(x)^2)^(1/2))
```

3.12.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.36

$$\int \sqrt{-1-\cot^2(x)}\,dx = i\log\left(e^{(ix)}+1\right) - i\log\left(e^{(ix)}-1\right)$$

input
```
integrate((-1-cot(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
I*log(e^(I*x) + 1) - I*log(e^(I*x) - 1)
```

3.12.6 Sympy [F]

$$\int \sqrt{-1-\cot^2(x)}\,dx = \int \sqrt{-\cot^2(x)-1}\,dx$$

input
```
integrate((-1-cot(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(-cot(x)**2 - 1), x)
```

3.12.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.21

$$\int \sqrt{-1-\cot^2(x)}\,dx = -\arctan\left(\sin(x), \cos(x)+1\right) + \arctan\left(\sin(x), \cos(x)-1\right)$$

input
```
integrate((-1-cot(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-arctan2(sin(x), cos(x) + 1) + arctan2(sin(x), cos(x) - 1)
```

3.12.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

$$\int \sqrt{-1-\cot^2(x)}\,dx = i\,\log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right|\right)\,\mathrm{sgn}(\sin(x))$$

input
```
integrate((-1-cot(x)^2)^(1/2),x, algorithm="giac")
```

output
```
I*log(abs(tan(1/2*x)))*sgn(sin(x))
```

3.12.9 Mupad [B] (verification not implemented)

Time = 12.87 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \sqrt{-1-\cot^2(x)}\,dx = \operatorname{atan}\left(\frac{\cot(x)}{\sqrt{-\cot(x)^2-1}}\right)$$

input
```
int((- cot(x)^2 - 1)^(1/2),x)
```

output
```
atan(cot(x)/(- cot(x)^2 - 1)^(1/2))
```

3.13 $\int \frac{1}{\sqrt{-1-\cot^2(x)}}\,dx$

3.13.1 Optimal result

Integrand size = 12, antiderivative size = 14

$$\int \frac{1}{\sqrt{-1-\cot^2(x)}}\,dx = -\frac{\cot(x)}{\sqrt{-\csc^2(x)}}$$

output
```
-cot(x)/(-csc(x)^2)^(1/2)
```

3.13.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-1-\cot^2(x)}}\,dx = -\frac{\cot(x)}{\sqrt{-\csc^2(x)}}$$

input
```
Integrate[1/Sqrt[-1 - Cot[x]^2],x]
```

output
```
-(Cot[x]/Sqrt[-Csc[x]^2])
```

3.13.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3042, 4140, 3042, 4610, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-\cot^2(x)-1}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sqrt{-\tan\left(x+\frac{\pi}{2}\right)^2-1}}dx$$

$$\Big\downarrow \text{ 4140}$$

$$\int \frac{1}{\sqrt{-\csc^2(x)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sqrt{-\sec\left(x+\frac{\pi}{2}\right)^2}}dx$$

$$\Big\downarrow \text{ 4610}$$

$$\int \frac{1}{\left(-\cot^2(x)-1\right)^{3/2}}d\cot(x)$$

$$\Big\downarrow \text{ 208}$$

$$-\frac{\cot(x)}{\sqrt{-\cot^2(x)-1}}$$

input
```
Int[1/Sqrt[-1 - Cot[x]^2],x]
```

output
```
-(Cot[x]/Sqrt[-1 - Cot[x]^2])
```

3.13.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

3.13.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

method	result	size
derivativedivides	$-\frac{\cot(x)}{\sqrt{-1-\cot(x)^2}}$	15
default	$-\frac{\cot(x)}{\sqrt{-1-\cot(x)^2}}$	15
risch	$-\frac{i\mathrm{e}^{2ix}}{2\sqrt{\frac{\mathrm{e}^{2ix}}{\left(\mathrm{e}^{2ix}-1\right)^2}}\,(\mathrm{e}^{2ix}-1)}-\frac{i}{2(\mathrm{e}^{2ix}-1)\sqrt{\frac{\mathrm{e}^{2ix}}{\left(\mathrm{e}^{2ix}-1\right)^2}}}$	65

input
```
int(1/(-1-cot(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-cot(x)/(-1-cot(x)^2)^(1/2)
```

3.13.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-1-\cot^2(x)}}\,dx = \frac{1}{2}\left(-i\,e^{(2i\,x)} - i\right)e^{(-i\,x)}$$

input
```
integrate(1/(-1-cot(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(-I*e^(2*I*x) - I)*e^(-I*x)
```

3.13.6 Sympy [F]

$$\int \frac{1}{\sqrt{-1-\cot^2(x)}}\,dx = \int \frac{1}{\sqrt{-\cot^2(x)-1}}\,dx$$

input
```
integrate(1/(-1-cot(x)**2)**(1/2),x)
```

output
```
Integral(1/sqrt(-cot(x)**2 - 1), x)
```

3.13.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{-1-\cot^2(x)}}\,dx = -\frac{1}{\sqrt{-\tan(x)^2-1}}$$

input
```
integrate(1/(-1-cot(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/sqrt(-tan(x)^2 - 1)
```

3.13.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.00

$$\int \frac{1}{\sqrt{-1-\cot^2(x)}}\,dx = -\frac{2i}{\left(\frac{\cos(x)-1}{\cos(x)+1}-1\right)\operatorname{sgn}\left(\sin\left(x\right)\right)} - 2i\operatorname{sgn}(\sin\left(x\right))$$

input
```
integrate(1/(-1-cot(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-2*I/(((cos(x) - 1)/(cos(x) + 1) - 1)*sgn(sin(x))) - 2*I*sgn(sin(x))
```

3.13.9 Mupad [B] (verification not implemented)

Time = 13.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{1}{\sqrt{-1-\cot^2(x)}}\,dx = \frac{\sin\left(2\,x\right)\,1\mathrm{i}}{2\,\sqrt{{\sin\left(x\right)}^2}}$$

input
```
int(1/(- cot(x)^2 - 1)^(1/2),x)
```

output
```
(sin(2*x)*1i)/(2*(sin(x)^2)^(1/2))
```

3.14 $\int \frac{\cot^3(x)}{\sqrt{a+a\cot^2(x)}}\,dx$

3.14.1 Optimal result

Integrand size = 17, antiderivative size = 28

$$\int \frac{\cot^3(x)}{\sqrt{a+a\cot^2(x)}}\,dx = -\frac{1}{\sqrt{a\csc^2(x)}} - \frac{\sqrt{a\csc^2(x)}}{a}$$

output
```
-1/(a*csc(x)^2)^(1/2)-(a*csc(x)^2)^(1/2)/a
```

3.14.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.68

$$\int \frac{\cot^3(x)}{\sqrt{a+a\cot^2(x)}}\,dx = \frac{-1-\csc^2(x)}{\sqrt{a\csc^2(x)}}$$

input
```
Integrate[Cot[x]^3/Sqrt[a + a*Cot[x]^2],x]
```

output
```
(-1 - Csc[x]^2)/Sqrt[a*Csc[x]^2]
```

3.14.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.588, Rules used = {3042, 25, 4140, 25, 3042, 25, 4612, 25, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(x)}{\sqrt{a\cot^2(x)+a}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{\tan\left(x+\frac{\pi}{2}\right)^3}{\sqrt{a\tan\left(x+\frac{\pi}{2}\right)^2+a}}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{\tan\left(x+\frac{\pi}{2}\right)^3}{\sqrt{a\tan\left(x+\frac{\pi}{2}\right)^2+a}}dx$$

$$\Big\downarrow \text{ 4140}$$

$$-\int -\frac{\cot^3(x)}{\sqrt{a\csc^2(x)}}dx$$

$$\Big\downarrow \text{ 25}$$

$$\int \frac{\cot^3(x)}{\sqrt{a\csc^2(x)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{\tan\left(x+\frac{\pi}{2}\right)^3}{\sqrt{a\sec\left(x+\frac{\pi}{2}\right)^2}}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{\tan\left(x+\frac{\pi}{2}\right)^3}{\sqrt{a\sec\left(x+\frac{\pi}{2}\right)^2}}dx$$

$$\Big\downarrow \text{ 4612}$$

$$-\frac{1}{2}a\int -\frac{1-\csc^2(x)}{(a\csc^2(x))^{3/2}}d\csc^2(x)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{2}a\int \frac{1-\csc^2(x)}{(a\csc^2(x))^{3/2}}d\csc^2(x)$$

$$\Big\downarrow \textbf{53}$$

$$\frac{1}{2}a\int \left(\frac{1}{(a\csc^2(x))^{3/2}}-\frac{1}{a\sqrt{a\csc^2(x)}}\right)d\csc^2(x)$$

$$\Big\downarrow \textbf{2009}$$

$$-\frac{1}{2}a\left(\frac{2\sqrt{a\csc^2(x)}}{a^2}+\frac{2}{a\sqrt{a\csc^2(x)}}\right)$$

input
```
Int[Cot[x]^3/Sqrt[a + a*Cot[x]^2],x]
```

output
```
-1/2*(a*(2/(a*Sqrt[a*Csc[x]^2]) + (2*Sqrt[a*Csc[x]^2])/a^2))
```

3.14.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

3.14. $\int \frac{\cot^3(x)}{\sqrt{a+a\cot^2(x)}}\,dx$

rule 4612
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.),
x_Symbol] :> Simp[b/(2*f)   Subst[Int[(-1 + x)^((m - 1)/2)*(b*x)^(p - 1), x
], x, Sec[e + f*x]^2], x] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p] && Int
egerQ[(m - 1)/2]
```

3.14.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$-\frac{\sqrt{a+a\cot(x)^2}}{a}-\frac{1}{\sqrt{a+a\cot(x)^2}}$	29
default	$-\frac{\sqrt{a+a\cot(x)^2}}{a}-\frac{1}{\sqrt{a+a\cot(x)^2}}$	29
risch	$-\frac{e^{4ix}-6\,e^{2ix}+1}{2\sqrt{-\frac{a\,e^{2ix}}{\left(e^{2ix}-1\right)^2}}\,\left(e^{2ix}-1\right)^2}$	45

input
```
int(cot(x)^3/(a+a*cot(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/a*(a+a*cot(x)^2)^(1/2)-1/(a+a*cot(x)^2)^(1/2)
```

3.14.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{\cot^3(x)}{\sqrt{a+a\cot^2(x)}}\,dx=\frac{\sqrt{2}\sqrt{-\frac{a}{\cos(2x)-1}}\,(\cos(2x)-3)}{2a}$$

input
```
integrate(cot(x)^3/(a+a*cot(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*sqrt(-a/(cos(2*x) - 1))*(cos(2*x) - 3)/a
```

3.14.6 Sympy [F]

$$\int \frac{\cot^3(x)}{\sqrt{a + a\cot^2(x)}}\,dx = \int \frac{\cot^3(x)}{\sqrt{a\left(\cot^2(x) + 1\right)}}\,dx$$

input
```
integrate(cot(x)**3/(a+a*cot(x)**2)**(1/2),x)
```

output
```
Integral(cot(x)**3/sqrt(a*(cot(x)**2 + 1)), x)
```

3.14.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{\cot^3(x)}{\sqrt{a + a\cot^2(x)}}\,dx = -\frac{1}{\sqrt{\frac{a}{\sin(x)^2}}} - \frac{\sqrt{\frac{a}{\sin(x)^2}}}{a}$$

input
```
integrate(cot(x)^3/(a+a*cot(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/sqrt(a/sin(x)^2) - sqrt(a/sin(x)^2)/a
```

3.14.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \frac{\cot^3(x)}{\sqrt{a + a\cot^2(x)}}\,dx = -\frac{\sqrt{a}\sin(x) + \frac{\sqrt{a}}{\sin(x)}}{a\mathrm{sgn}(\sin(x))}$$

input
```
integrate(cot(x)^3/(a+a*cot(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-(sqrt(a)*sin(x) + sqrt(a)/sin(x))/(a*sgn(sin(x)))
```

3.14.9 Mupad [B] (verification not implemented)

Time = 13.10 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.61

$$\int \frac{\cot^3(x)}{\sqrt{a + a\cot^2(x)}}\, dx = -\frac{\sin{(x)}^2 + 1}{\sqrt{a}\,\sqrt{\sin{(x)}^2}}$$

input
```
int(cot(x)^3/(a + a*cot(x)^2)^(1/2),x)
```

output
```
-(sin(x)^2 + 1)/(a^(1/2)*(sin(x)^2)^(1/2))
```

3.15 $\int \frac{\cot^2(x)}{\sqrt{a+a\cot^2(x)}}\,dx$

3.15.1 Optimal result

Integrand size = 17, antiderivative size = 31

$$\int \frac{\cot^2(x)}{\sqrt{a+a\cot^2(x)}}\,dx=\frac{\cot(x)}{\sqrt{a\csc^2(x)}}-\frac{\operatorname{arctanh}(\cos(x))\csc(x)}{\sqrt{a\csc^2(x)}}$$

output
```
cot(x)/(a*csc(x)^2)^(1/2)-arctanh(cos(x))*csc(x)/(a*csc(x)^2)^(1/2)
```

3.15.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{\cot^2(x)}{\sqrt{a+a\cot^2(x)}}\,dx=\frac{\csc(x)\left(\cos(x)-\log\left(\cos\left(\frac{x}{2}\right)\right)+\log\left(\sin\left(\frac{x}{2}\right)\right)\right)}{\sqrt{a\csc^2(x)}}$$

input
```
Integrate[Cot[x]^2/Sqrt[a + a*Cot[x]^2],x]
```

output
```
(Csc[x]*(Cos[x] - Log[Cos[x/2]] + Log[Sin[x/2]]))/Sqrt[a*Csc[x]^2]
```

3.15.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {3042, 4140, 3042, 4613, 3042, 25, 3072, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(x)}{\sqrt{a\cot^2(x)+a}}dx$$

$$\Big\downarrow \textbf{3042}$$

$$\int \frac{\tan\left(x+\frac{\pi}{2}\right)^2}{\sqrt{a\tan\left(x+\frac{\pi}{2}\right)^2+a}}dx$$

$$\Big\downarrow \textbf{4140}$$

$$\int \frac{\cot^2(x)}{\sqrt{a\csc^2(x)}}dx$$

$$\Big\downarrow \textbf{3042}$$

$$\int \frac{\tan\left(x+\frac{\pi}{2}\right)^2}{\sqrt{a\sec\left(x+\frac{\pi}{2}\right)^2}}dx$$

$$\Big\downarrow \textbf{4613}$$

$$\frac{\csc(x)\int \cos(x)\cot(x)dx}{\sqrt{a\csc^2(x)}}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\csc(x)\int -\sin\left(x+\frac{\pi}{2}\right)\tan\left(x+\frac{\pi}{2}\right)dx}{\sqrt{a\csc^2(x)}}$$

$$\Big\downarrow \textbf{25}$$

$$-\frac{\csc(x)\int \sin\left(x+\frac{\pi}{2}\right)\tan\left(x+\frac{\pi}{2}\right)dx}{\sqrt{a\csc^2(x)}}$$

$$\Big\downarrow \textbf{3072}$$

$$-\frac{\csc(x)\int \frac{\cos^2(x)}{1-\cos^2(x)}d\cos(x)}{\sqrt{a\csc^2(x)}}$$

$$\Big\downarrow \textbf{262}$$

$$-\frac{\csc(x)\left(\int \frac{1}{1-\cos^2(x)}d\cos(x)-\cos(x)\right)}{\sqrt{a\csc^2(x)}}$$

$$\Big\downarrow \; 219$$

$$-\frac{\csc(x)(\operatorname{arctanh}(\cos(x))-\cos(x))}{\sqrt{a\csc^2(x)}}$$

input
```
Int[Cot[x]^2/Sqrt[a + a*Cot[x]^2],x]
```

output
```
-(((ArcTanh[Cos[x]] - Cos[x])*Csc[x])/Sqrt[a*Csc[x]^2])
```

3.15.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

rule 4613
```
Int[(u_.)*((b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sec[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sec[e + f*x]^
n)^FracPart[p]/(Sec[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Se
c[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.15.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.23

method	result	si
derivativedivides	$-\frac{\ln\left(\sqrt{a}\,\cot(x)+\sqrt{a+a\cot(x)^{2}}\right)}{\sqrt{a}}+\frac{\cot(x)}{\sqrt{a+a\cot(x)^{2}}}$	38
default	$-\frac{\ln\left(\sqrt{a}\,\cot(x)+\sqrt{a+a\cot(x)^{2}}\right)}{\sqrt{a}}+\frac{\cot(x)}{\sqrt{a+a\cot(x)^{2}}}$	38
risch	$\frac{i\,e^{2ix}}{2\sqrt{-\frac{a\,e^{2ix}}{\left(e^{2ix}-1\right)^{2}}}\,\left(e^{2ix}-1\right)}+\frac{i}{2\left(e^{2ix}-1\right)\sqrt{-\frac{a\,e^{2ix}}{\left(e^{2ix}-1\right)^{2}}}}-\frac{i\,e^{ix}\ln\left(e^{ix}+1\right)}{\sqrt{-\frac{a\,e^{2ix}}{\left(e^{2ix}-1\right)^{2}}}\,\left(e^{2ix}-1\right)}+\frac{i\,e^{ix}\ln\left(e^{ix}-1\right)}{\sqrt{-\frac{a\,e^{2ix}}{\left(e^{2ix}-1\right)^{2}}}\,\left(e^{2ix}-1\right)}$	15

input
```
int(cot(x)^2/(a+a*cot(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-ln(a^(1/2)*cot(x)+(a+a*cot(x)^2)^(1/2))/a^(1/2)+cot(x)/(a+a*cot(x)^2)^(1/
2)
```

3.15.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(27) = 54$.

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.48

$$\int \frac{\cot^2(x)}{\sqrt{a+a\cot^2(x)}}\,dx = \frac{\sqrt{2}\sqrt{-\frac{a}{\cos(2x)-1}}\sin(2x)+\sqrt{a}\log\left(\frac{2\sqrt{2}\sqrt{a}\sqrt{-\frac{a}{\cos(2x)-1}}\sin(2x)-a\cos(2x)-3a}{\cos(2x)-1}\right)}{2a}$$

input
```
integrate(cot(x)^2/(a+a*cot(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(sqrt(2)*sqrt(-a/(cos(2*x) - 1))*sin(2*x) + sqrt(a)*log((2*sqrt(2)*sqr
t(a)*sqrt(-a/(cos(2*x) - 1))*sin(2*x) - a*cos(2*x) - 3*a)/(cos(2*x) - 1)))
/a
```

3.15.6 Sympy [F]

$$\int \frac{\cot^2(x)}{\sqrt{a+a\cot^2(x)}}\,dx = \int \frac{\cot^2(x)}{\sqrt{a(\cot^2(x)+1)}}\,dx$$

input
```
integrate(cot(x)**2/(a+a*cot(x)**2)**(1/2),x)
```

output
```
Integral(cot(x)**2/sqrt(a*(cot(x)**2 + 1)), x)
```

3.15.7 Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{\cot^2(x)}{\sqrt{a+a\cot^2(x)}}\,dx = -\frac{\sqrt{-a}(\arctan(\sin(x),\cos(x)+1)-\arctan(\sin(x),\cos(x)-1))}{a}$$

input
```
integrate(cot(x)^2/(a+a*cot(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(-a)*(arctan2(sin(x), cos(x) + 1) - arctan2(sin(x), cos(x) - 1))/a
```

3.15.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.58

$$\int \frac{\cot^2(x)}{\sqrt{a+a\cot^2(x)}}\,dx = \frac{1}{2}\sqrt{a}\left(\frac{2\,\cos(x)}{a\mathrm{sgn}(\sin(x))} - \frac{\log(\cos(x)+1)}{a\mathrm{sgn}(\sin(x))} + \frac{\log(-\cos(x)+1)}{a\mathrm{sgn}(\sin(x))}\right)$$

input
```
integrate(cot(x)^2/(a+a*cot(x)^2)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(a)*(2*cos(x)/(a*sgn(sin(x))) - log(cos(x) + 1)/(a*sgn(sin(x))) +
log(-cos(x) + 1)/(a*sgn(sin(x))))
```

3.15.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^2(x)}{\sqrt{a+a\cot^2(x)}}\,dx = \int \frac{\cot(x)^2}{\sqrt{a\cot(x)^2+a}}\,dx$$

input
```
int(cot(x)^2/(a + a*cot(x)^2)^(1/2),x)
```

output
```
int(cot(x)^2/(a + a*cot(x)^2)^(1/2), x)
```

3.16 $\int \frac{\cot(x)}{\sqrt{a+a\cot^2(x)}}\,dx$

3.16.1 Optimal result

Integrand size = 15, antiderivative size = 10

$$\int \frac{\cot(x)}{\sqrt{a+a\cot^2(x)}}\,dx=\frac{1}{\sqrt{a\csc^2(x)}}$$

output
```
1/(a*csc(x)^2)^(1/2)
```

3.16.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\cot(x)}{\sqrt{a+a\cot^2(x)}}\,dx=\frac{1}{\sqrt{a\csc^2(x)}}$$

input
```
Integrate[Cot[x]/Sqrt[a + a*Cot[x]^2],x]
```

output
```
1/Sqrt[a*Csc[x]^2]
```

3.16.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {3042, 25, 4140, 25, 3042, 25, 4612, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(x)}{\sqrt{a\cot^2(x)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\tan\left(x+\frac{\pi}{2}\right)}{\sqrt{a\tan\left(x+\frac{\pi}{2}\right)^2+a}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\tan\left(x+\frac{\pi}{2}\right)}{\sqrt{a\tan\left(x+\frac{\pi}{2}\right)^2+a}}dx$$

$$\Big\downarrow \text{4140}$$

$$-\int -\frac{\cot(x)}{\sqrt{a\csc^2(x)}}dx$$

$$\Big\downarrow \text{25}$$

$$\int \frac{\cot(x)}{\sqrt{a\csc^2(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\tan\left(x+\frac{\pi}{2}\right)}{\sqrt{a\sec\left(x+\frac{\pi}{2}\right)^2}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\tan\left(x+\frac{\pi}{2}\right)}{\sqrt{a\sec\left(x+\frac{\pi}{2}\right)^2}}dx$$

$$\Big\downarrow \text{4612}$$

$$-\frac{1}{2}a\int \frac{1}{\left(a\csc^2(x)\right)^{3/2}}d\csc^2(x)$$

$$\Big\downarrow \text{17}$$

$$\frac{1}{\sqrt{a \csc^2(x)}}$$

input
```
Int[Cot[x]/Sqrt[a + a*Cot[x]^2],x]
```

output
```
1/Sqrt[a*Csc[x]^2]
```

3.16.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

rule 4612
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.),
x_Symbol] :> Simp[b/(2*f)   Subst[Int[(-1 + x)^((m - 1)/2)*(b*x)^(p - 1), x
], x, Sec[e + f*x]^2], x] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p] && Int
egerQ[(m - 1)/2]
```

3.16.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
derivativedivides	$\frac{1}{\sqrt{a+a\cot(x)^2}}$	11
default	$\frac{1}{\sqrt{a+a\cot(x)^2}}$	11
risch	$\frac{e^{2ix}}{2\sqrt{-\frac{a e^{2ix}}{(e^{2ix}-1)^2}}\,(e^{2ix}-1)}-\frac{1}{2(e^{2ix}-1)\sqrt{-\frac{a e^{2ix}}{(e^{2ix}-1)^2}}}$	67

input
```
int(cot(x)/(a+a*cot(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(a+a*cot(x)^2)^(1/2)
```

3.16.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(8) = 16$.

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.70

$$\int \frac{\cot(x)}{\sqrt{a+a\cot^2(x)}}\,dx = -\frac{\sqrt{2}\sqrt{-\frac{a}{\cos(2x)-1}}(\cos(2x)-1)}{2a}$$

input
```
integrate(cot(x)/(a+a*cot(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(2)*sqrt(-a/(cos(2*x) - 1))*(cos(2*x) - 1)/a
```

3.16.6 Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\cot(x)}{\sqrt{a+a\cot^2(x)}}\,dx = \frac{1}{\sqrt{a\cot^2(x)+a}}$$

input
```
integrate(cot(x)/(a+a*cot(x)**2)**(1/2),x)
```

output
```
1/sqrt(a*cot(x)**2 + a)
```

3.16.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\cot(x)}{\sqrt{a + a\cot^2(x)}}\, dx = \frac{1}{\sqrt{\frac{a}{\sin(x)^2}}}$$

input
```
integrate(cot(x)/(a+a*cot(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/sqrt(a/sin(x)^2)
```

3.16.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{\cot(x)}{\sqrt{a + a\cot^2(x)}}\, dx = \frac{\sin(x)}{\sqrt{a}\,\mathrm{sgn}(\sin(x))}$$

input
```
integrate(cot(x)/(a+a*cot(x)^2)^(1/2),x, algorithm="giac")
```

output
```
sin(x)/(sqrt(a)*sgn(sin(x)))
```

3.16.9 Mupad [B] (verification not implemented)

Time = 12.98 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\cot(x)}{\sqrt{a + a\cot^2(x)}}\, dx = \frac{\sqrt{\sin(x)^2}}{\sqrt{a}}$$

input
```
int(cot(x)/(a + a*cot(x)^2)^(1/2),x)
```

output
```
(sin(x)^2)^(1/2)/a^(1/2)
```

3.17 $\int \frac{\tan(x)}{\sqrt{a+a\cot^2(x)}}\,dx$

3.17.1 Optimal result

Integrand size = 15, antiderivative size = 36

$$\int \frac{\tan(x)}{\sqrt{a+a\cot^2(x)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a\csc^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{1}{\sqrt{a\csc^2(x)}}$$

output
```
arctanh((a*csc(x)^2)^(1/2)/a^(1/2))/a^(1/2)-1/(a*csc(x)^2)^(1/2)
```

3.17.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.53

$$\int \frac{\tan(x)}{\sqrt{a+a\cot^2(x)}}\,dx = \frac{-1+\operatorname{arctanh}(\sin(x))\csc(x)}{\sqrt{a\csc^2(x)}}$$

input
```
Integrate[Tan[x]/Sqrt[a + a*Cot[x]^2],x]
```

output
```
(-1 + ArcTanh[Sin[x]]*Csc[x])/Sqrt[a*Csc[x]^2]
```

3.17.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.25, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.733, Rules used = {3042, 25, 4140, 25, 3042, 25, 4612, 25, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(x)}{\sqrt{a\cot^2(x)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\tan\left(x+\frac{\pi}{2}\right)\sqrt{a\tan\left(x+\frac{\pi}{2}\right)^2+a}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\tan\left(x+\frac{\pi}{2}\right)\sqrt{a\tan\left(x+\frac{\pi}{2}\right)^2+a}}dx$$

$$\Big\downarrow \text{4140}$$

$$-\int -\frac{\tan(x)}{\sqrt{a\csc^2(x)}}dx$$

$$\Big\downarrow \text{25}$$

$$\int \frac{\tan(x)}{\sqrt{a\csc^2(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\tan\left(x+\frac{\pi}{2}\right)\sqrt{a\sec\left(x+\frac{\pi}{2}\right)^2}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\sqrt{a\sec\left(x+\frac{\pi}{2}\right)^2}\tan\left(x+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4612}$$

$$-\frac{1}{2}a\int -\frac{1}{(a\csc^2(x))^{3/2}(1-\csc^2(x))}d\csc^2(x)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}a\int \frac{1}{\left(a\csc^2(x)\right)^{3/2}\left(1-\csc^2(x)\right)}d\csc^2(x)$$

$$\Big\downarrow \ 61$$

$$-\frac{1}{2}a\left(\frac{2}{a\sqrt{a\csc^2(x)}}-\frac{\int \frac{1}{\sqrt{a\csc^2(x)}(1-\csc^2(x))}d\csc^2(x)}{a}\right)$$

$$\Big\downarrow \ 73$$

$$-\frac{1}{2}a\left(\frac{2}{a\sqrt{a\csc^2(x)}}-\frac{2\int \frac{1}{1-\frac{\csc^4(x)}{a}}d\sqrt{a\csc^2(x)}}{a^2}\right)$$

$$\Big\downarrow \ 219$$

$$-\frac{1}{2}a\left(\frac{2}{a\sqrt{a\csc^2(x)}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a\csc^2(x)}}{\sqrt{a}}\right)}{a^{3/2}}\right)$$

input
```
Int[Tan[x]/Sqrt[a + a*Cot[x]^2],x]
```

output
```
-1/2*(a*((-2*ArcTanh[Sqrt[a*Csc[x]^2]/Sqrt[a]])/a^(3/2) + 2/(a*Sqrt[a*Csc[
x]^2])))
```

3.17.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

rule 4612
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.),
x_Symbol] :> Simp[b/(2*f)   Subst[Int[(-1 + x)^((m - 1)/2)*(b*x)^(p - 1), x
], x, Sec[e + f*x]^2], x] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p] && Int
egerQ[(m - 1)/2]
```

3.17.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

method	result	size
default	$-\frac{\sqrt{4}\,(\sin(x)+\ln(-\cot(x)+\csc(x)-1)-\ln(-\cot(x)+\csc(x)+1))\csc(x)}{2\sqrt{a\csc(x)^{2}}}$	39
risch	$-\frac{{\mathrm e}^{2 i x}}{2\sqrt{-\frac{a\,{\mathrm e}^{2 i x}}{\left({\mathrm e}^{2 i x}-1\right)^{2}}}\,\left({\mathrm e}^{2 i x}-1\right)}+\frac{1}{2\left({\mathrm e}^{2 i x}-1\right)\sqrt{-\frac{a\,{\mathrm e}^{2 i x}}{\left({\mathrm e}^{2 i x}-1\right)^{2}}}}-\frac{i {\mathrm e}^{i x} \ln\left({\mathrm e}^{i x}-i\right)}{\sqrt{-\frac{a\,{\mathrm e}^{2 i x}}{\left({\mathrm e}^{2 i x}-1\right)^{2}}}\,\left({\mathrm e}^{2 i x}-1\right)}+\frac{i {\mathrm e}^{i x} \ln\left({\mathrm e}^{i x}+i\right)}{\sqrt{-\frac{a\,{\mathrm e}^{2 i x}}{\left({\mathrm e}^{2 i x}-1\right)^{2}}}\,\left({\mathrm e}^{2 i x}-1\right)}$	157

input
```
int(tan(x)/(a+a*cot(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*4^(1/2)*(sin(x)+ln(-cot(x)+csc(x)-1)-ln(-cot(x)+csc(x)+1))/(a*csc(x)^
2)^(1/2)*csc(x)
```

3.17.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(28) = 56$.

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.17

$$\int \frac{\tan(x)}{\sqrt{a+a\cot^2(x)}}\,dx = \frac{\left(\tan(x)^2+1\right)\sqrt{a}\log\left(2\,a\tan(x)^2+2\sqrt{a}\sqrt{\frac{a\tan(x)^2+a}{\tan(x)^2}}\tan(x)^2+a\right)-2\sqrt{\frac{a\tan(x)^2+a}{\tan(x)^2}}\tan(x)^2}{2\left(a\tan(x)^2+a\right)}$$

input
```
integrate(tan(x)/(a+a*cot(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*((tan(x)^2 + 1)*sqrt(a)*log(2*a*tan(x)^2 + 2*sqrt(a)*sqrt((a*tan(x)^2
+ a)/tan(x)^2)*tan(x)^2 + a) - 2*sqrt((a*tan(x)^2 + a)/tan(x)^2)*tan(x)^2)
/(a*tan(x)^2 + a)
```

3.17.6 Sympy [F]

$$\int \frac{\tan(x)}{\sqrt{a+a\cot^2(x)}}\,dx = \int \frac{\tan(x)}{\sqrt{a\left(\cot^2(x)+1\right)}}\,dx$$

input
```
integrate(tan(x)/(a+a*cot(x)**2)**(1/2),x)
```

output
```
Integral(tan(x)/sqrt(a*(cot(x)**2 + 1)), x)
```

3.17.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.44

$$\int \frac{\tan(x)}{\sqrt{a+a\cot^2(x)}}\,dx = -\frac{1}{2}\,a\left(\frac{\log\left(-\frac{\sqrt{a}-\sqrt{\frac{a}{\sin(x)^2}}}{\sqrt{a}+\sqrt{\frac{a}{\sin(x)^2}}}\right)}{a^{\frac{3}{2}}}+\frac{2}{a\sqrt{\frac{a}{\sin(x)^2}}}\right)$$

input
```
integrate(tan(x)/(a+a*cot(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*a*(log(-(sqrt(a) - sqrt(a/sin(x)^2))/(sqrt(a) + sqrt(a/sin(x)^2)))/a^
(3/2) + 2/(a*sqrt(a/sin(x)^2)))
```

3.17.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.33

$$\int \frac{\tan(x)}{\sqrt{a+a\cot^2(x)}}\,dx = -\frac{\sin(x)}{\sqrt{a}\,\mathrm{sgn}(\sin(x))}$$

input
```
integrate(tan(x)/(a+a*cot(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-sin(x)/(sqrt(a)*sgn(sin(x)))
```

3.17.9 Mupad [B] (verification not implemented)

Time = 13.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.56

$$\int \frac{\tan(x)}{\sqrt{a+a\cot^2(x)}}\,dx = \frac{\mathrm{atanh}\left(\sqrt{\frac{1}{\sin(x)^2}}\right)-\sqrt{\sin(x)^2}}{\sqrt{a}}$$

input
```
int(tan(x)/(a + a*cot(x)^2)^(1/2),x)
```

output
```
(atanh((1/sin(x)^2)^(1/2)) - (sin(x)^2)^(1/2))/a^(1/2)
```

3.18 $\int \frac{\tan^2(x)}{\sqrt{a+a\cot^2(x)}}\,dx$

3.18.1 Optimal result

Integrand size = 17, antiderivative size = 29

$$\int \frac{\tan^2(x)}{\sqrt{a+a\cot^2(x)}}\,dx=\frac{\cot(x)}{\sqrt{a\csc^2(x)}}+\frac{\csc(x)\sec(x)}{\sqrt{a\csc^2(x)}}$$

output
```
cot(x)/(a*csc(x)^2)^(1/2)+csc(x)*sec(x)/(a*csc(x)^2)^(1/2)
```

3.18.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

$$\int \frac{\tan^2(x)}{\sqrt{a+a\cot^2(x)}}\,dx=\frac{\cot(x)+\csc(x)\sec(x)}{\sqrt{a\csc^2(x)}}$$

input
```
Integrate[Tan[x]^2/Sqrt[a + a*Cot[x]^2],x]
```

output
```
(Cot[x] + Csc[x]*Sec[x])/Sqrt[a*Csc[x]^2]
```

3.18.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {3042, 4140, 3042, 4613, 3042, 3070, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(x)}{\sqrt{a\cot^2(x)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan\left(x+\frac{\pi}{2}\right)^2\sqrt{a\tan\left(x+\frac{\pi}{2}\right)^2+a}}dx$$

$$\Big\downarrow \text{4140}$$

$$\int \frac{\tan^2(x)}{\sqrt{a\csc^2(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan\left(x+\frac{\pi}{2}\right)^2\sqrt{a\sec\left(x+\frac{\pi}{2}\right)^2}}dx$$

$$\Big\downarrow \text{4613}$$

$$\frac{\csc(x)\int \sin(x)\tan^2(x)dx}{\sqrt{a\csc^2(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\csc(x)\int \sin(x)\tan(x)^2dx}{\sqrt{a\csc^2(x)}}$$

$$\Big\downarrow \text{3070}$$

$$-\frac{\csc(x)\int \left(1-\cos^2(x)\right)\sec^2(x)d\cos(x)}{\sqrt{a\csc^2(x)}}$$

$$\Big\downarrow \text{244}$$

$$-\frac{\csc(x)\int \left(\sec^2(x)-1\right)d\cos(x)}{\sqrt{a\csc^2(x)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\csc(x)(-\cos(x)-\sec(x))}{\sqrt{a\csc^2(x)}}$$

input
```
Int[Tan[x]^2/Sqrt[a + a*Cot[x]^2],x]
```

output
```
-((Csc[x]*(-Cos[x] - Sec[x]))/Sqrt[a*Csc[x]^2])
```

3.18.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

rule 4613
```
Int[(u_.)*((b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sec[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sec[e + f*x]^
n)^FracPart[p]/(Sec[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Se
c[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.18.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{\sqrt{4}\,(\cos(x)+1)^2 \sec(x)\csc(x)}{2\sqrt{a\csc(x)^2}}$	24
risch	$\frac{i(\mathrm{e}^{4ix}+6\,\mathrm{e}^{2ix}+1)}{2\sqrt{-\frac{a\,\mathrm{e}^{2ix}}{(\mathrm{e}^{2ix}-1)^2}}\,(\mathrm{e}^{2ix}-1)(\mathrm{e}^{2ix}+1)}$	55

input
```
int(tan(x)^2/(a+a*cot(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*4^(1/2)*(cos(x)+1)^2/(a*csc(x)^2)^(1/2)*sec(x)*csc(x)
```

3.18.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.21

$$\int \frac{\tan^2(x)}{\sqrt{a+a\cot^2(x)}}\,dx = \frac{\left(\tan\left(x\right)^3+2\,\tan\left(x\right)\right)\sqrt{\frac{a\tan(x)^2+a}{\tan(x)^2}}}{a\tan\left(x\right)^2+a}$$

input
```
integrate(tan(x)^2/(a+a*cot(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
(tan(x)^3 + 2*tan(x))*sqrt((a*tan(x)^2 + a)/tan(x)^2)/(a*tan(x)^2 + a)
```

3.18.6 Sympy [F]

$$\int \frac{\tan^2(x)}{\sqrt{a+a\cot^2(x)}}\,dx = \int \frac{\tan^2\left(x\right)}{\sqrt{a\left(\cot^2\left(x\right)+1\right)}}\,dx$$

input
```
integrate(tan(x)**2/(a+a*cot(x)**2)**(1/2),x)
```

output
```
Integral(tan(x)**2/sqrt(a*(cot(x)**2 + 1)), x)
```

3.18.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.62

$$\int \frac{\tan^2(x)}{\sqrt{a+a\cot^2(x)}}\,dx = \frac{\tan(x)^2+2}{\sqrt{\tan(x)^2+1}\sqrt{a}}$$

input
```
integrate(tan(x)^2/(a+a*cot(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
(tan(x)^2 + 2)/(sqrt(tan(x)^2 + 1)*sqrt(a))
```

3.18.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{\tan^2(x)}{\sqrt{a+a\cot^2(x)}}\,dx = -\frac{2\,\mathrm{sgn}(\sin(x))}{\sqrt{a}} + \frac{\frac{1}{\cos(x)}+\cos(x)}{\sqrt{a}\mathrm{sgn}(\sin(x))}$$

input
```
integrate(tan(x)^2/(a+a*cot(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-2*sgn(sin(x))/sqrt(a) + (1/cos(x) + cos(x))/(sqrt(a)*sgn(sin(x)))
```

3.18.9 Mupad [B] (verification not implemented)

Time = 13.35 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.17

$$\int \frac{\tan^2(x)}{\sqrt{a+a\cot^2(x)}}\,dx = \frac{\tan(x)^3\sqrt{\frac{1}{\tan(x)^2}}+2\tan(x)\sqrt{\frac{1}{\tan(x)^2}}}{\sqrt{a}\sqrt{\tan(x)^2+1}}$$

input
```
int(tan(x)^2/(a + a*cot(x)^2)^(1/2),x)
```

output
```
(tan(x)^3*(1/tan(x)^2)^(1/2) + 2*tan(x)*(1/tan(x)^2)^(1/2))/(a^(1/2)*(tan(
x)^2 + 1)^(1/2))
```

3.19 $\int \cot^3(x)\sqrt{a+b\cot^2(x)}\,dx$

3.19.1 Optimal result

Integrand size = 17, antiderivative size = 66

$$\int \cot^3(x)\sqrt{a+b\cot^2(x)}\,dx = -\sqrt{a-b}\text{arctanh}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right) + \sqrt{a+b\cot^2(x)} - \frac{(a+b\cot^2(x))^{3/2}}{3b}$$

output
```
-1/3*(a+b*cot(x)^2)^(3/2)/b-arctanh((a+b*cot(x)^2)^(1/2)/(a-b)^(1/2))*(a-b
)^(1/2)+(a+b*cot(x)^2)^(1/2)
```

3.19.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.98

$$\int \cot^3(x)\sqrt{a+b\cot^2(x)}\,dx = -\sqrt{a-b}\text{arctanh}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right) - \frac{\sqrt{a+b\cot^2(x)}(a-3b+b\cot^2(x))}{3b}$$

input
```
Integrate[Cot[x]^3*Sqrt[a + b*Cot[x]^2],x]
```

output
```
-(Sqrt[a - b]*ArcTanh[Sqrt[a + b*Cot[x]^2]/Sqrt[a - b]]) - (Sqrt[a + b*Cot
[x]^2]*(a - 3*b + b*Cot[x]^2))/(3*b)
```

3.19.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {3042, 25, 4153, 25, 354, 90, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(x)\sqrt{a+b\cot^2(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\tan\left(x+\frac{\pi}{2}\right)^3\sqrt{a+b\tan\left(x+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \tan\left(x+\frac{\pi}{2}\right)^3\sqrt{b\tan\left(x+\frac{\pi}{2}\right)^2+a}dx$$

$$\Big\downarrow \text{4153}$$

$$\int -\frac{\cot^3(x)\sqrt{a+b\cot^2(x)}}{\cot^2(x)+1}d\cot(x)$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\cot^3(x)\sqrt{b\cot^2(x)+a}}{\cot^2(x)+1}d\cot(x)$$

$$\Big\downarrow \text{354}$$

$$-\frac{1}{2}\int \frac{\cot^2(x)\sqrt{b\cot^2(x)+a}}{\cot^2(x)+1}d\cot^2(x)$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{2}\left(\int \frac{\sqrt{b\cot^2(x)+a}}{\cot^2(x)+1}d\cot^2(x)-\frac{2\left(a+b\cot^2(x)\right)^{3/2}}{3b}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left((a-b)\int \frac{1}{\left(\cot^2(x)+1\right)\sqrt{b\cot^2(x)+a}}d\cot^2(x)-\frac{2\left(a+b\cot^2(x)\right)^{3/2}}{3b}+2\sqrt{a+b\cot^2(x)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{2(a-b)\int \frac{1}{\frac{\cot^4(x)}{b}-\frac{a}{b}+1}d\sqrt{b\cot^2(x)+a}}{b}-\frac{2\left(a+b\cot^2(x)\right)^{3/2}}{3b}+2\sqrt{a+b\cot^2(x)}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{2}\left(-2\sqrt{a-b}\text{arctanh}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)-\frac{2\left(a+b\cot^2(x)\right)^{3/2}}{3b}+2\sqrt{a+b\cot^2(x)}\right)$$

input
```
Int[Cot[x]^3*Sqrt[a + b*Cot[x]^2],x]
```

output
```
(-2*Sqrt[a - b]*ArcTanh[Sqrt[a + b*Cot[x]^2]/Sqrt[a - b]] + 2*Sqrt[a + b*C
ot[x]^2] - (2*(a + b*Cot[x]^2)^(3/2))/(3*b))/2
```

3.19.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.19.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.27

method	result	size
derivativedivides	$-\frac{\left(a+b\cot\left(x\right)^{2}\right)^{\frac{3}{2}}}{3b}+\sqrt{a+b\cot\left(x\right)^{2}}-\frac{b\arctan\left(\frac{\sqrt{a+b\cot\left(x\right)^{2}}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}+\frac{a\arctan\left(\frac{\sqrt{a+b\cot\left(x\right)^{2}}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}$	84
default	$-\frac{\left(a+b\cot\left(x\right)^{2}\right)^{\frac{3}{2}}}{3b}+\sqrt{a+b\cot\left(x\right)^{2}}-\frac{b\arctan\left(\frac{\sqrt{a+b\cot\left(x\right)^{2}}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}+\frac{a\arctan\left(\frac{\sqrt{a+b\cot\left(x\right)^{2}}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}$	84

input
```
int(cot(x)^3*(a+b*cot(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(a+b*cot(x)^2)^(3/2)/b+(a+b*cot(x)^2)^(1/2)-b/(-a+b)^(1/2)*arctan((a+
b*cot(x)^2)^(1/2)/(-a+b)^(1/2))+a/(-a+b)^(1/2)*arctan((a+b*cot(x)^2)^(1/2)
/(-a+b)^(1/2))
```

3.19.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(54) = 108$.

Time = 0.34 (sec) , antiderivative size = 330, normalized size of antiderivative = 5.00

$$\int \cot^3(x)\sqrt{a+b\cot^2(x)}\,dx$$

$$= \left[\frac{3\,(b\cos(2\,x)-b)\sqrt{a-b}\log\left(-2\,(a^2-2\,ab+b^2)\cos(2\,x)^2-2\,a^2+b^2+2\,((a-b)\cos(2\,x)^2-(2\,a-b)\right.}{12\,(} \right.$$

$$-\frac{3\,(b\cos(2\,x)-b)\sqrt{-a+b}\arctan\left(-\frac{\sqrt{-a+b}\sqrt{\frac{(a-b)\cos(2\,x)-a-b}{\cos(2\,x)-1}}(\cos(2\,x)-1)}{(a-b)\cos(2\,x)-a}\right)+2\,((a-4\,b)\cos(2\,x)-a+2\,b)}{6\,(b\cos(2\,x)-b)}$$

input
```
integrate(cot(x)^3*(a+b*cot(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/12*(3*(b*cos(2*x) - b)*sqrt(a - b)*log(-2*(a^2 - 2*a*b + b^2)*cos(2*x)^
2 - 2*a^2 + b^2 + 2*((a - b)*cos(2*x)^2 - (2*a - b)*cos(2*x) + a)*sqrt(a -
 b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1)) + 4*(a^2 - a*b)*cos(2*
x)) - 4*((a - 4*b)*cos(2*x) - a + 2*b)*sqrt(((a - b)*cos(2*x) - a - b)/(co
s(2*x) - 1)))/(b*cos(2*x) - b), -1/6*(3*(b*cos(2*x) - b)*sqrt(-a + b)*arct
an(-sqrt(-a + b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))*(cos(2*x)
 - 1)/((a - b)*cos(2*x) - a)) + 2*((a - 4*b)*cos(2*x) - a + 2*b)*sqrt(((a
- b)*cos(2*x) - a - b)/(cos(2*x) - 1)))/(b*cos(2*x) - b)]
```

3.19.6 Sympy [F]

$$\int \cot^3(x)\sqrt{a+b\cot^2(x)}\,dx = \int \sqrt{a+b\cot^2(x)}\cot^3(x)\,dx$$

input
```
integrate(cot(x)**3*(a+b*cot(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*cot(x)**2)*cot(x)**3, x)
```

3.19.7 Maxima [F(-2)]

Exception generated.

$$\int \cot^3(x)\sqrt{a+b\cot^2(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cot(x)^3*(a+b*cot(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for m
ore detail
```

3.19.8 Giac [F(-2)]

Exception generated.

$$\int \cot^3(x)\sqrt{a+b\cot^2(x)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(x)^3*(a+b*cot(x)^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to convert to real sageVARb
Error: Bad Argument ValueUnable to convert to real sageVARb Error: Bad Arg
ument Val
```

3.19.9 Mupad [B] (verification not implemented)

Time = 15.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00

$$\int \cot^3(x)\sqrt{a+b\cot^2(x)}\,dx = \sqrt{b\cot(x)^2+a} - \frac{\left(b\cot(x)^2+a\right)^{3/2}}{3\,b} + 2\,\mathrm{atan}\left(\frac{2\sqrt{b\cot(x)^2+a}\,\sqrt{\frac{b}{4}-\frac{a}{4}}}{a-b}\right)\sqrt{\frac{b}{4}-\frac{a}{4}}$$

input
```
int(cot(x)^3*(a + b*cot(x)^2)^(1/2),x)
```

output
```
(a + b*cot(x)^2)^(1/2) - (a + b*cot(x)^2)^(3/2)/(3*b) + 2*atan((2*(a + b*c
ot(x)^2)^(1/2)*(b/4 - a/4)^(1/2))/(a - b))*(b/4 - a/4)^(1/2)
```

3.19. $\int \cot^3(x)\sqrt{a+b\cot^2(x)}\,dx$

3.20 $\int \cot(x)\sqrt{a+b\cot^2(x)}\,dx$

3.20.1 Optimal result

Integrand size = 15, antiderivative size = 48

$$\int \cot(x)\sqrt{a+b\cot^2(x)}\,dx = \sqrt{a-b}\,\text{arctanh}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)-\sqrt{a+b\cot^2(x)}$$

output
```
arctanh((a+b*cot(x)^2)^(1/2)/(a-b)^(1/2))*(a-b)^(1/2)-(a+b*cot(x)^2)^(1/2)
```

3.20.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \cot(x)\sqrt{a+b\cot^2(x)}\,dx = \sqrt{a-b}\,\text{arctanh}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)-\sqrt{a+b\cot^2(x)}$$

input
```
Integrate[Cot[x]*Sqrt[a + b*Cot[x]^2],x]
```

output
```
Sqrt[a - b]*ArcTanh[Sqrt[a + b*Cot[x]^2]/Sqrt[a - b]] - Sqrt[a + b*Cot[x]^
2]
```

3.20.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {3042, 25, 4153, 25, 353, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(x)\sqrt{a+b\cot^2(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\tan\left(x+\frac{\pi}{2}\right)\sqrt{a+b\tan\left(x+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \tan\left(x+\frac{\pi}{2}\right)\sqrt{b\tan\left(x+\frac{\pi}{2}\right)^2+a}dx$$

$$\Big\downarrow \text{4153}$$

$$\int -\frac{\cot(x)\sqrt{a+b\cot^2(x)}}{\cot^2(x)+1}d\cot(x)$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\cot(x)\sqrt{b\cot^2(x)+a}}{\cot^2(x)+1}d\cot(x)$$

$$\Big\downarrow \text{353}$$

$$-\frac{1}{2}\int \frac{\sqrt{b\cot^2(x)+a}}{\cot^2(x)+1}d\cot^2(x)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(-\left((a-b)\int \frac{1}{\left(\cot^2(x)+1\right)\sqrt{b\cot^2(x)+a}}d\cot^2(x)\right)-2\sqrt{a+b\cot^2(x)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(-\frac{2(a-b)\int \frac{1}{\frac{\cot^4(x)}{b}-\frac{a}{b}+1}d\sqrt{b\cot^2(x)+a}}{b}-2\sqrt{a+b\cot^2(x)}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(2\sqrt{a-b}\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)-2\sqrt{a+b\cot^2(x)}\right)$$

input
```
Int[Cot[x]*Sqrt[a + b*Cot[x]^2],x]
```

output
```
(2*Sqrt[a - b]*ArcTanh[Sqrt[a + b*Cot[x]^2]/Sqrt[a - b]] - 2*Sqrt[a + b*Co
t[x]^2])/2
```

3.20.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.20.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.48

method	result	size
derivativedivides	$-\sqrt{a+b\cot\left(x\right)^{2}}+\frac{b\arctan\left(\frac{\sqrt{a+b\cot(x)^{2}}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}-\frac{a\arctan\left(\frac{\sqrt{a+b\cot(x)^{2}}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}$	71
default	$-\sqrt{a+b\cot\left(x\right)^{2}}+\frac{b\arctan\left(\frac{\sqrt{a+b\cot(x)^{2}}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}-\frac{a\arctan\left(\frac{\sqrt{a+b\cot(x)^{2}}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}$	71

input
```
int(cot(x)*(a+b*cot(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(a+b*cot(x)^2)^(1/2)+b/(-a+b)^(1/2)*arctan((a+b*cot(x)^2)^(1/2)/(-a+b)^(1
/2))-a/(-a+b)^(1/2)*arctan((a+b*cot(x)^2)^(1/2)/(-a+b)^(1/2))
```

3.20.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(40) = 80.

Time = 0.34 (sec) , antiderivative size = 248, normalized size of antiderivative = 5.17

$$\int \cot(x)\sqrt{a+b\cot^2(x)}\,dx = \left[\frac{1}{4}\sqrt{a-b}\log\left(-2\left(a^2-2ab+b^2\right)\cos(2x)^2-2a^2+b^2 -2\left((a-b)\cos(2x)^2-(2a-b)\cos(2x)+a\right)\sqrt{a-b}\sqrt{\frac{(a-b)\cos(2x)-a-b}{\cos(2x)-1}} +4\left(a^2-ab\right)\cos(2x)\right) -\sqrt{\frac{(a-b)\cos(2x)-a-b}{\cos(2x)-1}}, \frac{1}{2}\sqrt{-a+b}\arctan\left(-\frac{\sqrt{-a+b}\sqrt{\frac{(a-b)\cos(2x)-a-b}{\cos(2x)-1}}(\cos(2x)-1)}{(a-b)\cos(2x)-a}\right) -\sqrt{\frac{(a-b)\cos(2x)-a-b}{\cos(2x)-1}}\right]$$

input
```
integrate(cot(x)*(a+b*cot(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*sqrt(a - b)*log(-2*(a^2 - 2*a*b + b^2)*cos(2*x)^2 - 2*a^2 + b^2 - 2*(
(a - b)*cos(2*x)^2 - (2*a - b)*cos(2*x) + a)*sqrt(a - b)*sqrt(((a - b)*cos
(2*x) - a - b)/(cos(2*x) - 1)) + 4*(a^2 - a*b)*cos(2*x)) - sqrt(((a - b)*c
os(2*x) - a - b)/(cos(2*x) - 1)), 1/2*sqrt(-a + b)*arctan(-sqrt(-a + b)*sq
rt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))*(cos(2*x) - 1)/((a - b)*cos(
2*x) - a)) - sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))]
```

3.20.6 Sympy [F]

$$\int \cot(x)\sqrt{a+b\cot^2(x)}\,dx = \int \sqrt{a+b\cot^2(x)}\cot(x)\,dx$$

input
```
integrate(cot(x)*(a+b*cot(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*cot(x)**2)*cot(x), x)
```

3.20.7 Maxima [F(-2)]

Exception generated.

$$\int \cot(x)\sqrt{a+b\cot^2(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cot(x)*(a+b*cot(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for m
ore detail
```

3.20.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(40) = 80$.

Time = 0.34 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.98

$$\int \cot(x)\sqrt{a+b\cot^2(x)}\,dx = -\frac{1}{2}\left(\sqrt{a-b}\log\left(\left(\sqrt{a-b}\sin(x)-\sqrt{a\sin(x)^2-b\sin(x)^2+b}\right)^2\right)-\frac{4\sqrt{a-b}b}{\left(\sqrt{a-b}\sin(x)-\sqrt{a\sin(x)^2-\cdots}\right.}\right.$$

input
```
integrate(cot(x)*(a+b*cot(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*(sqrt(a - b)*log((sqrt(a - b)*sin(x) - sqrt(a*sin(x)^2 - b*sin(x)^2 +
 b))^2) - 4*sqrt(a - b)*b/((sqrt(a - b)*sin(x) - sqrt(a*sin(x)^2 - b*sin(x
)^2 + b))^2 - b))*sgn(sin(x))
```

3.20.9 Mupad [B] (verification not implemented)

Time = 13.96 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10

$$\int \cot(x)\sqrt{a+b\cot^2(x)}\,dx$$
$$= -\sqrt{b\cot(x)^2+a} - 2\,\mathrm{atan}\left(\frac{2\sqrt{b\cot(x)^2+a}\,\sqrt{\frac{b}{4}-\frac{a}{4}}}{a-b}\right)\sqrt{\frac{b}{4}-\frac{a}{4}}$$

input
```
int(cot(x)*(a + b*cot(x)^2)^(1/2),x)
```

output
```
- (a + b*cot(x)^2)^(1/2) - 2*atan((2*(a + b*cot(x)^2)^(1/2)*(b/4 - a/4)^(1
/2))/(a - b))*(b/4 - a/4)^(1/2)
```

3.21 $\int \sqrt{a+b\cot^2(x)}\tan(x)\,dx$

3.21.1 Optimal result

Integrand size = 15, antiderivative size = 60

$$\int \sqrt{a+b\cot^2(x)}\tan(x)\,dx = \sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a}}\right) - \sqrt{a-b}\text{arctanh}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)$$

output
```
arctanh((a+b*cot(x)^2)^(1/2)/a^(1/2))*a^(1/2)-arctanh((a+b*cot(x)^2)^(1/2)
/(a-b)^(1/2))*(a-b)^(1/2)
```

3.21.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int \sqrt{a+b\cot^2(x)}\tan(x)\,dx = \sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a}}\right) - \sqrt{a-b}\text{arctanh}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)$$

input
```
Integrate[Sqrt[a + b*Cot[x]^2]*Tan[x],x]
```

output
```
Sqrt[a]*ArcTanh[Sqrt[a + b*Cot[x]^2]/Sqrt[a]] - Sqrt[a - b]*ArcTanh[Sqrt[a
 + b*Cot[x]^2]/Sqrt[a - b]]
```

3.21.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {3042, 25, 4153, 25, 354, 94, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(x)\sqrt{a+b\cot^2(x)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{\sqrt{a+b\tan\left(x+\frac{\pi}{2}\right)^2}}{\tan\left(x+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 25$$

$$-\int \frac{\sqrt{b\tan\left(x+\frac{\pi}{2}\right)^2+a}}{\tan\left(x+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 4153$$

$$\int -\frac{\tan(x)\sqrt{a+b\cot^2(x)}}{\cot^2(x)+1}d\cot(x)$$

$$\Big\downarrow \; 25$$

$$-\int \frac{\sqrt{b\cot^2(x)+a}\tan(x)}{\cot^2(x)+1}d\cot(x)$$

$$\Big\downarrow \; 354$$

$$-\frac{1}{2}\int \frac{\sqrt{b\cot^2(x)+a}\tan(x)}{\cot^2(x)+1}d\cot^2(x)$$

$$\Big\downarrow \; 94$$

$$\frac{1}{2}\left((a-b)\int \frac{1}{\left(\cot^2(x)+1\right)\sqrt{b\cot^2(x)+a}}d\cot^2(x)-a\int \frac{\tan(x)}{\sqrt{b\cot^2(x)+a}}d\cot^2(x)\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{2}\left(\frac{2(a-b)\int \frac{1}{\frac{\cot^4(x)}{b}-\frac{a}{b}+1}d\sqrt{b\cot^2(x)+a}}{b}-\frac{2a\int \frac{1}{\frac{\cot^4(x)}{b}-\frac{a}{b}}d\sqrt{b\cot^2(x)+a}}{b}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{2}\left(2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a}}\right)-2\sqrt{a-b}\text{arctanh}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)\right)$$

input
```
Int[Sqrt[a + b*Cot[x]^2]*Tan[x],x]
```

output
```
(2*Sqrt[a]*ArcTanh[Sqrt[a + b*Cot[x]^2]/Sqrt[a]] - 2*Sqrt[a - b]*ArcTanh[S
qrt[a + b*Cot[x]^2]/Sqrt[a - b]])/2
```

3.21.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 94
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(a + b*x), x],
x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(c + d*x), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.21.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(48) = 96$.

Time = 7.15 (sec) , antiderivative size = 194, normalized size of antiderivative = 3.23

method	result
default	$\frac{\sqrt{4}\,\sin(x)\left(\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{a\cos(x)^2-\cos(x)^2 b-a}{(\cos(x)+1)^2}}\,(\cot(x)+\csc(x))}{\sqrt{a}}\right)\sqrt{-a+b}+\arctan\left(\frac{\sqrt{-\frac{a\cos(x)^2-\cos(x)^2 b-a}{(\cos(x)+1)^2}}\,(\cot(x)+\csc(x))}{\sqrt{-a+b}}\right)a-\text{ar}}{2\sqrt{-a+b}\,(\cos(x)+1)\sqrt{-\frac{a\cos(x)^2-\cos(x)^2 b-a}{(\cos(x)+1)^2}}}$

input
```
int((a+b*cot(x)^2)^(1/2)*tan(x),x,method=_RETURNVERBOSE)
```

output
```
1/2*4^(1/2)/(-a+b)^(1/2)*sin(x)*(a^(1/2)*arctanh(1/a^(1/2)*(-(a*cos(x)^2-c
os(x)^2*b-a)/(cos(x)+1)^2)^(1/2)*(cot(x)+csc(x)))*(-a+b)^(1/2)+arctan(1/(-
a+b)^(1/2)*(-(a*cos(x)^2-cos(x)^2*b-a)/(cos(x)+1)^2)^(1/2)*(cot(x)+csc(x))
)*a-arctan(1/(-a+b)^(1/2)*(-(a*cos(x)^2-cos(x)^2*b-a)/(cos(x)+1)^2)^(1/2)*
(cot(x)+csc(x)))*b)*(a+b*cot(x)^2)^(1/2)/(cos(x)+1)/(-(a*cos(x)^2-cos(x)^2
*b-a)/(cos(x)+1)^2)^(1/2)
```

3.21.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 351, normalized size of antiderivative = 5.85

$$\int \sqrt{a+b\cot^2(x)}\tan(x)\,dx$$
$$= \left[\frac{1}{2}\sqrt{a}\log\left(2\,a\tan(x)^2+2\,\sqrt{a}\sqrt{\frac{a\tan(x)^2+b}{\tan(x)^2}}\tan(x)^2+b\right)+\frac{1}{2}\sqrt{a-b}\log\left(\frac{(2\,a-b)\tan(x)^2-2\,\sqrt{a-b}\sqrt{\frac{a\tan(x)^2+b}{\tan(x)^2}}\tan(x)^2+b}{\tan(x)^2+1}\right),\right.$$
$$-\sqrt{-a+b}\arctan\left(-\frac{\sqrt{-a+b}\sqrt{\frac{a\tan(x)^2+b}{\tan(x)^2}}}{a-b}\right)+\frac{1}{2}\sqrt{a}\log\left(2\,a\tan(x)^2+2\,\sqrt{a}\sqrt{\frac{a\tan(x)^2+b}{\tan(x)^2}}\tan(x)^2+b\right),$$
$$-\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{a\tan(x)^2+b}{\tan(x)^2}}}{a}\right)+\frac{1}{2}\sqrt{a-b}\log\left(\frac{(2\,a-b)\tan(x)^2-2\,\sqrt{a-b}\sqrt{\frac{a\tan(x)^2+b}{\tan(x)^2}}\tan(x)^2+b}{\tan(x)^2+1}\right),$$
$$\left.-\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{a\tan(x)^2+b}{\tan(x)^2}}}{a}\right)-\sqrt{-a+b}\arctan\left(-\frac{\sqrt{-a+b}\sqrt{\frac{a\tan(x)^2+b}{\tan(x)^2}}}{a-b}\right)\right]$$

input
```
integrate((a+b*cot(x)^2)^(1/2)*tan(x),x, algorithm="fricas")
```

output
```
[1/2*sqrt(a)*log(2*a*tan(x)^2 + 2*sqrt(a)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*
tan(x)^2 + b) + 1/2*sqrt(a - b)*log(((2*a - b)*tan(x)^2 - 2*sqrt(a - b)*sq
rt((a*tan(x)^2 + b)/tan(x)^2)*tan(x)^2 + b)/(tan(x)^2 + 1)), -sqrt(-a + b)
*arctan(-sqrt(-a + b)*sqrt((a*tan(x)^2 + b)/tan(x)^2)/(a - b)) + 1/2*sqrt(
a)*log(2*a*tan(x)^2 + 2*sqrt(a)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x)^2 +
 b), -sqrt(-a)*arctan(sqrt(-a)*sqrt((a*tan(x)^2 + b)/tan(x)^2)/a) + 1/2*sq
rt(a - b)*log(((2*a - b)*tan(x)^2 - 2*sqrt(a - b)*sqrt((a*tan(x)^2 + b)/ta
n(x)^2)*tan(x)^2 + b)/(tan(x)^2 + 1)), -sqrt(-a)*arctan(sqrt(-a)*sqrt((a*t
an(x)^2 + b)/tan(x)^2)/a) - sqrt(-a + b)*arctan(-sqrt(-a + b)*sqrt((a*tan(
x)^2 + b)/tan(x)^2)/(a - b))]
```

3.21.6 Sympy [F]

$$\int \sqrt{a+b\cot^2(x)}\tan(x)\,dx = \int \sqrt{a+b\cot^2{(x)}}\tan{(x)}\,dx$$

input
```
integrate((a+b*cot(x)**2)**(1/2)*tan(x),x)
```

output
```
Integral(sqrt(a + b*cot(x)**2)*tan(x), x)
```

3.21.7 Maxima [F]

$$\int \sqrt{a+b\cot^2(x)}\tan(x)\,dx = \int \sqrt{b\cot{(x)}^2+a}\tan{(x)}\ dx$$

input
```
integrate((a+b*cot(x)^2)^(1/2)*tan(x),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*cot(x)^2 + a)*tan(x), x)
```

3.21.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(48) = 96.

Time = 0.33 (sec) , antiderivative size = 187, normalized size of antiderivative = 3.12

$$\int \sqrt{a+b\cot^2(x)}\tan(x)\,dx$$
$$= \frac{1}{2}\left(\frac{2\sqrt{a-b}a\arctan\left(\frac{\left(\sqrt{a-b}\sin(x)-\sqrt{a\sin(x)^2-b\sin(x)^2+b}\right)^2-2a+b}{2\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}}+\sqrt{a-b}\log\left(\left(\sqrt{a-b}\sin{(x)}-\sqrt{a\sin\right.\right.$$
$$-\frac{\left(2\sqrt{a-b}a\arctan\left(-\frac{a-b}{\sqrt{-a^2+ab}}\right)+\sqrt{-a^2+ab}\sqrt{a-b}\log{(b)}\right)\mathrm{sgn}(\sin{(x)})}{2\sqrt{-a^2+ab}}$$

input
```
integrate((a+b*cot(x)^2)^(1/2)*tan(x),x, algorithm="giac")
```

output
```
1/2*(2*sqrt(a - b)*a*arctan(1/2*((sqrt(a - b)*sin(x) - sqrt(a*sin(x)^2 - b
*sin(x)^2 + b))^2 - 2*a + b)/sqrt(-a^2 + a*b))/sqrt(-a^2 + a*b) + sqrt(a -
 b)*log((sqrt(a - b)*sin(x) - sqrt(a*sin(x)^2 - b*sin(x)^2 + b))^2))*sgn(s
in(x)) - 1/2*(2*sqrt(a - b)*a*arctan(-(a - b)/sqrt(-a^2 + a*b)) + sqrt(-a^
2 + a*b)*sqrt(a - b)*log(b))*sgn(sin(x))/sqrt(-a^2 + a*b)
```

3.21.9 Mupad [B] (verification not implemented)

Time = 13.78 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.15

$$\int \sqrt{a+b\cot^2(x)}\tan(x)\,dx = \operatorname{atanh}\left(\frac{2\,a\,b^3\,\sqrt{a-b}\,\sqrt{a+\frac{b}{\tan(x)^2}}}{2\,a\,b^4-2\,a^2\,b^3}\right)\,\sqrt{a-b} + \sqrt{a}\,\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{\tan(x)^2}}}{\sqrt{a}}\right)$$

input
```
int(tan(x)*(a + b*cot(x)^2)^(1/2),x)
```

output
```
atanh((2*a*b^3*(a - b)^(1/2)*(a + b/tan(x)^2)^(1/2))/(2*a*b^4 - 2*a^2*b^3)
)*(a - b)^(1/2) + a^(1/2)*atanh((a + b/tan(x)^2)^(1/2)/a^(1/2))
```

3.22 $\int \cot^2(x)\sqrt{a+b\cot^2(x)}\,dx$

3.22.1 Optimal result

Integrand size = 17, antiderivative size = 89

$$\int \cot^2(x)\sqrt{a+b\cot^2(x)}\,dx=\sqrt{a-b}\arctan\left(\frac{\sqrt{a-b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)-\frac{(a-2b)\text{arctanh}\left(\frac{\sqrt{b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{2\sqrt{b}}-\frac{1}{2}\cot(x)\sqrt{a+b\cot^2(x)}$$

output
```
arctan(cot(x)*(a-b)^(1/2)/(a+b*cot(x)^2)^(1/2))*(a-b)^(1/2)-1/2*(a-2*b)*ar
ctanh(cot(x)*b^(1/2)/(a+b*cot(x)^2)^(1/2))/b^(1/2)-1/2*cot(x)*(a+b*cot(x)^
2)^(1/2)
```

3.22.2 Mathematica [A] (verified)

Time = 4.41 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.57

$$\int \cot^2(x)\sqrt{a+b\cot^2(x)}\,dx=-\frac{1}{2}\sqrt{\frac{-a-b+a\cos(2x)-b\cos(2x)}{-1+\cos(2x)}}\cot(x)-\frac{\left(2\sqrt{a-b}\sqrt{b}\arctan\left(\frac{\sqrt{b+a\tan^2(x)}}{\sqrt{a-b}}\right)+(a-2b)\text{arctanh}\left(\frac{\sqrt{b+a\tan^2(x)}}{\sqrt{b}}\right)\right)\sqrt{a+b\cot^2(x)}\tan(x)}{2\sqrt{b}\sqrt{b+a\tan^2(x)}}$$

input
```
Integrate[Cot[x]^2*Sqrt[a + b*Cot[x]^2],x]
```

output
```
-1/2*(Sqrt[(-a - b + a*Cos[2*x] - b*Cos[2*x])/(-1 + Cos[2*x])]*Cot[x]) - (
(2*Sqrt[a - b]*Sqrt[b]*ArcTan[Sqrt[b + a*Tan[x]^2]/Sqrt[a - b]] + (a - 2*b
)*ArcTanh[Sqrt[b + a*Tan[x]^2]/Sqrt[b]])*Sqrt[a + b*Cot[x]^2*Tan[x])/(2*S
qrt[b]*Sqrt[b + a*Tan[x]^2])
```

3.22.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {3042, 4153, 380, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(x)\sqrt{a+b\cot^2(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan\left(x+\frac{\pi}{2}\right)^2\sqrt{a+b\tan\left(x+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{4153}$$

$$-\int \frac{\cot^2(x)\sqrt{b\cot^2(x)+a}}{\cot^2(x)+1}d\cot(x)$$

$$\Big\downarrow \text{380}$$

$$\frac{1}{2}\int \frac{a-(a-2b)\cot^2(x)}{\left(\cot^2(x)+1\right)\sqrt{b\cot^2(x)+a}}d\cot(x)-\frac{1}{2}\cot(x)\sqrt{a+b\cot^2(x)}$$

$$\Big\downarrow \text{398}$$

$$\frac{1}{2}\left(2(a-b)\int \frac{1}{\left(\cot^2(x)+1\right)\sqrt{b\cot^2(x)+a}}d\cot(x)-(a-2b)\int \frac{1}{\sqrt{b\cot^2(x)+a}}d\cot(x)\right)-\frac{1}{2}\cot(x)\sqrt{a+b\cot^2(x)}$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{2}\left(2(a-b)\int \frac{1}{\left(\cot^2(x)+1\right)\sqrt{b\cot^2(x)+a}}d\cot(x)-(a-2b)\int \frac{1}{1-\frac{b\cot^2(x)}{b\cot^2(x)+a}}d\frac{\cot(x)}{\sqrt{b\cot^2(x)+a}}\right)-\frac{1}{2}\cot(x)\sqrt{a+b\cot^2(x)}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(2(a-b)\int \frac{1}{(\cot^2(x)+1)\sqrt{b\cot^2(x)+a}}d\cot(x)-\frac{(a-2b)\text{arctanh}\left(\frac{\sqrt{b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{\sqrt{b}}\right)-\frac{1}{2}\cot(x)\sqrt{a+b\cot^2(x)}$$

$$\Big\downarrow \text{291}$$

$$\frac{1}{2}\left(2(a-b)\int \frac{1}{1-\frac{(b-a)\cot^2(x)}{b\cot^2(x)+a}}d\frac{\cot(x)}{\sqrt{b\cot^2(x)+a}}-\frac{(a-2b)\text{arctanh}\left(\frac{\sqrt{b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{\sqrt{b}}\right)-\frac{1}{2}\cot(x)\sqrt{a+b\cot^2(x)}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(2\sqrt{a-b}\arctan\left(\frac{\sqrt{a-b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)-\frac{(a-2b)\text{arctanh}\left(\frac{\sqrt{b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{\sqrt{b}}\right)-\frac{1}{2}\cot(x)\sqrt{a+b\cot^2(x)}$$

input
```
Int[Cot[x]^2*Sqrt[a + b*Cot[x]^2],x]
```

output
```
(2*Sqrt[a - b]*ArcTan[(Sqrt[a - b]*Cot[x])/Sqrt[a + b*Cot[x]^2]] - ((a - 2
*b)*ArcTanh[(Sqrt[b]*Cot[x])/Sqrt[a + b*Cot[x]^2]])/Sqrt[b])/2 - (Cot[x]*S
qrt[a + b*Cot[x]^2])/2
```

3.22.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 380
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*
(m + 2*(p + q) + 1))), x] - Simp[e^2/(b*(m + 2*(p + q) + 1))   Int[(e*x)^(m
 - 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[a*c*(m - 1) + (a*d*(m - 1) - 2
*q*(b*c - a*d))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
- a*d, 0] && GtQ[q, 0] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p,
 q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.22. $\int \cot^2(x)\sqrt{a+b\cot^2(x)}\,dx$

3.22.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. 2(71) = 142.

Time = 0.06 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.96

method	result
derivativedivides	$-\frac{\cot(x)\sqrt{a+b\cot(x)^{2}}}{2}-\frac{a\ln\left(\sqrt{b}\,\cot(x)+\sqrt{a+b\cot(x)^{2}}\right)}{2\sqrt{b}}+\sqrt{b}\,\ln\left(\sqrt{b}\,\cot\left(x\right)+\sqrt{a+b\cot\left(x\right)^{2}}\right)$
default	$-\frac{\cot(x)\sqrt{a+b\cot(x)^{2}}}{2}-\frac{a\ln\left(\sqrt{b}\,\cot(x)+\sqrt{a+b\cot(x)^{2}}\right)}{2\sqrt{b}}+\sqrt{b}\,\ln\left(\sqrt{b}\,\cot\left(x\right)+\sqrt{a+b\cot\left(x\right)^{2}}\right)$

input
```
int(cot(x)^2*(a+b*cot(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*cot(x)*(a+b*cot(x)^2)^(1/2)-1/2*a/b^(1/2)*ln(b^(1/2)*cot(x)+(a+b*cot(
x)^2)^(1/2))+b^(1/2)*ln(b^(1/2)*cot(x)+(a+b*cot(x)^2)^(1/2))-(b^4*(a-b))^(
1/2)/b/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*cot(x)^2)^(1/2)*cot(x
))+a*(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*c
ot(x)^2)^(1/2)*cot(x))
```

3.22.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(71) = 142.

Time = 0.31 (sec) , antiderivative size = 768, normalized size of antiderivative = 8.63

$$\int \cot^{2}(x)\sqrt{a+b\cot^{2}(x)}\,dx$$

$$=\left[\frac{2\sqrt{-a+b}b\log\left(-(a-b)\cos\left(2x\right)-\sqrt{-a+b}\sqrt{\frac{(a-b)\cos(2x)-a-b}{\cos(2x)-1}}\sin\left(2x\right)+b\right)\sin\left(2x\right)-(a-2b)\sqrt{b}\log}{4b\sin\left(2x\right)}\right.$$

input
```
integrate(cot(x)^2*(a+b*cot(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(2*sqrt(-a + b)*b*log(-(a - b)*cos(2*x) - sqrt(-a + b)*sqrt(((a - b)*
cos(2*x) - a - b)/(cos(2*x) - 1))*sin(2*x) + b)*sin(2*x) - (a - 2*b)*sqrt(
b)*log(((a - 2*b)*cos(2*x) - 2*sqrt(b)*sqrt(((a - b)*cos(2*x) - a - b)/(co
s(2*x) - 1))*sin(2*x) - a - 2*b)/(cos(2*x) - 1))*sin(2*x) - 2*(b*cos(2*x)
+ b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1)))/(b*sin(2*x)), 1/4*(4
*sqrt(a - b)*b*arctan(-sqrt(a - b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*
x) - 1))*sin(2*x)/((a - b)*cos(2*x) + a - b))*sin(2*x) - (a - 2*b)*sqrt(b)
*log(((a - 2*b)*cos(2*x) - 2*sqrt(b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(
2*x) - 1))*sin(2*x) - a - 2*b)/(cos(2*x) - 1))*sin(2*x) - 2*(b*cos(2*x) +
b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1)))/(b*sin(2*x)), 1/2*((a
- 2*b)*sqrt(-b)*arctan(sqrt(-b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x)
- 1))*sin(2*x)/(b*cos(2*x) + b))*sin(2*x) + sqrt(-a + b)*b*log(-(a - b)*co
s(2*x) - sqrt(-a + b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))*sin(
2*x) + b)*sin(2*x) - (b*cos(2*x) + b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos
(2*x) - 1)))/(b*sin(2*x)), 1/2*(2*sqrt(a - b)*b*arctan(-sqrt(a - b)*sqrt((
(a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))*sin(2*x)/((a - b)*cos(2*x) + a -
 b))*sin(2*x) + (a - 2*b)*sqrt(-b)*arctan(sqrt(-b)*sqrt(((a - b)*cos(2*x)
- a - b)/(cos(2*x) - 1))*sin(2*x)/(b*cos(2*x) + b))*sin(2*x) - (b*cos(2*x)
 + b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1)))/(b*sin(2*x))]
```

3.22.6 Sympy [F]

$$\int \cot^2(x)\sqrt{a+b\cot^2(x)}\,dx = \int \sqrt{a+b\cot^2{(x)}}\cot^2{(x)}\,dx$$

input
```
integrate(cot(x)**2*(a+b*cot(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*cot(x)**2)*cot(x)**2, x)
```

3.22.7 Maxima [F]

$$\int \cot^2(x)\sqrt{a+b\cot^2(x)}\,dx = \int \sqrt{b\cot{(x)}^2+a}\cot{(x)}^2\,dx$$

input
```
integrate(cot(x)^2*(a+b*cot(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*cot(x)^2 + a)*cot(x)^2, x)
```

3.22.8 Giac [F(-2)]

Exception generated.

$$\int \cot^2(x)\sqrt{a+b\cot^2(x)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(x)^2*(a+b*cot(x)^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.22.9 Mupad [F(-1)]

Timed out.

$$\int \cot^2(x)\sqrt{a+b\cot^2(x)}\,dx = \int \cot(x)^2\,\sqrt{b\cot(x)^2+a}\,dx$$

input
```
int(cot(x)^2*(a + b*cot(x)^2)^(1/2),x)
```

output
```
int(cot(x)^2*(a + b*cot(x)^2)^(1/2), x)
```

3.23 $\int \sqrt{a+b\cot^2(x)}\,dx$

3.23.1 Optimal result

Integrand size = 12, antiderivative size = 65

$$\int \sqrt{a+b\cot^2(x)}\,dx=-\sqrt{a-b}\arctan\left(\frac{\sqrt{a-b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)-\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)$$

output
```
-arctan(cot(x)*(a-b)^(1/2)/(a+b*cot(x)^2)^(1/2))*(a-b)^(1/2)-arctanh(cot(x
)*b^(1/2)/(a+b*cot(x)^2)^(1/2))*b^(1/2)
```

3.23.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.20

$$\int \sqrt{a+b\cot^2(x)}\,dx=\sqrt{a-b}\arctan\left(\frac{-\cot(x)\sqrt{a+b\cot^2(x)}+\sqrt{b}\csc^2(x)}{\sqrt{a-b}}\right)+\sqrt{b}\log\left(-\sqrt{b}\cot(x)+\sqrt{a+b\cot^2(x)}\right)$$

input
```
Integrate[Sqrt[a + b*Cot[x]^2],x]
```

output
```
Sqrt[a - b]*ArcTan[(-(Cot[x]*Sqrt[a + b*Cot[x]^2]) + Sqrt[b]*Csc[x]^2)/Sqr
t[a - b]] + Sqrt[b]*Log[-(Sqrt[b]*Cot[x]) + Sqrt[a + b*Cot[x]^2]]
```

3.23.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {3042, 4144, 301, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\cot^2(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a+b\tan\left(x+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{4144}$$

$$-\int \frac{\sqrt{b\cot^2(x)+a}}{\cot^2(x)+1}d\cot(x)$$

$$\Big\downarrow \text{301}$$

$$-b\int \frac{1}{\sqrt{b\cot^2(x)+a}}d\cot(x)-(a-b)\int \frac{1}{\left(\cot^2(x)+1\right)\sqrt{b\cot^2(x)+a}}d\cot(x)$$

$$\Big\downarrow \text{224}$$

$$-\left((a-b)\int \frac{1}{\left(\cot^2(x)+1\right)\sqrt{b\cot^2(x)+a}}d\cot(x)\right)-b\int \frac{1}{1-\frac{b\cot^2(x)}{b\cot^2(x)+a}}d\frac{\cot(x)}{\sqrt{b\cot^2(x)+a}}$$

$$\Big\downarrow \text{219}$$

$$-(a-b)\int \frac{1}{\left(\cot^2(x)+1\right)\sqrt{b\cot^2(x)+a}}d\cot(x)-\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)$$

$$\Big\downarrow \text{291}$$

$$-(a-b)\int \frac{1}{1-\frac{(b-a)\cot^2(x)}{b\cot^2(x)+a}}d\frac{\cot(x)}{\sqrt{b\cot^2(x)+a}}-\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)$$

$$\Big\downarrow \text{216}$$

$$-\sqrt{a-b}\arctan\left(\frac{\sqrt{a-b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)-\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)$$

input
```
Int[Sqrt[a + b*Cot[x]^2],x]
```

output
```
-(Sqrt[a - b]*ArcTan[(Sqrt[a - b]*Cot[x])/Sqrt[a + b*Cot[x]^2]]) - Sqrt[b]
*ArcTanh[(Sqrt[b]*Cot[x])/Sqrt[a + b*Cot[x]^2]]
```

3.23.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

3.23.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(53) = 106$.

Time = 0.05 (sec) , antiderivative size = 137, normalized size of antiderivative = 2.11

method	result
derivativedivides	$-\sqrt{b}\,\ln\left(\sqrt{b}\,\cot\left(x\right)+\sqrt{a+b\cot\left(x\right)^{2}}\right)+\frac{\sqrt{b^{4}(a-b)}\,\arctan\left(\frac{b^{2}(a-b)\cot(x)}{\sqrt{b^{4}(a-b)}\,\sqrt{a+b\cot(x)^{2}}}\right)}{b(a-b)}-\frac{a\sqrt{b^{4}(a-}}{}$
default	$-\sqrt{b}\,\ln\left(\sqrt{b}\,\cot\left(x\right)+\sqrt{a+b\cot\left(x\right)^{2}}\right)+\frac{\sqrt{b^{4}(a-b)}\,\arctan\left(\frac{b^{2}(a-b)\cot(x)}{\sqrt{b^{4}(a-b)}\,\sqrt{a+b\cot(x)^{2}}}\right)}{b(a-b)}-\frac{a\sqrt{b^{4}(a-}}{}$

input
```
int((a+b*cot(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-b^(1/2)*ln(b^(1/2)*cot(x)+(a+b*cot(x)^2)^(1/2))+(b^4*(a-b))^(1/2)/b/(a-b)
*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*cot(x)^2)^(1/2)*cot(x))-a*(b^4*(a
-b))^(1/2)/b^2/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*cot(x)^2)^(1/
2)*cot(x))
```

3.23.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(53) = 106$.

Time = 0.28 (sec) , antiderivative size = 515, normalized size of antiderivative = 7.92

$$\int \sqrt{a+b\cot^2(x)}\,dx$$

$$= \left[\frac{1}{2}\sqrt{-a+b}\log\left(-(a-b)\cos(2x)+\sqrt{-a+b}\sqrt{\frac{(a-b)\cos(2x)-a-b}{\cos(2x)-1}}\sin(2x)+b\right) + \frac{1}{2}\sqrt{b}\log\left(\frac{(a-2b)\cos(2x)+2\sqrt{b}\sqrt{\frac{(a-b)\cos(2x)-a-b}{\cos(2x)-1}}\sin(2x)-a-2b}{\cos(2x)-1}\right), -\sqrt{a-b}\arctan\left(-\frac{\sqrt{a-b}\sqrt{\frac{(a-b)\cos(2x)-a-b}{\cos(2x)-1}}\sin(2x)}{(a-b)\cos(2x)+a-b}\right) + \frac{1}{2}\sqrt{b}\log\left(\frac{(a-2b)\cos(2x)+2\sqrt{b}\sqrt{\frac{(a-b)\cos(2x)-a-b}{\cos(2x)-1}}\sin(2x)-a-2b}{\cos(2x)-1}\right), \sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{(a-b)c\ldots}{co\ldots}}}{b\cos\ldots}\right) + \frac{1}{2}\sqrt{-a+b}\log\left(-(a-b)\cos(2x)+\sqrt{-a+b}\sqrt{\frac{(a-b)\cos(2x)-a-b}{\cos(2x)-1}}\sin(2x)+b\right), -\sqrt{a-b}\arctan\left(-\frac{\sqrt{a-b}\sqrt{\frac{(a-b)\cos(2x)-a-b}{\cos(2x)-1}}\sin(2x)}{(a-b)\cos(2x)+a-b}\right) + \sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{(a-b)\cos(2x)-a-b}{\cos(2x)-1}}\sin(2x)}{b\cos(2x)+b}\right)\right]$$

input
```
integrate((a+b*cot(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(-a + b)*log(-(a - b)*cos(2*x) + sqrt(-a + b)*sqrt(((a - b)*cos(2
*x) - a - b)/(cos(2*x) - 1))*sin(2*x) + b) + 1/2*sqrt(b)*log(((a - 2*b)*co
s(2*x) + 2*sqrt(b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))*sin(2*x
) - a - 2*b)/(cos(2*x) - 1)), -sqrt(a - b)*arctan(-sqrt(a - b)*sqrt(((a -
b)*cos(2*x) - a - b)/(cos(2*x) - 1))*sin(2*x)/((a - b)*cos(2*x) + a - b))
+ 1/2*sqrt(b)*log(((a - 2*b)*cos(2*x) + 2*sqrt(b)*sqrt(((a - b)*cos(2*x) -
 a - b)/(cos(2*x) - 1))*sin(2*x) - a - 2*b)/(cos(2*x) - 1)), sqrt(-b)*arct
an(sqrt(-b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))*sin(2*x)/(b*co
s(2*x) + b)) + 1/2*sqrt(-a + b)*log(-(a - b)*cos(2*x) + sqrt(-a + b)*sqrt(
((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))*sin(2*x) + b), -sqrt(a - b)*arc
tan(-sqrt(a - b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))*sin(2*x)/
((a - b)*cos(2*x) + a - b)) + sqrt(-b)*arctan(sqrt(-b)*sqrt(((a - b)*cos(2
*x) - a - b)/(cos(2*x) - 1))*sin(2*x)/(b*cos(2*x) + b))]
```

3.23. $\int \sqrt{a+b\cot^2(x)}\,dx$

3.23.6 Sympy [F]

$$\int \sqrt{a+b\cot^2(x)}\,dx = \int \sqrt{a+b\cot^2{(x)}}\,dx$$

input
```
integrate((a+b*cot(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*cot(x)**2), x)
```

3.23.7 Maxima [F(-2)]

Exception generated.

$$\int \sqrt{a+b\cot^2(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*cot(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

3.23.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. $2(53) = 106$.

Time = 0.50 (sec) , antiderivative size = 210, normalized size of antiderivative = 3.23

$$\int \sqrt{a+b\cot^2(x)}\,dx = -\frac{1}{2}\left(\frac{2\sqrt{-a+b}b\arctan\left(\frac{\left(\sqrt{-a+b}\cos(x)-\sqrt{-a\cos(x)^2+b\cos(x)^2+a}\right)^2+a-2b}{2\sqrt{ab-b^2}}\right)}{\sqrt{ab-b^2}}+\sqrt{-a+b}\log\left(\left(\sqrt{-a+b}\cos(x\right.\right.\right.$$

$$-\frac{\left(2\sqrt{-a+b}b\arctan\left(\frac{\sqrt{-a+b}\sqrt{b}}{\sqrt{ab-b^2}}\right)-\sqrt{ab-b^2}\sqrt{-a+b}\log\left(-a-2\sqrt{-a+b}\sqrt{b}+2b\right)\right)\mathrm{sgn}(\sin{(x)})}{2\sqrt{ab-b^2}}$$

input
```
integrate((a+b*cot(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*(2*sqrt(-a + b)*b*arctan(1/2*((sqrt(-a + b)*cos(x) - sqrt(-a*cos(x)^2
 + b*cos(x)^2 + a))^2 + a - 2*b)/sqrt(a*b - b^2))/sqrt(a*b - b^2) + sqrt(-
a + b)*log((sqrt(-a + b)*cos(x) - sqrt(-a*cos(x)^2 + b*cos(x)^2 + a))^2))*
sgn(sin(x)) - 1/2*(2*sqrt(-a + b)*b*arctan(sqrt(-a + b)*sqrt(b)/sqrt(a*b -
 b^2)) - sqrt(a*b - b^2)*sqrt(-a + b)*log(-a - 2*sqrt(-a + b)*sqrt(b) + 2*
b))*sgn(sin(x))/sqrt(a*b - b^2)
```

3.23.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\cot^2(x)}\,dx=\int \sqrt{b\cot\left(x\right)^2+a}\,dx$$

input
```
int((a + b*cot(x)^2)^(1/2),x)
```

output
```
int((a + b*cot(x)^2)^(1/2), x)
```

3.24 $\int \sqrt{a+b\cot^2(x)}\tan^2(x)\,dx$

3.24.1 Optimal result

Integrand size = 17, antiderivative size = 51

$$\int \sqrt{a+b\cot^2(x)}\tan^2(x)\,dx = \sqrt{a-b}\arctan\left(\frac{\sqrt{a-b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)+\sqrt{a+b\cot^2(x)}\tan(x)$$

output
```
arctan(cot(x)*(a-b)^(1/2)/(a+b*cot(x)^2)^(1/2))*(a-b)^(1/2)+(a+b*cot(x)^2)
^(1/2)*tan(x)
```

3.24.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.86

$$\int \sqrt{a+b\cot^2(x)}\tan^2(x)\,dx = \sqrt{a+b\cot^2(x)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\frac{(a-b)\cot^2(x)}{a+b\cot^2(x)}\right)\tan(x)$$

input
```
Integrate[Sqrt[a + b*Cot[x]^2]*Tan[x]^2,x]
```

output
```
Sqrt[a + b*Cot[x]^2]*Hypergeometric2F1[-1/2, 1, 1/2, -(((a - b)*Cot[x]^2)/
(a + b*Cot[x]^2))]*Tan[x]
```

3.24.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {3042, 4153, 377, 25, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(x)\sqrt{a+b\cot^2(x)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sqrt{a+b\tan\left(x+\frac{\pi}{2}\right)^2}}{\tan\left(x+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \; 4153$$

$$-\int \frac{\sqrt{b\cot^2(x)+a}\tan^2(x)}{\cot^2(x)+1}d\cot(x)$$

$$\Big\downarrow \; 377$$

$$\tan(x)\sqrt{a+b\cot^2(x)}-\int -\frac{a-b}{\left(\cot^2(x)+1\right)\sqrt{b\cot^2(x)+a}}d\cot(x)$$

$$\Big\downarrow \; 25$$

$$\int \frac{a-b}{\left(\cot^2(x)+1\right)\sqrt{b\cot^2(x)+a}}d\cot(x)+\tan(x)\sqrt{a+b\cot^2(x)}$$

$$\Big\downarrow \; 27$$

$$(a-b)\int \frac{1}{\left(\cot^2(x)+1\right)\sqrt{b\cot^2(x)+a}}d\cot(x)+\tan(x)\sqrt{a+b\cot^2(x)}$$

$$\Big\downarrow \; 291$$

$$(a-b)\int \frac{1}{1-\frac{(b-a)\cot^2(x)}{b\cot^2(x)+a}}d\frac{\cot(x)}{\sqrt{b\cot^2(x)+a}}+\tan(x)\sqrt{a+b\cot^2(x)}$$

$$\Big\downarrow \; 216$$

$$\sqrt{a-b}\arctan\left(\frac{\sqrt{a-b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)+\tan(x)\sqrt{a+b\cot^2(x)}$$

input
```
Int[Sqrt[a + b*Cot[x]^2]*Tan[x]^2,x]
```

output
```
Sqrt[a - b]*ArcTan[(Sqrt[a - b]*Cot[x])/Sqrt[a + b*Cot[x]^2]] + Sqrt[a + b
*Cot[x]^2]*Tan[x]
```

3.24.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 377
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(a*e*(
m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[b*c*(m + 1) + 2*(b*c*(p + 1) + a*d*q) + d*(b*(m + 1)
+ 2*b*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
*c - a*d, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m
, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.24.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 311 vs. 2(43) = 86.

Time = 0.87 (sec) , antiderivative size = 312, normalized size of antiderivative = 6.12

method	result
default	$\frac{\sqrt{4}\,\sqrt{a+b\cot(x)^{2}}\,\left(\ln\left(4\cos(x)\sqrt{-a+b}\,\sqrt{-\frac{a\cos(x)^{2}-\cos(x)^{2}b-a}{(\cos(x)+1)^{2}}}-4\cos(x)a+4b\cos(x)+4\sqrt{-a+b}\,\sqrt{-\frac{a\cos(x)^{2}-\cos(x)^{2}b-a}{(\cos(x)+1)^{2}}}\right)a\sin(x)}{}$

input
```
int((a+b*cot(x)^2)^(1/2)*tan(x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*4^(1/2)/(-a+b)^(1/2)*(a+b*cot(x)^2)^(1/2)/(-(a*cos(x)^2-cos(x)^2*b-a)/
(cos(x)+1)^2)^(1/2)/(cos(x)+1)*(ln(4*cos(x)*(-a+b)^(1/2)*(-(a*cos(x)^2-cos
(x)^2*b-a)/(cos(x)+1)^2)^(1/2)-4*cos(x)*a+4*b*cos(x)+4*(-a+b)^(1/2)*(-(a*c
os(x)^2-cos(x)^2*b-a)/(cos(x)+1)^2)^(1/2))*a*sin(x)-ln(4*cos(x)*(-a+b)^(1/
2)*(-(a*cos(x)^2-cos(x)^2*b-a)/(cos(x)+1)^2)^(1/2)-4*cos(x)*a+4*b*cos(x)+4
*(-a+b)^(1/2)*(-(a*cos(x)^2-cos(x)^2*b-a)/(cos(x)+1)^2)^(1/2))*b*sin(x)+(-
a+b)^(1/2)*(-(a*cos(x)^2-cos(x)^2*b-a)/(cos(x)+1)^2)^(1/2)*sin(x)+(-a+b)^(
1/2)*(-(a*cos(x)^2-cos(x)^2*b-a)/(cos(x)+1)^2)^(1/2)*tan(x))
```

3.24.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 193, normalized size of antiderivative = 3.78

$$\int \sqrt{a+b\cot^{2}(x)}\tan^{2}(x)\,dx$$
$$=\left[\frac{1}{4}\sqrt{-a+b}\log\left(-\frac{a^{2}\tan(x)^{4}-2\,(3\,a^{2}-4\,ab)\tan(x)^{2}+a^{2}-8\,ab+8\,b^{2}-4\,\left(a\tan(x)^{3}-(a-2\,b)\tan\right.}{\tan(x)^{4}+2\,\tan(x)^{2}+1}\right.\right.$$
$$\left.+\sqrt{\frac{a\tan(x)^{2}+b}{\tan(x)^{2}}}\tan(x)\,,\,\frac{1}{2}\sqrt{a-b}\arctan\left(\frac{2\sqrt{a-b}\sqrt{\frac{a\tan(x)^{2}+b}{\tan(x)^{2}}}\tan(x)}{a\tan(x)^{2}-a+2\,b}\right)\right.$$
$$\left.+\sqrt{\frac{a\tan(x)^{2}+b}{\tan(x)^{2}}}\tan(x)\right]$$

input
```
integrate((a+b*cot(x)^2)^(1/2)*tan(x)^2,x, algorithm="fricas")
```

output
```
[1/4*sqrt(-a + b)*log(-(a^2*tan(x)^4 - 2*(3*a^2 - 4*a*b)*tan(x)^2 + a^2 -
8*a*b + 8*b^2 - 4*(a*tan(x)^3 - (a - 2*b)*tan(x))*sqrt(-a + b)*sqrt((a*tan
(x)^2 + b)/tan(x)^2))/(tan(x)^4 + 2*tan(x)^2 + 1)) + sqrt((a*tan(x)^2 + b)
/tan(x)^2)*tan(x), 1/2*sqrt(a - b)*arctan(2*sqrt(a - b)*sqrt((a*tan(x)^2 +
 b)/tan(x)^2)*tan(x)/(a*tan(x)^2 - a + 2*b)) + sqrt((a*tan(x)^2 + b)/tan(x
)^2)*tan(x)]
```

3.24.6 Sympy [F]

$$\int \sqrt{a+b\cot^2(x)}\tan^2(x)\,dx = \int \sqrt{a+b\cot^2{(x)}}\tan^2{(x)}\,dx$$

input
```
integrate((a+b*cot(x)**2)**(1/2)*tan(x)**2,x)
```

output
```
Integral(sqrt(a + b*cot(x)**2)*tan(x)**2, x)
```

3.24.7 Maxima [F]

$$\int \sqrt{a+b\cot^2(x)}\tan^2(x)\,dx = \int \sqrt{b\cot{(x)}^2+a}\tan{(x)}^2\,dx$$

input
```
integrate((a+b*cot(x)^2)^(1/2)*tan(x)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*cot(x)^2 + a)*tan(x)^2, x)
```

3.24.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(43) = 86$.

Time = 0.29 (sec) , antiderivative size = 239, normalized size of antiderivative = 4.69

$$\int \sqrt{a+b\cot^2(x)}\tan^2(x)\,dx$$

$$= \frac{1}{2}\left(\sqrt{-a+b}\log\left(\left(\sqrt{-a+b}\cos(x)-\sqrt{-a\cos(x)^2+b\cos(x)^2+a}\right)^2\right)-\frac{4\,a\sqrt{\ }}{\left(\sqrt{-a+b}\cos(x)-\sqrt{-a\,c}\right.}\right.$$

$$-\frac{\left(a\sqrt{-a+b}\log\left(-a-2\sqrt{-a+b}\sqrt{b}+2\,b\right)-a\sqrt{b}\log\left(-a-2\sqrt{-a+b}\sqrt{b}+2\,b\right)-\sqrt{-a+b}b\log\left(-\right.\right.}{2\left(a+\sqrt{-a+b}\sqrt{b}\right.}$$

input
```
integrate((a+b*cot(x)^2)^(1/2)*tan(x)^2,x, algorithm="giac")
```

output
```
1/2*(sqrt(-a + b)*log((sqrt(-a + b)*cos(x) - sqrt(-a*cos(x)^2 + b*cos(x)^2
 + a))^2) - 4*a*sqrt(-a + b)/((sqrt(-a + b)*cos(x) - sqrt(-a*cos(x)^2 + b*
cos(x)^2 + a))^2 - a))*sgn(sin(x)) - 1/2*(a*sqrt(-a + b)*log(-a - 2*sqrt(-
a + b)*sqrt(b) + 2*b) - a*sqrt(b)*log(-a - 2*sqrt(-a + b)*sqrt(b) + 2*b) -
 sqrt(-a + b)*b*log(-a - 2*sqrt(-a + b)*sqrt(b) + 2*b) + b^(3/2)*log(-a -
2*sqrt(-a + b)*sqrt(b) + 2*b) + 2*a*sqrt(-a + b))*sgn(sin(x))/(a + sqrt(-a
 + b)*sqrt(b) - b)
```

3.24.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\cot^2(x)}\tan^2(x)\,dx = \int \tan(x)^2\sqrt{b\cot(x)^2+a}\,dx$$

input
```
int(tan(x)^2*(a + b*cot(x)^2)^(1/2),x)
```

output
```
int(tan(x)^2*(a + b*cot(x)^2)^(1/2), x)
```

3.25 $\int \sqrt{a+b\cot^2(x)}\tan^4(x)\,dx$

3.25.1 Optimal result

Integrand size = 17, antiderivative size = 85

$$\int \sqrt{a+b\cot^2(x)}\tan^4(x)\,dx = -\sqrt{a-b}\arctan\left(\frac{\sqrt{a-b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right) - \frac{(3a-b)\sqrt{a+b\cot^2(x)}\tan(x)}{3a} + \frac{1}{3}\sqrt{a+b\cot^2(x)}\tan^3(x)$$

output
```
-arctan(cot(x)*(a-b)^(1/2)/(a+b*cot(x)^2)^(1/2))*(a-b)^(1/2)-1/3*(3*a-b)*(
a+b*cot(x)^2)^(1/2)*tan(x)/a+1/3*(a+b*cot(x)^2)^(1/2)*tan(x)^3
```

3.25.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.76 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.05

$$\int \sqrt{a+b\cot^2(x)}\tan^4(x)\,dx = \frac{1}{3}\sqrt{a+b\cot^2(x)}\left(1+\frac{b\cot^2(x)}{a}\right)\sin^2(x)\left(-\frac{4(a-b)\cos^2(x)\left(a+b\cot^2(x)\right)\text{Hypergeometric2F1}\left(2,2,\frac{3}{2},\frac{(a-b)\cos^2(x)}{a}\right)}{a^2} + \frac{\left(a-2b\cot^2(x)\right)\csc^2(x)\left(\arcsin\left(\sqrt{\frac{(a-b)\cos^2(x)}{a}}\right)\sqrt{\frac{(a-b)\cos^2(x)}{a}}+\sqrt{\frac{b\cos^2(x)}{a}+\sin^2(x)}\right)}{\left(a+b\cot^2(x)\right)\sqrt{\frac{b\cos^2(x)}{a}+\sin^2(x)}}\right)\tan^3(x)$$

input
```
Integrate[Sqrt[a + b*Cot[x]^2]*Tan[x]^4,x]
```

output
```
(Sqrt[a + b*Cot[x]^2]*(1 + (b*Cot[x]^2)/a)*Sin[x]^2*((-4*(a - b)*Cos[x]^2*
(a + b*Cot[x]^2)*Hypergeometric2F1[2, 2, 3/2, ((a - b)*Cos[x]^2)/a])/a^2 +
 ((a - 2*b*Cot[x]^2)*Csc[x]^2*(ArcSin[Sqrt[((a - b)*Cos[x]^2)/a]]*Sqrt[((a
 - b)*Cos[x]^2)/a] + Sqrt[(b*Cos[x]^2)/a + Sin[x]^2]))/((a + b*Cot[x]^2)*S
qrt[(b*Cos[x]^2)/a + Sin[x]^2]))*Tan[x]^3)/3
```

3.25.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {3042, 4153, 377, 25, 445, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(x)\sqrt{a+b\cot^2(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\tan\left(x+\frac{\pi}{2}\right)^2}}{\tan\left(x+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{4153}$$

$$-\int \frac{\sqrt{b\cot^2(x)+a}\tan^4(x)}{\cot^2(x)+1}d\cot(x)$$

$$\Big\downarrow \text{377}$$

$$\frac{1}{3}\tan^3(x)\sqrt{a+b\cot^2(x)}-\frac{1}{3}\int -\frac{\left(2b\cot^2(x)+3a-b\right)\tan^2(x)}{\left(\cot^2(x)+1\right)\sqrt{b\cot^2(x)+a}}d\cot(x)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\int \frac{\left(2b\cot^2(x)+3a-b\right)\tan^2(x)}{\left(\cot^2(x)+1\right)\sqrt{b\cot^2(x)+a}}d\cot(x)+\frac{1}{3}\tan^3(x)\sqrt{a+b\cot^2(x)}$$

$$\Big\downarrow \text{445}$$

$$\frac{1}{3}\left(-\frac{\int \frac{3a(a-b)}{(\cot^2(x)+1)\sqrt{b\cot^2(x)+a}}d\cot(x)}{a}-\frac{(3a-b)\tan(x)\sqrt{a+b\cot^2(x)}}{a}\right)+\frac{1}{3}\tan^3(x)\sqrt{a+b\cot^2(x)}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{3}\left(-3(a-b)\int \frac{1}{(\cot^2(x)+1)\sqrt{b\cot^2(x)+a}}d\cot(x)-\frac{(3a-b)\tan(x)\sqrt{a+b\cot^2(x)}}{a}\right)+\frac{1}{3}\tan^3(x)\sqrt{a+b\cot^2(x)}$$

$$\Big\downarrow \ 291$$

$$\frac{1}{3}\left(-3(a-b)\int \frac{1}{1-\frac{(b-a)\cot^2(x)}{b\cot^2(x)+a}}d\frac{\cot(x)}{\sqrt{b\cot^2(x)+a}}-\frac{(3a-b)\tan(x)\sqrt{a+b\cot^2(x)}}{a}\right)+\frac{1}{3}\tan^3(x)\sqrt{a+b\cot^2(x)}$$

$$\Big\downarrow \ 216$$

$$\frac{1}{3}\left(-3\sqrt{a-b}\arctan\left(\frac{\sqrt{a-b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)-\frac{(3a-b)\tan(x)\sqrt{a+b\cot^2(x)}}{a}\right)+\frac{1}{3}\tan^3(x)\sqrt{a+b\cot^2(x)}$$

input
```
Int[Sqrt[a + b*Cot[x]^2]*Tan[x]^4,x]
```

output
```
(Sqrt[a + b*Cot[x]^2]*Tan[x]^3)/3 + (-3*Sqrt[a - b]*ArcTan[(Sqrt[a - b]*Co
t[x])/Sqrt[a + b*Cot[x]^2]] - ((3*a - b)*Sqrt[a + b*Cot[x]^2]*Tan[x])/a)/3
```

3.25.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 377
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(a*e*(
m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[b*c*(m + 1) + 2*(b*c*(p + 1) + a*d*q) + d*(b*(m + 1)
+ 2*b*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
*c - a*d, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m
, 2, p, q, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.25. $\int \sqrt{a+b\cot^2(x)}\tan^4(x)\,dx$

3.25.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 494 vs. $2(71) = 142$.

Time = 1.51 (sec) , antiderivative size = 495, normalized size of antiderivative = 5.82

method	result
default	$-\frac{\sqrt{4}\left(4\sqrt{-a+b}\,\sqrt{-\frac{a\cos(x)^2-\cos(x)^2 b-a}{(\cos(x)+1)^2}}\,a\cos(x)^3-\cos(x)^3\sqrt{-\frac{a\cos(x)^2-\cos(x)^2 b-a}{(\cos(x)+1)^2}}\,\sqrt{-a+b}\,b+3\ln\left(4\cos(x)\sqrt{-a+b}\,\sqrt{-\frac{a\cos(x)^2-}{(\cos(}}\right.\right.}{}$

input
```
int((a+b*cot(x)^2)^(1/2)*tan(x)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/6*4^(1/2)/a/(-a+b)^(1/2)*(4*(-a+b)^(1/2)*(-(a*cos(x)^2-cos(x)^2*b-a)/(c
os(x)+1)^2)^(1/2)*a*cos(x)^3-cos(x)^3*(-(a*cos(x)^2-cos(x)^2*b-a)/(cos(x)+
1)^2)^(1/2)*(-a+b)^(1/2)*b+3*ln(4*cos(x)*(-a+b)^(1/2)*(-(a*cos(x)^2-cos(x)
^2*b-a)/(cos(x)+1)^2)^(1/2)-4*cos(x)*a+4*b*cos(x)+4*(-a+b)^(1/2)*(-(a*cos(
x)^2-cos(x)^2*b-a)/(cos(x)+1)^2)^(1/2))*cos(x)^3*a^2-3*ln(4*cos(x)*(-a+b)^
(1/2)*(-(a*cos(x)^2-cos(x)^2*b-a)/(cos(x)+1)^2)^(1/2)-4*cos(x)*a+4*b*cos(x
)+4*(-a+b)^(1/2)*(-(a*cos(x)^2-cos(x)^2*b-a)/(cos(x)+1)^2)^(1/2))*cos(x)^3
*a*b+4*(-(a*cos(x)^2-cos(x)^2*b-a)/(cos(x)+1)^2)^(1/2)*cos(x)^2*(-a+b)^(1/
2)*a-cos(x)^2*(-(a*cos(x)^2-cos(x)^2*b-a)/(cos(x)+1)^2)^(1/2)*(-a+b)^(1/2)
*b-(-(a*cos(x)^2-cos(x)^2*b-a)/(cos(x)+1)^2)^(1/2)*(-a+b)^(1/2)*a*cos(x)-(
-(a*cos(x)^2-cos(x)^2*b-a)/(cos(x)+1)^2)^(1/2)*(-a+b)^(1/2)*a)*(a+b*cot(x)
^2)^(1/2)/(cos(x)+1)/(-(a*cos(x)^2-cos(x)^2*b-a)/(cos(x)+1)^2)^(1/2)*tan(x
)*sec(x)^2
```

3.25.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 239, normalized size of antiderivative = 2.81

$$\int \sqrt{a+b\cot^2(x)}\tan^4(x)\,dx$$
$$= \left[\frac{3\,a\sqrt{-a+b}\log\left(-\frac{a^2\tan(x)^4-2\,(3\,a^2-4\,ab)\tan(x)^2+a^2-8\,ab+8\,b^2+4\,\left(a\tan(x)^3-(a-2\,b)\tan(x)\right)\sqrt{-a+b}\sqrt{\frac{a\tan(x)^2+b}{\tan(x)^2}}}{\tan(x)^4+2\tan(x)^2+1}\right)+4\,(a}{12\,a}\right.$$
$$\left.-\frac{3\sqrt{a-b}a\arctan\left(\frac{2\sqrt{a-b}\sqrt{\frac{a\tan(x)^2+b}{\tan(x)^2}}\tan(x)}{a\tan(x)^2-a+2\,b}\right)-2\left(a\tan(x)^3-(3\,a-b)\tan(x)\right)\sqrt{\frac{a\tan(x)^2+b}{\tan(x)^2}}}{6\,a}\right]$$

input
```
integrate((a+b*cot(x)^2)^(1/2)*tan(x)^4,x, algorithm="fricas")
```

output
```
[1/12*(3*a*sqrt(-a + b)*log(-(a^2*tan(x)^4 - 2*(3*a^2 - 4*a*b)*tan(x)^2 +
a^2 - 8*a*b + 8*b^2 + 4*(a*tan(x)^3 - (a - 2*b)*tan(x))*sqrt(-a + b)*sqrt(
(a*tan(x)^2 + b)/tan(x)^2))/(tan(x)^4 + 2*tan(x)^2 + 1)) + 4*(a*tan(x)^3 -
 (3*a - b)*tan(x))*sqrt((a*tan(x)^2 + b)/tan(x)^2))/a, -1/6*(3*sqrt(a - b)
*a*arctan(2*sqrt(a - b)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x)/(a*tan(x)^2
 - a + 2*b)) - 2*(a*tan(x)^3 - (3*a - b)*tan(x))*sqrt((a*tan(x)^2 + b)/tan
(x)^2))/a]
```

3.25.6 Sympy [F]

$$\int \sqrt{a+b\cot^2(x)}\tan^4(x)\,dx = \int \sqrt{a+b\cot^2(x)}\tan^4(x)\,dx$$

input
```
integrate((a+b*cot(x)**2)**(1/2)*tan(x)**4,x)
```

output
```
Integral(sqrt(a + b*cot(x)**2)*tan(x)**4, x)
```

3.25.7 Maxima [F]

$$\int \sqrt{a+b\cot^2(x)}\tan^4(x)\,dx = \int \sqrt{b\cot(x)^2+a}\tan(x)^4\,dx$$

input
```
integrate((a+b*cot(x)^2)^(1/2)*tan(x)^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*cot(x)^2 + a)*tan(x)^4, x)
```

3.25.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 476 vs. 2(71) = 142.

Time = 0.33 (sec) , antiderivative size = 476, normalized size of antiderivative = 5.60

$$\int \sqrt{a+b\cot^2(x)}\tan^4(x)\,dx =$$

$$-\frac{1}{6}\left(3\sqrt{-a+b}\log\left(\left(\sqrt{-a+b}\cos(x)-\sqrt{-a\cos(x)^2+b\cos(x)^2+a}\right)^2\right)-\frac{4\left(3\left(\sqrt{-a+b}\cos(x)\right.\right.}{}\right.$$

$$+\frac{\left(3\,a^2\sqrt{-a+b}\log\left(-a-2\sqrt{-a+b}\sqrt{b}+2\,b\right)-9\,a^2\sqrt{b}\log\left(-a-2\sqrt{-a+b}\sqrt{b}+2\,b\right)-15\,a\sqrt{-a+\right.}{}$$

input
```
integrate((a+b*cot(x)^2)^(1/2)*tan(x)^4,x, algorithm="giac")
```

output
```
-1/6*(3*sqrt(-a + b)*log((sqrt(-a + b)*cos(x) - sqrt(-a*cos(x)^2 + b*cos(x
)^2 + a))^2) - 4*(3*(sqrt(-a + b)*cos(x) - sqrt(-a*cos(x)^2 + b*cos(x)^2 +
 a))^4*(2*a - b)*sqrt(-a + b) - 6*(sqrt(-a + b)*cos(x) - sqrt(-a*cos(x)^2
+ b*cos(x)^2 + a))^2*a^2*sqrt(-a + b) + (4*a^3 - a^2*b)*sqrt(-a + b))/((sq
rt(-a + b)*cos(x) - sqrt(-a*cos(x)^2 + b*cos(x)^2 + a))^2 - a)^3)*sgn(sin(
x)) + 1/6*(3*a^2*sqrt(-a + b)*log(-a - 2*sqrt(-a + b)*sqrt(b) + 2*b) - 9*a
^2*sqrt(b)*log(-a - 2*sqrt(-a + b)*sqrt(b) + 2*b) - 15*a*sqrt(-a + b)*b*lo
g(-a - 2*sqrt(-a + b)*sqrt(b) + 2*b) + 21*a*b^(3/2)*log(-a - 2*sqrt(-a + b
)*sqrt(b) + 2*b) + 12*sqrt(-a + b)*b^2*log(-a - 2*sqrt(-a + b)*sqrt(b) + 2
*b) - 12*b^(5/2)*log(-a - 2*sqrt(-a + b)*sqrt(b) + 2*b) + 8*a^2*sqrt(-a +
b) - 18*a^2*sqrt(b) - 24*a*sqrt(-a + b)*b + 30*a*b^(3/2) + 12*sqrt(-a + b)
*b^2 - 12*b^(5/2))*sgn(sin(x))/(a^2 + 3*a*sqrt(-a + b)*sqrt(b) - 5*a*b - 4
*sqrt(-a + b)*b^(3/2) + 4*b^2)
```

3.25.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\cot^2(x)}\tan^4(x)\,dx=\int \tan(x)^4\sqrt{b\cot{(x)}^2+a}\,dx$$

input
```
int(tan(x)^4*(a + b*cot(x)^2)^(1/2),x)
```

output
```
int(tan(x)^4*(a + b*cot(x)^2)^(1/2), x)
```

3.26 $\int \cot^3(x) \left(a+b\cot^2(x)\right)^{3/2} dx$

3.26.1 Optimal result

Integrand size = 17, antiderivative size = 88

$$\int \cot^3(x) \left(a+b\cot^2(x)\right)^{3/2} dx = -(a-b)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right) + (a-b)\sqrt{a+b\cot^2(x)} + \frac{1}{3}\left(a+b\cot^2(x)\right)^{3/2} - \frac{\left(a+b\cot^2(x)\right)^{5/2}}{5b}$$

output
```
-(a-b)^(3/2)*arctanh((a+b*cot(x)^2)^(1/2)/(a-b)^(1/2))+1/3*(a+b*cot(x)^2)^
(3/2)-1/5*(a+b*cot(x)^2)^(5/2)/b+(a-b)*(a+b*cot(x)^2)^(1/2)
```

3.26.2 Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.03

$$\int \cot^3(x) \left(a+b\cot^2(x)\right)^{3/2} dx = -(a-b)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right) - \frac{\sqrt{a+b\cot^2(x)}(3a^2-20ab+15b^2+(6a-5b)b\cot^2(x)+3b^2\cot^4(x))}{15b}$$

input
```
Integrate[Cot[x]^3*(a + b*Cot[x]^2)^(3/2),x]
```

output
```
-((a - b)^(3/2)*ArcTanh[Sqrt[a + b*Cot[x]^2]/Sqrt[a - b]]) - (Sqrt[a + b*C
ot[x]^2]*(3*a^2 - 20*a*b + 15*b^2 + (6*a - 5*b)*b*Cot[x]^2 + 3*b^2*Cot[x]^
4))/(15*b)
```

3.26.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.08, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.588, Rules used = {3042, 25, 4153, 25, 354, 90, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(x)\left(a+b\cot^2(x)\right)^{3/2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\tan\left(x+\frac{\pi}{2}\right)^3\left(a+b\tan\left(x+\frac{\pi}{2}\right)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \tan\left(x+\frac{\pi}{2}\right)^3\left(b\tan\left(x+\frac{\pi}{2}\right)^2+a\right)^{3/2}dx$$

$$\Big\downarrow \text{ 4153}$$

$$\int -\frac{\cot^3(x)\left(a+b\cot^2(x)\right)^{3/2}}{\cot^2(x)+1}d\cot(x)$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{\cot^3(x)\left(b\cot^2(x)+a\right)^{3/2}}{\cot^2(x)+1}d\cot(x)$$

$$\Big\downarrow \text{ 354}$$

$$-\frac{1}{2}\int \frac{\cot^2(x)\left(b\cot^2(x)+a\right)^{3/2}}{\cot^2(x)+1}d\cot^2(x)$$

$$\Big\downarrow \text{ 90}$$

$$\frac{1}{2}\left(\int \frac{\left(b\cot^2(x)+a\right)^{3/2}}{\cot^2(x)+1}d\cot^2(x)-\frac{2\left(a+b\cot^2(x)\right)^{5/2}}{5b}\right)$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{2}\left((a-b)\int \frac{\sqrt{b\cot^2(x)+a}}{\cot^2(x)+1}d\cot^2(x)-\frac{2\left(a+b\cot^2(x)\right)^{5/2}}{5b}+\frac{2}{3}\left(a+b\cot^2(x)\right)^{3/2}\right)$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{2}\left((a-b)\left((a-b)\int \frac{1}{\left(\cot^2(x)+1\right)\sqrt{b\cot^2(x)+a}}d\cot^2(x)+2\sqrt{a+b\cot^2(x)}\right)-\frac{2\left(a+b\cot^2(x)\right)^{5/2}}{5b}+\frac{2}{3}(a+$$

$$\Big\downarrow 73$$

$$\frac{1}{2}\left((a-b)\left(\frac{2(a-b)\int \frac{1}{\frac{\cot^4(x)}{b}-\frac{a}{b}+1}d\sqrt{b\cot^2(x)+a}}{b}+2\sqrt{a+b\cot^2(x)}\right)-\frac{2\left(a+b\cot^2(x)\right)^{5/2}}{5b}+\frac{2}{3}(a+b\cot^2(x$$

$$\Big\downarrow 221$$

$$\frac{1}{2}\left((a-b)\left(2\sqrt{a+b\cot^2(x)}-2\sqrt{a-b}\,\text{barctanh}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)\right)-\frac{2\left(a+b\cot^2(x)\right)^{5/2}}{5b}+\frac{2}{3}\left(a+b\cot^2(x)\right)^{3/2}$$

input
```
Int[Cot[x]^3*(a + b*Cot[x]^2)^(3/2),x]
```

output
```
((2*(a + b*Cot[x]^2)^(3/2))/3 - (2*(a + b*Cot[x]^2)^(5/2))/(5*b) + (a - b)
*(-2*Sqrt[a - b]*ArcTanh[Sqrt[a + b*Cot[x]^2]/Sqrt[a - b]] + 2*Sqrt[a + b*
Cot[x]^2]))/2
```

3.26.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.26.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(72) = 144$.

Time = 0.04 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.70

method	result
derivativedivides	$-\frac{\left(a+b\cot(x)^2\right)^{\frac{5}{2}}}{5b}+\frac{b\cot(x)^2\sqrt{a+b\cot(x)^2}}{3}+\frac{4a\sqrt{a+b\cot(x)^2}}{3}-b\sqrt{a+b\cot(x)^2}+\frac{b^2\arctan\left(\frac{\sqrt{a+b\cot\ldots}}{\sqrt{-a\ldots}}\right.}{\sqrt{-a+b}}$
default	$-\frac{\left(a+b\cot(x)^2\right)^{\frac{5}{2}}}{5b}+\frac{b\cot(x)^2\sqrt{a+b\cot(x)^2}}{3}+\frac{4a\sqrt{a+b\cot(x)^2}}{3}-b\sqrt{a+b\cot(x)^2}+\frac{b^2\arctan\left(\frac{\sqrt{a+b\cot\ldots}}{\sqrt{-a\ldots}}\right.}{\sqrt{-a+b}}$

input
```
int(cot(x)^3*(a+b*cot(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5*(a+b*cot(x)^2)^(5/2)/b+1/3*b*cot(x)^2*(a+b*cot(x)^2)^(1/2)+4/3*a*(a+b
*cot(x)^2)^(1/2)-b*(a+b*cot(x)^2)^(1/2)+b^2/(-a+b)^(1/2)*arctan((a+b*cot(x
)^2)^(1/2)/(-a+b)^(1/2))-2*a*b/(-a+b)^(1/2)*arctan((a+b*cot(x)^2)^(1/2)/(-
a+b)^(1/2))+a^2/(-a+b)^(1/2)*arctan((a+b*cot(x)^2)^(1/2)/(-a+b)^(1/2))
```

3.26.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(72) = 144$.

Time = 0.35 (sec) , antiderivative size = 486, normalized size of antiderivative = 5.52

$$\int \cot^3(x) \left(a + b\cot^2(x)\right)^{3/2} dx = \left[-\frac{15\left((ab-b^2)\cos(2x)^2 + ab - b^2 - 2(ab-b^2)\cos(2x)\right)\sqrt{a-b}\log\left(-2\left(a^2-2ab+\right.\right.}{} \right.$$

$$-\frac{15\left((ab-b^2)\cos(2x)^2 + ab - b^2 - 2(ab-b^2)\cos(2x)\right)\sqrt{-a+b}\arctan\left(-\frac{\sqrt{-a+b}\sqrt{\frac{(a-b)\cos(2x)-a-b}{\cos(2x)-1}}(\cos(2x)-1}{(a-b)\cos(2x)-a}\right.}{30\left(b\cos(2x\right.}$$

input
```
integrate(cot(x)^3*(a+b*cot(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/60*(15*((a*b - b^2)*cos(2*x)^2 + a*b - b^2 - 2*(a*b - b^2)*cos(2*x))*s
qrt(a - b)*log(-2*(a^2 - 2*a*b + b^2)*cos(2*x)^2 - 2*a^2 + b^2 - 2*((a - b
)*cos(2*x)^2 - (2*a - b)*cos(2*x) + a)*sqrt(a - b)*sqrt(((a - b)*cos(2*x)
- a - b)/(cos(2*x) - 1)) + 4*(a^2 - a*b)*cos(2*x)) + 4*((3*a^2 - 26*a*b +
23*b^2)*cos(2*x)^2 + 3*a^2 - 14*a*b + 13*b^2 - 2*(3*a^2 - 20*a*b + 12*b^2)
*cos(2*x))*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1)))/(b*cos(2*x)^2
- 2*b*cos(2*x) + b), -1/30*(15*((a*b - b^2)*cos(2*x)^2 + a*b - b^2 - 2*(a*
b - b^2)*cos(2*x))*sqrt(-a + b)*arctan(-sqrt(-a + b)*sqrt(((a - b)*cos(2*x
) - a - b)/(cos(2*x) - 1))*(cos(2*x) - 1)/((a - b)*cos(2*x) - a)) + 2*((3*
a^2 - 26*a*b + 23*b^2)*cos(2*x)^2 + 3*a^2 - 14*a*b + 13*b^2 - 2*(3*a^2 - 2
0*a*b + 12*b^2)*cos(2*x))*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1)))
/(b*cos(2*x)^2 - 2*b*cos(2*x) + b)]
```

3.26.6 Sympy [F]

$$\int \cot^3(x) \left(a+b\cot^2(x)\right)^{3/2} \, dx = \int \left(a+b\cot^2\left(x\right)\right)^{\frac{3}{2}} \cot^3\left(x\right) dx$$

input
```
integrate(cot(x)**3*(a+b*cot(x)**2)**(3/2),x)
```

output
```
Integral((a + b*cot(x)**2)**(3/2)*cot(x)**3, x)
```

3.26.7 Maxima [F(-2)]

Exception generated.

$$\int \cot^3(x) \left(a+b\cot^2(x)\right)^{3/2} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(cot(x)^3*(a+b*cot(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for m
ore detail
```

3.26.8 Giac [F(-2)]

Exception generated.

$$\int \cot^3(x) \left(a+b\cot^2(x)\right)^{3/2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(x)^3*(a+b*cot(x)^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to convert to real sageVARb
Error: Bad Argument ValueUnable to convert to real sageVARb Error: Bad Arg
ument Val
```

3.26.9 Mupad [B] (verification not implemented)

Time = 24.41 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.36

$$\int \cot^3(x) \left(a+b\cot^2(x)\right)^{3/2} dx = \left(\frac{a}{3\,b}-\frac{a-b}{3\,b}\right)\left(b\cot(x)^2+a\right)^{3/2}-\frac{\left(b\cot(x)^2+a\right)^{5/2}}{5\,b}$$
$$+(a-b)\left(\frac{a}{b}-\frac{a-b}{b}\right)\sqrt{b\cot\left(x\right)^2+a}+\mathrm{atan}\left(\frac{(a-b)^{3/2}\sqrt{b\cot\left(x\right)^2+a}\,1\mathrm{i}}{a^2-2\,a\,b+b^2}\right)(a-b)^{3/2}\,1\mathrm{i}$$

input
```
int(cot(x)^3*(a + b*cot(x)^2)^(3/2),x)
```

output
```
atan(((a - b)^(3/2)*(a + b*cot(x)^2)^(1/2)*1i)/(a^2 - 2*a*b + b^2))*(a - b
)^(3/2)*1i - (a + b*cot(x)^2)^(5/2)/(5*b) + (a/(3*b) - (a - b)/(3*b))*(a +
 b*cot(x)^2)^(3/2) + (a - b)*(a/b - (a - b)/b)*(a + b*cot(x)^2)^(1/2)
```

3.27 $\int \cot^2(x) \left(a+b\cot^2(x)\right)^{3/2} \, dx$

3.27.1 Optimal result

Integrand size = 17, antiderivative size = 127

$$\int \cot^2(x) \left(a+b\cot^2(x)\right)^{3/2} \, dx = (a-b)^{3/2}\arctan\left(\frac{\sqrt{a-b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right) - \frac{(3a^2-12ab+8b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{8\sqrt{b}} - \frac{1}{8}(5a-4b)\cot(x)\sqrt{a+b\cot^2(x)} - \frac{1}{4}b\cot^3(x)\sqrt{a+b\cot^2(x)}$$

output
```
(a-b)^(3/2)*arctan(cot(x)*(a-b)^(1/2)/(a+b*cot(x)^2)^(1/2))-1/8*(3*a^2-12*
a*b+8*b^2)*arctanh(cot(x)*b^(1/2)/(a+b*cot(x)^2)^(1/2))/b^(1/2)-1/8*(5*a-4
*b)*cot(x)*(a+b*cot(x)^2)^(1/2)-1/4*b*cot(x)^3*(a+b*cot(x)^2)^(1/2)
```

3.27.2 Mathematica [A] (verified)

Time = 1.32 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.99

$$\int \cot^2(x) \left(a+b\cot^2(x)\right)^{3/2} \, dx = \frac{\sqrt{-a-b+(a-b)\cos(2x)}\csc(x)\left(8\sqrt{2}(a-b)^2\sqrt{-b}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{a-b}\cos(x)}{\sqrt{-a-b+(a-b)\cos(2x)}}\right)+\sqrt{a}\right.}{8\sqrt{2}\sqrt{}}$$

input
```
Integrate[Cot[x]^2*(a + b*Cot[x]^2)^(3/2),x]
```

output
```
(Sqrt[-a - b + (a - b)*Cos[2*x]]*Csc[x]*(8*Sqrt[2]*(a - b)^2*Sqrt[-b]*ArcT
anh[(Sqrt[2]*Sqrt[a - b]*Cos[x])/Sqrt[-a - b + (a - b)*Cos[2*x]]] + Sqrt[a
 - b]*(-(Sqrt[2]*(3*a^2 - 12*a*b + 8*b^2)*ArcTanh[(Sqrt[2]*Sqrt[-b]*Cos[x]
)/Sqrt[-a - b + (a - b)*Cos[2*x]]]) + Sqrt[-b]*Sqrt[-a - b + (a - b)*Cos[2
*x]]*Cot[x]*Csc[x]*(5*a - 6*b + 2*b*Csc[x]^2))))/(8*Sqrt[2]*Sqrt[a - b]*Sq
rt[-b]*Sqrt[-((-a - b + (a - b)*Cos[2*x])*Csc[x]^2)])
```

3.27.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.588, Rules used = {3042, 4153, 379, 444, 27, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(x)\left(a+b\cot^2(x)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan\left(x+\frac{\pi}{2}\right)^2\left(a+b\tan\left(x+\frac{\pi}{2}\right)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{4153}$$

$$-\int \frac{\cot^2(x)\left(b\cot^2(x)+a\right)^{3/2}}{\cot^2(x)+1}d\cot(x)$$

$$\Big\downarrow \text{379}$$

$$-\frac{1}{4}\int \frac{\cot^2(x)\left((5a-4b)b\cot^2(x)+a(4a-3b)\right)}{\left(\cot^2(x)+1\right)\sqrt{b\cot^2(x)+a}}d\cot(x)-\frac{1}{4}b\cot^3(x)\sqrt{a+b\cot^2(x)}$$

$$\Big\downarrow \text{444}$$

$$\frac{1}{4}\left(\frac{\int \frac{b\left(a(5a-4b)-\left(3a^2-12ba+8b^2\right)\cot^2(x)\right)}{\left(\cot^2(x)+1\right)\sqrt{b\cot^2(x)+a}}d\cot(x)}{2b}-\frac{1}{2}(5a-4b)\cot(x)\sqrt{a+b\cot^2(x)}\right)-\frac{1}{4}b\cot^3(x)\sqrt{a+b\cot^2(x)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{a(5a-4b)-\left(3a^2-12ba+8b^2\right)\cot^2(x)}{\left(\cot^2(x)+1\right)\sqrt{b\cot^2(x)+a}}d\cot(x)-\frac{1}{2}(5a-4b)\cot(x)\sqrt{a+b\cot^2(x)}\right)-\frac{1}{4}b\cot^3(x)\sqrt{a+b\cot^2(x)}$$

$\downarrow$ 398

$$\frac{1}{4}\left(\frac{1}{2}\left(8(a-b)^2\int \frac{1}{\left(\cot^2(x)+1\right)\sqrt{b\cot^2(x)+a}}d\cot(x)-\left(3a^2-12ab+8b^2\right)\int \frac{1}{\sqrt{b\cot^2(x)+a}}d\cot(x)\right)-\frac{1}{2}(5 \right. \frac{1}{4}b\cot^3(x)\sqrt{a+b\cot^2(x)}$$

$\downarrow$ 224

$$\frac{1}{4}\left(\frac{1}{2}\left(8(a-b)^2\int \frac{1}{\left(\cot^2(x)+1\right)\sqrt{b\cot^2(x)+a}}d\cot(x)-\left(3a^2-12ab+8b^2\right)\int \frac{1}{1-\frac{b\cot^2(x)}{b\cot^2(x)+a}}d\frac{\cot(x)}{\sqrt{b\cot^2(x)+a}} \right.\right. \frac{1}{4}b\cot^3(x)\sqrt{a+b\cot^2(x)}$$

$\downarrow$ 219

$$\frac{1}{4}\left(\frac{1}{2}\left(8(a-b)^2\int \frac{1}{\left(\cot^2(x)+1\right)\sqrt{b\cot^2(x)+a}}d\cot(x)-\frac{\left(3a^2-12ab+8b^2\right)\text{arctanh}\left(\frac{\sqrt{b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{\sqrt{b}}\right)-\frac{1}{2}(5a \right. \frac{1}{4}b\cot^3(x)\sqrt{a+b\cot^2(x)}$$

$\downarrow$ 291

$$\frac{1}{4}\left(\frac{1}{2}\left(8(a-b)^2\int \frac{1}{1-\frac{(b-a)\cot^2(x)}{b\cot^2(x)+a}}d\frac{\cot(x)}{\sqrt{b\cot^2(x)+a}}-\frac{\left(3a^2-12ab+8b^2\right)\text{arctanh}\left(\frac{\sqrt{b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{\sqrt{b}}\right)-\frac{1}{2}(5a-4b) \right. \frac{1}{4}b\cot^3(x)\sqrt{a+b\cot^2(x)}$$

$\downarrow$ 216

$$\frac{1}{4}\left(\frac{1}{2}\left(8(a-b)^{3/2}\arctan\left(\frac{\sqrt{a-b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)-\frac{\left(3a^2-12ab+8b^2\right)\text{arctanh}\left(\frac{\sqrt{b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{\sqrt{b}}\right)-\frac{1}{2}(5a-4b)\cot(x) \right. \frac{1}{4}b\cot^3(x)\sqrt{a+b\cot^2(x)}$$

3.27. $\int \cot^2(x)\left(a+b\cot^2(x)\right)^{3/2}dx$

input
```
Int[Cot[x]^2*(a + b*Cot[x]^2)^(3/2),x]
```

output
```
-1/4*(b*Cot[x]^3*Sqrt[a + b*Cot[x]^2]) + ((8*(a - b)^(3/2)*ArcTan[(Sqrt[a
- b]*Cot[x])/Sqrt[a + b*Cot[x]^2]] - ((3*a^2 - 12*a*b + 8*b^2)*ArcTanh[(Sq
rt[b]*Cot[x])/Sqrt[a + b*Cot[x]^2]])/Sqrt[b])/2 - ((5*a - 4*b)*Cot[x]*Sqrt
[a + b*Cot[x]^2])/2)/4
```

3.27.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 379
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q -
1)/(b*e*(m + 2*(p + q) + 1))), x] + Simp[1/(b*(m + 2*(p + q) + 1))   Int[(e
*x)^m*(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*((b*c - a*d)*(m + 1) + b*c*2
*(p + q)) + (d*(b*c - a*d)*(m + 1) + d*2*(q - 1)*(b*c - a*d) + b*c*d*2*(p +
 q))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d, 0
] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.27.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 285 vs. $2(105) = 210$.

Time = 0.03 (sec) , antiderivative size = 286, normalized size of antiderivative = 2.25

method	result
derivativedivides	$-\frac{\cot(x)\left(a+b\cot(x)^2\right)^{\frac{3}{2}}}{4}-\frac{3a\cot(x)\sqrt{a+b\cot(x)^2}}{8}-\frac{3a^2\ln\left(\sqrt{b}\,\cot(x)+\sqrt{a+b\cot(x)^2}\right)}{8\sqrt{b}}-b^{\frac{3}{2}}\ln\left(\sqrt{b}\,\cot(\right.$
default	$-\frac{\cot(x)\left(a+b\cot(x)^2\right)^{\frac{3}{2}}}{4}-\frac{3a\cot(x)\sqrt{a+b\cot(x)^2}}{8}-\frac{3a^2\ln\left(\sqrt{b}\,\cot(x)+\sqrt{a+b\cot(x)^2}\right)}{8\sqrt{b}}-b^{\frac{3}{2}}\ln\left(\sqrt{b}\,\cot(\right.$

input
```
int(cot(x)^2*(a+b*cot(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*cot(x)*(a+b*cot(x)^2)^(3/2)-3/8*a*cot(x)*(a+b*cot(x)^2)^(1/2)-3/8*a^2
/b^(1/2)*ln(b^(1/2)*cot(x)+(a+b*cot(x)^2)^(1/2))-b^(3/2)*ln(b^(1/2)*cot(x)
+(a+b*cot(x)^2)^(1/2))+1/2*b*cot(x)*(a+b*cot(x)^2)^(1/2)+3/2*b^(1/2)*a*ln(
b^(1/2)*cot(x)+(a+b*cot(x)^2)^(1/2))+(b^4*(a-b))^(1/2)/(a-b)*arctan(b^2*(a
-b)/(b^4*(a-b))^(1/2)/(a+b*cot(x)^2)^(1/2)*cot(x))-2*a/b*(b^4*(a-b))^(1/2)
/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*cot(x)^2)^(1/2)*cot(x))+a^2
*(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*cot(x
)^2)^(1/2)*cot(x))
```

3.27.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(105) = 210$.

Time = 0.33 (sec) , antiderivative size = 1134, normalized size of antiderivative = 8.93

$$\int \cot^2(x) \left(a + b\cot^2(x)\right)^{3/2} dx = \text{Too large to display}$$

input
```
integrate(cot(x)^2*(a+b*cot(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/16*(8*(a*b - b^2 - (a*b - b^2)*cos(2*x))*sqrt(-a + b)*log(-(a - b)*cos(
2*x) + sqrt(-a + b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))*sin(2*
x) + b)*sin(2*x) - (3*a^2 - 12*a*b + 8*b^2 - (3*a^2 - 12*a*b + 8*b^2)*cos(
2*x))*sqrt(b)*log(((a - 2*b)*cos(2*x) + 2*sqrt(b)*sqrt(((a - b)*cos(2*x) -
 a - b)/(cos(2*x) - 1))*sin(2*x) - a - 2*b)/(cos(2*x) - 1))*sin(2*x) + 2*(
4*b^2*cos(2*x) - (5*a*b - 6*b^2)*cos(2*x)^2 + 5*a*b - 2*b^2)*sqrt(((a - b)
*cos(2*x) - a - b)/(cos(2*x) - 1)))/((b*cos(2*x) - b)*sin(2*x)), -1/8*((3*
a^2 - 12*a*b + 8*b^2 - (3*a^2 - 12*a*b + 8*b^2)*cos(2*x))*sqrt(-b)*arctan(
sqrt(-b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))*sin(2*x)/(b*cos(2
*x) + b))*sin(2*x) - 4*(a*b - b^2 - (a*b - b^2)*cos(2*x))*sqrt(-a + b)*log
(-(a - b)*cos(2*x) + sqrt(-a + b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x
) - 1))*sin(2*x) + b)*sin(2*x) - (4*b^2*cos(2*x) - (5*a*b - 6*b^2)*cos(2*x
)^2 + 5*a*b - 2*b^2)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1)))/((b*
cos(2*x) - b)*sin(2*x)), -1/16*(16*(a*b - b^2 - (a*b - b^2)*cos(2*x))*sqrt
(a - b)*arctan(-sqrt(a - b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1)
)*sin(2*x)/((a - b)*cos(2*x) + a - b))*sin(2*x) + (3*a^2 - 12*a*b + 8*b^2
- (3*a^2 - 12*a*b + 8*b^2)*cos(2*x))*sqrt(b)*log(((a - 2*b)*cos(2*x) + 2*s
qrt(b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))*sin(2*x) - a - 2*b)
/(cos(2*x) - 1))*sin(2*x) - 2*(4*b^2*cos(2*x) - (5*a*b - 6*b^2)*cos(2*x)^2
 + 5*a*b - 2*b^2)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1)))/((b*...
```

3.27.6 Sympy [F]

$$\int \cot^2(x)\left(a+b\cot^2(x)\right)^{3/2}\,dx=\int \left(a+b\cot^2{(x)}\right)^{\frac{3}{2}}\cot^2{(x)}\,dx$$

input
```
integrate(cot(x)**2*(a+b*cot(x)**2)**(3/2),x)
```

output
```
Integral((a + b*cot(x)**2)**(3/2)*cot(x)**2, x)
```

3.27.7 Maxima [F]

$$\int \cot^2(x)\left(a+b\cot^2(x)\right)^{3/2}\,dx=\int \left(b\cot{(x)}^2+a\right)^{\frac{3}{2}}\cot{(x)}^2\,dx$$

input
```
integrate(cot(x)^2*(a+b*cot(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*cot(x)^2 + a)^(3/2)*cot(x)^2, x)
```

3.27.8 Giac [F(-2)]

Exception generated.

$$\int \cot^2(x)\left(a+b\cot^2(x)\right)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(cot(x)^2*(a+b*cot(x)^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.27.9 Mupad [F(-1)]

Timed out.

$$\int \cot^2(x) \left(a + b\cot^2(x)\right)^{3/2} dx = \int \cot(x)^2 \left(b\cot(x)^2 + a\right)^{3/2} dx$$

input
```
int(cot(x)^2*(a + b*cot(x)^2)^(3/2),x)
```

output
```
int(cot(x)^2*(a + b*cot(x)^2)^(3/2), x)
```

3.28 $\int \cot(x) \left(a + b\cot^2(x)\right)^{3/2} dx$

3.28.1 Optimal result

Integrand size = 15, antiderivative size = 69

$$\int \cot(x) \left(a + b\cot^2(x)\right)^{3/2} dx = (a-b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right) - (a-b)\sqrt{a+b\cot^2(x)} - \frac{1}{3}\left(a+b\cot^2(x)\right)^{3/2}$$

output
```
(a-b)^(3/2)*arctanh((a+b*cot(x)^2)^(1/2)/(a-b)^(1/2))-1/3*(a+b*cot(x)^2)^(
3/2)-(a-b)*(a+b*cot(x)^2)^(1/2)
```

3.28.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91

$$\int \cot(x) \left(a + b\cot^2(x)\right)^{3/2} dx = (a-b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right) - \frac{1}{3}\sqrt{a+b\cot^2(x)}\left(4a-3b+b\cot^2(x)\right)$$

input
```
Integrate[Cot[x]*(a + b*Cot[x]^2)^(3/2),x]
```

output
```
(a - b)^(3/2)*ArcTanh[Sqrt[a + b*Cot[x]^2]/Sqrt[a - b]] - (Sqrt[a + b*Cot[
x]^2]*(4*a - 3*b + b*Cot[x]^2))/3
```

3.28.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 25, 4153, 25, 353, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(x)\left(a+b\cot^2(x)\right)^{3/2}dx$$

$$\Big\downarrow \ 3042$$

$$\int -\tan\left(x+\frac{\pi}{2}\right)\left(a+b\tan\left(x+\frac{\pi}{2}\right)^2\right)^{3/2}dx$$

$$\Big\downarrow \ 25$$

$$-\int \tan\left(x+\frac{\pi}{2}\right)\left(b\tan\left(x+\frac{\pi}{2}\right)^2+a\right)^{3/2}dx$$

$$\Big\downarrow \ 4153$$

$$\int -\frac{\cot(x)\left(a+b\cot^2(x)\right)^{3/2}}{\cot^2(x)+1}d\cot(x)$$

$$\Big\downarrow \ 25$$

$$-\int \frac{\cot(x)\left(b\cot^2(x)+a\right)^{3/2}}{\cot^2(x)+1}d\cot(x)$$

$$\Big\downarrow \ 353$$

$$-\frac{1}{2}\int \frac{\left(b\cot^2(x)+a\right)^{3/2}}{\cot^2(x)+1}d\cot^2(x)$$

$$\Big\downarrow \ 60$$

$$\frac{1}{2}\left(-(a-b)\int \frac{\sqrt{b\cot^2(x)+a}}{\cot^2(x)+1}d\cot^2(x)-\frac{2}{3}\left(a+b\cot^2(x)\right)^{3/2}\right)$$

$$\Big\downarrow \ 60$$

$$\frac{1}{2}\left(-(a-b)\left((a-b)\int \frac{1}{\left(\cot^2(x)+1\right)\sqrt{b\cot^2(x)+a}}d\cot^2(x)+2\sqrt{a+b\cot^2(x)}\right)-\frac{2}{3}\left(a+b\cot^2(x)\right)^{3/2}\right)$$

$$\Big\downarrow \ 73$$

$$\frac{1}{2}\left(-(a-b)\left(\frac{2(a-b)\int \frac{1}{\frac{\cot^4(x)}{b}-\frac{a}{b}+1}d\sqrt{b\cot^2(x)+a}}{b}+2\sqrt{a+b\cot^2(x)}\right)-\frac{2}{3}\left(a+b\cot^2(x)\right)^{3/2}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{2}\left(-(a-b)\left(2\sqrt{a+b\cot^2(x)}-2\sqrt{a-b}\text{arctanh}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)\right)-\frac{2}{3}\left(a+b\cot^2(x)\right)^{3/2}\right)$$

input
```
Int[Cot[x]*(a + b*Cot[x]^2)^(3/2),x]
```

output
```
((-2*(a + b*Cot[x]^2)^(3/2))/3 - (a - b)*(-2*Sqrt[a - b]*ArcTanh[Sqrt[a +
b*Cot[x]^2]/Sqrt[a - b]] + 2*Sqrt[a + b*Cot[x]^2]))/2
```

3.28.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.28.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(57) = 114$.

Time = 0.02 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.97

method	result
derivativedivides	$-\frac{b\cot(x)^2\sqrt{a+b\cot(x)^2}}{3}-\frac{4a\sqrt{a+b\cot(x)^2}}{3}+b\sqrt{a+b\cot\left(x\right)^2}-\frac{b^2\arctan\left(\frac{\sqrt{a+b\cot(x)^2}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}+\frac{2ab\arctan}{}$
default	$-\frac{b\cot(x)^2\sqrt{a+b\cot(x)^2}}{3}-\frac{4a\sqrt{a+b\cot(x)^2}}{3}+b\sqrt{a+b\cot\left(x\right)^2}-\frac{b^2\arctan\left(\frac{\sqrt{a+b\cot(x)^2}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}+\frac{2ab\arctan}{}$

input
```
int(cot(x)*(a+b*cot(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*b*cot(x)^2*(a+b*cot(x)^2)^(1/2)-4/3*a*(a+b*cot(x)^2)^(1/2)+b*(a+b*cot
(x)^2)^(1/2)-b^2/(-a+b)^(1/2)*arctan((a+b*cot(x)^2)^(1/2)/(-a+b)^(1/2))+2*
a*b/(-a+b)^(1/2)*arctan((a+b*cot(x)^2)^(1/2)/(-a+b)^(1/2))-a^2/(-a+b)^(1/2
)*arctan((a+b*cot(x)^2)^(1/2)/(-a+b)^(1/2))
```

3.28.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(57) = 114$.

Time = 0.33 (sec) , antiderivative size = 330, normalized size of antiderivative = 4.78

$$\int \cot(x) \left(a + b\cot^2(x)\right)^{3/2} dx = \left[-\frac{3\left((a-b)\cos(2x) - a + b\right)\sqrt{a-b}\log\left(-2\left(a^2 - 2ab + b^2\right)\cos(2x)^2 - 2a^2 + b^2 + 2\right.}{}\right.$$

input
```
integrate(cot(x)*(a+b*cot(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/12*(3*((a - b)*cos(2*x) - a + b)*sqrt(a - b)*log(-2*(a^2 - 2*a*b + b^2
)*cos(2*x)^2 - 2*a^2 + b^2 + 2*((a - b)*cos(2*x)^2 - (2*a - b)*cos(2*x) +
a)*sqrt(a - b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1)) + 4*(a^2 -
a*b)*cos(2*x)) + 8*(2*(a - b)*cos(2*x) - 2*a + b)*sqrt(((a - b)*cos(2*x) -
 a - b)/(cos(2*x) - 1)))/(cos(2*x) - 1), 1/6*(3*((a - b)*cos(2*x) - a + b)
*sqrt(-a + b)*arctan(-sqrt(-a + b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*
x) - 1))*(cos(2*x) - 1)/((a - b)*cos(2*x) - a)) - 4*(2*(a - b)*cos(2*x) -
2*a + b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1)))/(cos(2*x) - 1)]
```

3.28.6 Sympy [F]

$$\int \cot(x) \left(a + b\cot^2(x)\right)^{3/2} dx = \int \left(a + b\cot^2(x)\right)^{\frac{3}{2}} \cot(x)\, dx$$

input
```
integrate(cot(x)*(a+b*cot(x)**2)**(3/2),x)
```

output
```
Integral((a + b*cot(x)**2)**(3/2)*cot(x), x)
```

3.28.7 Maxima [F(-2)]

Exception generated.

$$\int \cot(x) \left(a + b\cot^2(x)\right)^{3/2} dx = \text{Exception raised: ValueError}$$

input
```
integrate(cot(x)*(a+b*cot(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for m
ore detail
```

3.28.8 Giac [F(-2)]

Exception generated.

$$\int \cot(x) \left(a + b\cot^2(x)\right)^{3/2} dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(x)*(a+b*cot(x)^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to convert to real sageVARb
Error: Bad Argument ValueUnable to convert to real sageVARb Error: Bad Arg
ument Val
```

3.28.9 Mupad [B] (verification not implemented)

Time = 16.99 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.01

$$\int \cot(x) \left(a + b\cot^2(x)\right)^{3/2} dx = \operatorname{atanh}\left(\frac{(a-b)^{3/2}\sqrt{b\cot(x)^2+a}}{a^2-2ab+b^2}\right)(a-b)^{3/2} - \frac{\left(b\cot(x)^2+a\right)^{3/2}}{3} - (a-b)\sqrt{b\cot(x)^2+a}$$

input
```
int(cot(x)*(a + b*cot(x)^2)^(3/2),x)
```

output
```
atanh(((a - b)^(3/2)*(a + b*cot(x)^2)^(1/2))/(a^2 - 2*a*b + b^2))*(a - b)^
(3/2) - (a + b*cot(x)^2)^(3/2)/3 - (a - b)*(a + b*cot(x)^2)^(1/2)
```

3.28. $\int \cot(x) \left(a + b\cot^2(x)\right)^{3/2} \, dx$

3.29 $\int (a+b\cot^2(x))^{3/2}\tan(x)\,dx$

3.29.1 Optimal result

Integrand size = 15, antiderivative size = 75

$$\int (a+b\cot^2(x))^{3/2}\tan(x)\,dx = a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a}}\right) - (a-b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right) - b\sqrt{a+b\cot^2(x)}$$

output
```
a^(3/2)*arctanh((a+b*cot(x)^2)^(1/2)/a^(1/2))-(a-b)^(3/2)*arctanh((a+b*cot
(x)^2)^(1/2)/(a-b)^(1/2))-b*(a+b*cot(x)^2)^(1/2)
```

3.29.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int (a+b\cot^2(x))^{3/2}\tan(x)\,dx = a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a}}\right) - (a-b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right) - b\sqrt{a+b\cot^2(x)}$$

input
```
Integrate[(a + b*Cot[x]^2)^(3/2)*Tan[x],x]
```

output
```
a^(3/2)*ArcTanh[Sqrt[a + b*Cot[x]^2]/Sqrt[a]] - (a - b)^(3/2)*ArcTanh[Sqrt
[a + b*Cot[x]^2]/Sqrt[a - b]] - b*Sqrt[a + b*Cot[x]^2]
```

3.29.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 25, 4153, 25, 354, 95, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(x)\left(a+b\cot^2(x)\right)^{3/2}dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{\left(a+b\tan\left(x+\frac{\pi}{2}\right)^2\right)^{3/2}}{\tan\left(x+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 25$$

$$-\int \frac{\left(b\tan\left(x+\frac{\pi}{2}\right)^2+a\right)^{3/2}}{\tan\left(x+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 4153$$

$$\int -\frac{\tan(x)\left(a+b\cot^2(x)\right)^{3/2}}{\cot^2(x)+1}d\cot(x)$$

$$\Big\downarrow \; 25$$

$$-\int \frac{\left(b\cot^2(x)+a\right)^{3/2}\tan(x)}{\cot^2(x)+1}d\cot(x)$$

$$\Big\downarrow \; 354$$

$$-\frac{1}{2}\int \frac{\left(b\cot^2(x)+a\right)^{3/2}\tan(x)}{\cot^2(x)+1}d\cot^2(x)$$

$$\Big\downarrow \; 95$$

$$\frac{1}{2}\left(-\int \frac{\left(a^2+(2a-b)b\cot^2(x)\right)\tan(x)}{\left(\cot^2(x)+1\right)\sqrt{b\cot^2(x)+a}}d\cot^2(x)-2b\sqrt{a+b\cot^2(x)}\right)$$

$$\Big\downarrow \; 174$$

$$\frac{1}{2}\left(a^2\left(-\int \frac{\tan(x)}{\sqrt{b\cot^2(x)+a}}d\cot^2(x)\right)+(a-b)^2\int \frac{1}{\left(\cot^2(x)+1\right)\sqrt{b\cot^2(x)+a}}d\cot^2(x)-2b\sqrt{a+b\cot^2(x)}\right)$$

$$\Big\downarrow 73$$

$$\frac{1}{2}\left(-\frac{2a^2\int \frac{1}{\frac{\cot^4(x)}{b}-\frac{a}{b}}d\sqrt{b\cot^2(x)+a}}{b}+\frac{2(a-b)^2\int \frac{1}{\frac{\cot^4(x)}{b}-\frac{a}{b}+1}d\sqrt{b\cot^2(x)+a}}{b}-2b\sqrt{a+b\cot^2(x)}\right)$$

$$\Big\downarrow 221$$

$$\frac{1}{2}\left(2a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a}}\right)-2(a-b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)-2b\sqrt{a+b\cot^2(x)}\right)$$

input
```
Int[(a + b*Cot[x]^2)^(3/2)*Tan[x],x]
```

output
```
(2*a^(3/2)*ArcTanh[Sqrt[a + b*Cot[x]^2]/Sqrt[a]] - 2*(a - b)^(3/2)*ArcTanh
[Sqrt[a + b*Cot[x]^2]/Sqrt[a - b]] - 2*b*Sqrt[a + b*Cot[x]^2])/2
```

3.29.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 95
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[f*((e + f*x)^(p - 1)/(b*d*(p - 1))), x] + Simp[1/(b*d)   Int[(b
*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*((e + f*x)^(p - 2)/((a +
b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.29.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 340 vs. 2(61) = 122.

Time = 1.28 (sec) , antiderivative size = 341, normalized size of antiderivative = 4.55

method	result
default	$\frac{\sqrt{4}\left(\cos(x)-1\right)\left(a+b\cot(x)^2\right)^{\frac{3}{2}}\left(a^{\frac{3}{2}}\sqrt{-a+b}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{a\cos(x)^2-\cos(x)^2 b-a}{(\cos(x)+1)^2}}\,(\cot(x)+\csc(x))}{\sqrt{a}}\right)\sin(x)-\cos(x)\sqrt{-\frac{a\cos(x)^2-\cos(x)^2}{(\cos(x)+1)^2}}\right.}{}$

input
```
int((a+b*cot(x)^2)^(3/2)*tan(x),x,method=_RETURNVERBOSE)
```

output
```
1/2*4^(1/2)/(-a+b)^(1/2)*(cos(x)-1)*(a+b*cot(x)^2)^(3/2)*(a^(3/2)*(-a+b)^(
1/2)*arctanh(1/a^(1/2)*(-(a*cos(x)^2-cos(x)^2*b-a)/(cos(x)+1)^2)^(1/2)*(co
t(x)+csc(x)))*sin(x)-cos(x)*(-(a*cos(x)^2-cos(x)^2*b-a)/(cos(x)+1)^2)^(1/2
)*(-a+b)^(1/2)*b+arctan(1/(-a+b)^(1/2)*(-(a*cos(x)^2-cos(x)^2*b-a)/(cos(x)
+1)^2)^(1/2)*(cot(x)+csc(x)))*a^2*sin(x)-2*arctan(1/(-a+b)^(1/2)*(-(a*cos(
x)^2-cos(x)^2*b-a)/(cos(x)+1)^2)^(1/2)*(cot(x)+csc(x)))*a*b*sin(x)+arctan(
1/(-a+b)^(1/2)*(-(a*cos(x)^2-cos(x)^2*b-a)/(cos(x)+1)^2)^(1/2)*(cot(x)+csc
(x)))*b^2*sin(x)-(-(a*cos(x)^2-cos(x)^2*b-a)/(cos(x)+1)^2)^(1/2)*(-a+b)^(1
/2)*b)/(a*cos(x)^2-cos(x)^2*b-a)/(-(a*cos(x)^2-cos(x)^2*b-a)/(cos(x)+1)^2)
^(1/2)
```

3.29.5 Fricas [A] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 565, normalized size of antiderivative = 7.53

$$\int \left(a+b\cot^2(x)\right)^{3/2}\tan(x)\,dx = \left[\frac{1}{2}a^{\frac{3}{2}}\log\left(2\,a\tan(x)^2+2\sqrt{a}\sqrt{\frac{a\tan(x)^2+b}{\tan(x)^2}}\tan(x)^2+b\right)\right.$$

$$-\frac{1}{4}(a-b)^{\frac{3}{2}}\log\left(-\frac{(8\,a^2-8\,ab+b^2)\tan(x)^4+2\,(4\,ab-3\,b^2)\tan(x)^2+b^2+4\,((2\,a-b)\tan(x)^4+b\tan(}{\tan(x)^4+2\,\tan(x)^2+1}\right.$$

$$\left.-b\sqrt{\frac{a\tan(x)^2+b}{\tan(x)^2}},-\sqrt{-a}a\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{a\tan(x)^2+b}{\tan(x)^2}}\tan(x)^2}{a\tan(x)^2+b}\right)\right.$$

$$-\frac{1}{4}(a-b)^{\frac{3}{2}}\log\left(-\frac{(8\,a^2-8\,ab+b^2)\tan(x)^4+2\,(4\,ab-3\,b^2)\tan(x)^2+b^2+4\,((2\,a-b)\tan(x)^4+b\tan(}{\tan(x)^4+2\,\tan(x)^2+1}\right.$$

$$-b\sqrt{\frac{a\tan(x)^2+b}{\tan(x)^2}},\frac{1}{2}(-a+b)^{\frac{3}{2}}\arctan\left(-\frac{2\sqrt{-a+b}\sqrt{\frac{a\tan(x)^2+b}{\tan(x)^2}}\tan(x)^2}{(2\,a-b)\tan(x)^2+b}\right)$$

$$+\frac{1}{2}a^{\frac{3}{2}}\log\left(2\,a\tan(x)^2+2\sqrt{a}\sqrt{\frac{a\tan(x)^2+b}{\tan(x)^2}}\tan(x)^2+b\right)$$

$$-b\sqrt{\frac{a\tan(x)^2+b}{\tan(x)^2}},-\sqrt{-a}a\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{a\tan(x)^2+b}{\tan(x)^2}}\tan(x)^2}{a\tan(x)^2+b}\right)$$

$$\left.+\frac{1}{2}(-a+b)^{\frac{3}{2}}\arctan\left(-\frac{2\sqrt{-a+b}\sqrt{\frac{a\tan(x)^2+b}{\tan(x)^2}}\tan(x)^2}{(2\,a-b)\tan(x)^2+b}\right)-b\sqrt{\frac{a\tan(x)^2+b}{\tan(x)^2}}\right]$$

input
```
integrate((a+b*cot(x)^2)^(3/2)*tan(x),x, algorithm="fricas")
```

output
```
[1/2*a^(3/2)*log(2*a*tan(x)^2 + 2*sqrt(a)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*
tan(x)^2 + b) - 1/4*(a - b)^(3/2)*log(-((8*a^2 - 8*a*b + b^2)*tan(x)^4 + 2
*(4*a*b - 3*b^2)*tan(x)^2 + b^2 + 4*((2*a - b)*tan(x)^4 + b*tan(x)^2)*sqrt
(a - b)*sqrt((a*tan(x)^2 + b)/tan(x)^2))/(tan(x)^4 + 2*tan(x)^2 + 1)) - b*
sqrt((a*tan(x)^2 + b)/tan(x)^2), -sqrt(-a)*a*arctan(sqrt(-a)*sqrt((a*tan(x
)^2 + b)/tan(x)^2)*tan(x)^2/(a*tan(x)^2 + b)) - 1/4*(a - b)^(3/2)*log(-((8
*a^2 - 8*a*b + b^2)*tan(x)^4 + 2*(4*a*b - 3*b^2)*tan(x)^2 + b^2 + 4*((2*a
- b)*tan(x)^4 + b*tan(x)^2)*sqrt(a - b)*sqrt((a*tan(x)^2 + b)/tan(x)^2))/(
tan(x)^4 + 2*tan(x)^2 + 1)) - b*sqrt((a*tan(x)^2 + b)/tan(x)^2), 1/2*(-a +
 b)^(3/2)*arctan(-2*sqrt(-a + b)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x)^2/
((2*a - b)*tan(x)^2 + b)) + 1/2*a^(3/2)*log(2*a*tan(x)^2 + 2*sqrt(a)*sqrt(
(a*tan(x)^2 + b)/tan(x)^2)*tan(x)^2 + b) - b*sqrt((a*tan(x)^2 + b)/tan(x)^
2), -sqrt(-a)*a*arctan(sqrt(-a)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x)^2/(
a*tan(x)^2 + b)) + 1/2*(-a + b)^(3/2)*arctan(-2*sqrt(-a + b)*sqrt((a*tan(x
)^2 + b)/tan(x)^2)*tan(x)^2/((2*a - b)*tan(x)^2 + b)) - b*sqrt((a*tan(x)^2
 + b)/tan(x)^2)]
```

3.29.6 Sympy [F]

$$\int \left(a+b\cot^2(x)\right)^{3/2}\tan(x)\,dx=\int \left(a+b\cot^{2}\left(x\right)\right)^{\frac{3}{2}}\tan\left(x\right)dx$$

input
```
integrate((a+b*cot(x)**2)**(3/2)*tan(x),x)
```

output
```
Integral((a + b*cot(x)**2)**(3/2)*tan(x), x)
```

3.29.7 Maxima [F]

$$\int \left(a+b\cot^2(x)\right)^{3/2}\tan(x)\,dx=\int \left(b\cot\left(x\right)^{2}+a\right)^{\frac{3}{2}}\tan\left(x\right)\,dx$$

input
```
integrate((a+b*cot(x)^2)^(3/2)*tan(x),x, algorithm="maxima")
```

output
```
integrate((b*cot(x)^2 + a)^(3/2)*tan(x), x)
```

3.29.8 Giac [F(-2)]

Exception generated.

$$\int \left(a+b\cot^2(x)\right)^{3/2}\tan(x)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*cot(x)^2)^(3/2)*tan(x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.29.9 Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 506, normalized size of antiderivative = 6.75

$$\int \left(a+b\cot^2(x)\right)^{3/2}\tan(x)\,dx = \operatorname{atanh}\left(\frac{2\,b^6\,\sqrt{a^3}\,\sqrt{a+\frac{b}{\tan(x)^2}}}{-6\,a^5\,b^3+12\,a^4\,b^4-8\,a^3\,b^5+2\,a^2\,b^6} - \frac{8\,a\,b^5\,\sqrt{a^3}\,\sqrt{a+\frac{b}{\tan(x)^2}}}{-6\,a^5\,b^3+12\,a^4\,b^4-8\,a^3\,b^5+2\,a^2\,b^6} + \frac{12\,a^2\,b^4\,\sqrt{a^3}\,\sqrt{a+\frac{b}{\tan(x)^2}}}{-6\,a^5\,b^3+12\,a^4\,b^4-8\,a^3\,b^5+2\,a^2\,b^6} - \frac{6\,a^3\,b^3\,\sqrt{a^3}\,\sqrt{a+\frac{b}{\tan(x)^2}}}{-6\,a^5\,b^3+12\,a^4\,b^4-8\,a^3\,b^5+2\,a^2\,b^6}\right)\sqrt{a^3} - \operatorname{atanh}\left(\frac{2\,a\,b^5\,\sqrt{a+\frac{b}{\tan(x)^2}}\,\sqrt{a^3-3\,a^2\,b+3\,a\,b^2-b^3}}{6\,a^5\,b^3-18\,a^4\,b^4+20\,a^3\,b^5-10\,a^2\,b^6+2\,a\,b^7} - \frac{6\,a^2\,b^4\,\sqrt{a+\frac{b}{\tan(x)^2}}\,\sqrt{a^3-3\,a^2\,b+3\,a\,b^2-b^3}}{6\,a^5\,b^3-18\,a^4\,b^4+20\,a^3\,b^5-10\,a^2\,b^6+2\,a\,b^7} + \frac{6\,a^3\,b^3\,\sqrt{a+\frac{b}{\tan(x)^2}}\,\sqrt{a^3-3\,a^2\,b+3\,a\,b^2-b^3}}{6\,a^5\,b^3-18\,a^4\,b^4+20\,a^3\,b^5-10\,a^2\,b^6+2\,a\,b^7}\right)\sqrt{(a-b)^3} - b\,\sqrt{a+\frac{b}{\tan\left(x\right)^2}}$$

input
```
int(tan(x)*(a + b*cot(x)^2)^(3/2),x)
```

output
```
atanh((2*b^6*(a^3)^(1/2)*(a + b/tan(x)^2)^(1/2))/(2*a^2*b^6 - 8*a^3*b^5 +
12*a^4*b^4 - 6*a^5*b^3) - (8*a*b^5*(a^3)^(1/2)*(a + b/tan(x)^2)^(1/2))/(2*
a^2*b^6 - 8*a^3*b^5 + 12*a^4*b^4 - 6*a^5*b^3) + (12*a^2*b^4*(a^3)^(1/2)*(a
 + b/tan(x)^2)^(1/2))/(2*a^2*b^6 - 8*a^3*b^5 + 12*a^4*b^4 - 6*a^5*b^3) - (
6*a^3*b^3*(a^3)^(1/2)*(a + b/tan(x)^2)^(1/2))/(2*a^2*b^6 - 8*a^3*b^5 + 12*
a^4*b^4 - 6*a^5*b^3))*(a^3)^(1/2) - atanh((2*a*b^5*(a + b/tan(x)^2)^(1/2)*
(3*a*b^2 - 3*a^2*b + a^3 - b^3)^(1/2))/(2*a*b^7 - 10*a^2*b^6 + 20*a^3*b^5
- 18*a^4*b^4 + 6*a^5*b^3) - (6*a^2*b^4*(a + b/tan(x)^2)^(1/2)*(3*a*b^2 - 3
*a^2*b + a^3 - b^3)^(1/2))/(2*a*b^7 - 10*a^2*b^6 + 20*a^3*b^5 - 18*a^4*b^4
 + 6*a^5*b^3) + (6*a^3*b^3*(a + b/tan(x)^2)^(1/2)*(3*a*b^2 - 3*a^2*b + a^3
 - b^3)^(1/2))/(2*a*b^7 - 10*a^2*b^6 + 20*a^3*b^5 - 18*a^4*b^4 + 6*a^5*b^3
))*((a - b)^3)^(1/2) - b*(a + b/tan(x)^2)^(1/2)
```

3.29. $\int \left(a+b\cot^2(x)\right)^{3/2}\tan(x)\,dx$

3.30 $\int (a+b\cot^2(x))^{3/2}\tan^2(x)\,dx$

3.30.1 Optimal result

Integrand size = 17, antiderivative size = 80

$$\int (a+b\cot^2(x))^{3/2}\tan^2(x)\,dx = (a-b)^{3/2}\arctan\left(\frac{\sqrt{a-b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right) - b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right) + a\sqrt{a+b\cot^2(x)}\tan(x)$$

output
```
(a-b)^(3/2)*arctan(cot(x)*(a-b)^(1/2)/(a+b*cot(x)^2)^(1/2))-b^(3/2)*arctan
h(cot(x)*b^(1/2)/(a+b*cot(x)^2)^(1/2))+a*(a+b*cot(x)^2)^(1/2)*tan(x)
```

3.30.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 222 vs. 2(80) = 160.

Time = 0.82 (sec) , antiderivative size = 222, normalized size of antiderivative = 2.78

$$\int (a+b\cot^2(x))^{3/2}\tan^2(x)\,dx = \frac{\sqrt{-((-a-b+(a-b)\cos(2x))\csc^2(x))}\left(-\sqrt{2}(a-b)^2\sqrt{-b}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{a-b}\cdots}{\sqrt{-a-b+(a-\cdots}}\right.\right.}{\sqrt{2}\sqrt{a-\cdots}}$$

input
```
Integrate[(a + b*Cot[x]^2)^(3/2)*Tan[x]^2,x]
```

output
```
(Sqrt[-((-a - b + (a - b)*Cos[2*x])*Csc[x]^2)]*(-(Sqrt[2]*(a - b)^2*Sqrt[-
b]*ArcTanh[(Sqrt[2]*Sqrt[a - b]*Cos[x])/Sqrt[-a - b + (a - b)*Cos[2*x]]])
+ Sqrt[a - b]*(Sqrt[2]*b^2*ArcTanh[(Sqrt[2]*Sqrt[-b]*Cos[x])/Sqrt[-a - b +
 (a - b)*Cos[2*x]]] + a*Sqrt[-b]*Sqrt[-a - b + (a - b)*Cos[2*x]]*Sec[x]))*
Sin[x])/(Sqrt[2]*Sqrt[a - b]*Sqrt[-b]*Sqrt[-a - b + (a - b)*Cos[2*x]])
```

3.30.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {3042, 4153, 376, 25, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(x) \left(a+b\cot^2(x)\right)^{3/2} dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a+b\tan\left(x+\frac{\pi}{2}\right)^2\right)^{3/2}}{\tan\left(x+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \; 4153$$

$$-\int \frac{\left(b\cot^2(x)+a\right)^{3/2}\tan^2(x)}{\cot^2(x)+1}d\cot(x)$$

$$\Big\downarrow \; 376$$

$$a\tan(x)\sqrt{a+b\cot^2(x)}-\int -\frac{a(a-2b)-b^2\cot^2(x)}{\left(\cot^2(x)+1\right)\sqrt{b\cot^2(x)+a}}d\cot(x)$$

$$\Big\downarrow \; 25$$

$$\int \frac{a(a-2b)-b^2\cot^2(x)}{\left(\cot^2(x)+1\right)\sqrt{b\cot^2(x)+a}}d\cot(x)+a\tan(x)\sqrt{a+b\cot^2(x)}$$

$$\Big\downarrow \; 398$$

$$-b^2\int \frac{1}{\sqrt{b\cot^2(x)+a}}d\cot(x)+(a-b)^2\int \frac{1}{\left(\cot^2(x)+1\right)\sqrt{b\cot^2(x)+a}}d\cot(x)+ a\tan(x)\sqrt{a+b\cot^2(x)}$$

$$\Big\downarrow \; 224$$

$$-b^2\int \frac{1}{1-\frac{b\cot^2(x)}{b\cot^2(x)+a}}d\frac{\cot(x)}{\sqrt{b\cot^2(x)+a}}+(a-b)^2\int \frac{1}{\left(\cot^2(x)+1\right)\sqrt{b\cot^2(x)+a}}d\cot(x)+ a\tan(x)\sqrt{a+b\cot^2(x)}$$

$$\Big\downarrow \text{219}$$

$$(a-b)^2\int \frac{1}{\left(\cot^2(x)+1\right)\sqrt{b\cot^2(x)+a}}d\cot(x)-b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)+ a\tan(x)\sqrt{a+b\cot^2(x)}$$

$$\Big\downarrow \text{291}$$

$$(a-b)^2\int \frac{1}{1-\frac{(b-a)\cot^2(x)}{b\cot^2(x)+a}}d\frac{\cot(x)}{\sqrt{b\cot^2(x)+a}}-b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)+ a\tan(x)\sqrt{a+b\cot^2(x)}$$

$$\Big\downarrow \text{216}$$

$$(a-b)^{3/2}\arctan\left(\frac{\sqrt{a-b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)-b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)+a\tan(x)\sqrt{a+b\cot^2(x)}$$

input
```
Int[(a + b*Cot[x]^2)^(3/2)*Tan[x]^2,x]
```

output
```
(a - b)^(3/2)*ArcTan[(Sqrt[a - b]*Cot[x])/Sqrt[a + b*Cot[x]^2]] - b^(3/2)*
ArcTanh[(Sqrt[b]*Cot[x])/Sqrt[a + b*Cot[x]^2]] + a*Sqrt[a + b*Cot[x]^2]*Ta
n[x]
```

3.30.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 376
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1
)/(a*e*(m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^
2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b*c - a*d)*(m + 1) + 2*c*(b*c*(p + 1) + a*
d*(q - 1)) + d*((b*c - a*d)*(m + 1) + 2*b*c*(p + q))*x^2, x], x], x] /; Fre
eQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && GtQ[q, 1] && LtQ[m, -1] &
& IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.30. $\int (a+b\cot^2(x))^{3/2}\tan^2(x)\,dx$

3.30.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 641 vs. $2(66) = 132$.

Time = 0.82 (sec) , antiderivative size = 642, normalized size of antiderivative = 8.02

method	result
default	$\frac{\sqrt{4}\left(2\cos(x) b^{\frac{7}{2}} \ln\left(4\cos(x)\sqrt{-a+b}\sqrt{-\frac{a\cos(x)^2-\cos(x)^2 b-a}{(\cos(x)+1)^2}}-4\cos(x)a+4b\cos(x)+4\sqrt{-a+b}\sqrt{-\frac{a\cos(x)^2-\cos(x)^2 b-a}{(\cos(x)+1)^2}}\right)-4\cos(x)b\right.}{}$

input
```
int((a+b*cot(x)^2)^(3/2)*tan(x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*4^(1/2)/b^(3/2)/(-a+b)^(1/2)*(2*cos(x)*b^(7/2)*ln(4*cos(x)*(-a+b)^(1/2
)*(-(a*cos(x)^2-cos(x)^2*b-a)/(cos(x)+1)^2)^(1/2)-4*cos(x)*a+4*b*cos(x)+4*
(-a+b)^(1/2)*(-(a*cos(x)^2-cos(x)^2*b-a)/(cos(x)+1)^2)^(1/2))-4*cos(x)*b^(
5/2)*ln(4*cos(x)*(-a+b)^(1/2)*(-(a*cos(x)^2-cos(x)^2*b-a)/(cos(x)+1)^2)^(1
/2)-4*cos(x)*a+4*b*cos(x)+4*(-a+b)^(1/2)*(-(a*cos(x)^2-cos(x)^2*b-a)/(cos(
x)+1)^2)^(1/2))*a+2*cos(x)*b^(3/2)*(-(a*cos(x)^2-cos(x)^2*b-a)/(cos(x)+1)^
2)^(1/2)*(-a+b)^(1/2)*a+2*cos(x)*b^(3/2)*ln(4*cos(x)*(-a+b)^(1/2)*(-(a*cos
(x)^2-cos(x)^2*b-a)/(cos(x)+1)^2)^(1/2)-4*cos(x)*a+4*b*cos(x)+4*(-a+b)^(1/
2)*(-(a*cos(x)^2-cos(x)^2*b-a)/(cos(x)+1)^2)^(1/2))*a^2+2*b^(3/2)*(-(a*cos
(x)^2-cos(x)^2*b-a)/(cos(x)+1)^2)^(1/2)*(-a+b)^(1/2)*a-cos(x)*ln(-4*(b^(1/
2)*(-(a*cos(x)^2-cos(x)^2*b-a)/(cos(x)+1)^2)^(1/2)*cos(x)-cos(x)*a+b*cos(x
)+(-(a*cos(x)^2-cos(x)^2*b-a)/(cos(x)+1)^2)^(1/2)*b^(1/2)+a)/(cos(x)-1))*(
-a+b)^(1/2)*b^3+cos(x)*ln(2/b^(1/2)*(b^(1/2)*(-(a*cos(x)^2-cos(x)^2*b-a)/(
cos(x)+1)^2)^(1/2)*cos(x)+(-(a*cos(x)^2-cos(x)^2*b-a)/(cos(x)+1)^2)^(1/2)*
b^(1/2)+cos(x)*a-b*cos(x)+a)/(cos(x)+1))*(-a+b)^(1/2)*b^3)*(cos(x)-1)*(a+b
*cot(x)^2)^(3/2)/(a*cos(x)^2-cos(x)^2*b-a)/(-(a*cos(x)^2-cos(x)^2*b-a)/(co
s(x)+1)^2)^(1/2)*tan(x)
```

3.30.5 Fricas [A] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 543, normalized size of antiderivative = 6.79

$$\int (a + b\cot^2(x))^{3/2}\tan^2(x)\,dx = \left[\frac{1}{4}(-a+b)^{\frac{3}{2}}\log\left(-\frac{a^2\tan(x)^4 - 2(3a^2-4ab)\tan(x)^2 + a^2 - 8ab + 8b^2 + 4\cdots}{\tan(x)^4 + 2\tan\cdots}\right.\right.$$

$$+\frac{1}{2}b^{\frac{3}{2}}\log\left(\frac{a\tan(x)^2 - 2\sqrt{b}\sqrt{\frac{a\tan(x)^2+b}{\tan(x)^2}}\tan(x) + 2b}{\tan(x)^2}\right)$$

$$+a\sqrt{\frac{a\tan(x)^2+b}{\tan(x)^2}}\tan(x), \sqrt{-b}b\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{a\tan(x)^2+b}{\tan(x)^2}}\tan(x)}{b}\right)$$

$$+\frac{1}{4}(-a+b)^{\frac{3}{2}}\log\left(-\frac{a^2\tan(x)^4 - 2(3a^2-4ab)\tan(x)^2 + a^2 - 8ab + 8b^2 + 4\left(a\tan(x)^3 - (a-2b)\tan\cdots\right.}{\tan(x)^4 + 2\tan(x)^2 + 1}\right.$$

$$+a\sqrt{\frac{a\tan(x)^2+b}{\tan(x)^2}}\tan(x), \frac{1}{2}(a-b)^{\frac{3}{2}}\arctan\left(\frac{2\sqrt{a-b}\sqrt{\frac{a\tan(x)^2+b}{\tan(x)^2}}\tan(x)}{a\tan(x)^2 - a + 2b}\right)$$

$$+\frac{1}{2}b^{\frac{3}{2}}\log\left(\frac{a\tan(x)^2 - 2\sqrt{b}\sqrt{\frac{a\tan(x)^2+b}{\tan(x)^2}}\tan(x) + 2b}{\tan(x)^2}\right)$$

$$+a\sqrt{\frac{a\tan(x)^2+b}{\tan(x)^2}}\tan(x), \frac{1}{2}(a-b)^{\frac{3}{2}}\arctan\left(\frac{2\sqrt{a-b}\sqrt{\frac{a\tan(x)^2+b}{\tan(x)^2}}\tan(x)}{a\tan(x)^2 - a + 2b}\right)$$

$$\left.+\sqrt{-b}b\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{a\tan(x)^2+b}{\tan(x)^2}}\tan(x)}{b}\right) + a\sqrt{\frac{a\tan(x)^2+b}{\tan(x)^2}}\tan(x)\right]$$

input
```
integrate((a+b*cot(x)^2)^(3/2)*tan(x)^2,x, algorithm="fricas")
```

output
```
[1/4*(-a + b)^(3/2)*log(-(a^2*tan(x)^4 - 2*(3*a^2 - 4*a*b)*tan(x)^2 + a^2
- 8*a*b + 8*b^2 + 4*(a*tan(x)^3 - (a - 2*b)*tan(x))*sqrt(-a + b)*sqrt((a*t
an(x)^2 + b)/tan(x)^2))/(tan(x)^4 + 2*tan(x)^2 + 1)) + 1/2*b^(3/2)*log((a*
tan(x)^2 - 2*sqrt(b)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x) + 2*b)/tan(x)^
2) + a*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x), sqrt(-b)*b*arctan(sqrt(-b)*
sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x)/b) + 1/4*(-a + b)^(3/2)*log(-(a^2*t
an(x)^4 - 2*(3*a^2 - 4*a*b)*tan(x)^2 + a^2 - 8*a*b + 8*b^2 + 4*(a*tan(x)^3
 - (a - 2*b)*tan(x))*sqrt(-a + b)*sqrt((a*tan(x)^2 + b)/tan(x)^2))/(tan(x)
^4 + 2*tan(x)^2 + 1)) + a*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x), 1/2*(a -
 b)^(3/2)*arctan(2*sqrt(a - b)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x)/(a*t
an(x)^2 - a + 2*b)) + 1/2*b^(3/2)*log((a*tan(x)^2 - 2*sqrt(b)*sqrt((a*tan(
x)^2 + b)/tan(x)^2)*tan(x) + 2*b)/tan(x)^2) + a*sqrt((a*tan(x)^2 + b)/tan(
x)^2)*tan(x), 1/2*(a - b)^(3/2)*arctan(2*sqrt(a - b)*sqrt((a*tan(x)^2 + b)
/tan(x)^2)*tan(x)/(a*tan(x)^2 - a + 2*b)) + sqrt(-b)*b*arctan(sqrt(-b)*sqr
t((a*tan(x)^2 + b)/tan(x)^2)*tan(x)/b) + a*sqrt((a*tan(x)^2 + b)/tan(x)^2)
*tan(x)]
```

3.30.6 Sympy [F]

$$\int \left(a+b\cot^2(x)\right)^{3/2}\tan^2(x)\,dx=\int \left(a+b\cot^2\left(x\right)\right)^{\frac{3}{2}}\tan^2\left(x\right)dx$$

input
```
integrate((a+b*cot(x)**2)**(3/2)*tan(x)**2,x)
```

output
```
Integral((a + b*cot(x)**2)**(3/2)*tan(x)**2, x)
```

3.30.7 Maxima [F]

$$\int \left(a+b\cot^2(x)\right)^{3/2}\tan^2(x)\,dx=\int \left(b\cot\left(x\right)^2+a\right)^{\frac{3}{2}}\tan\left(x\right)^2\,dx$$

input
```
integrate((a+b*cot(x)^2)^(3/2)*tan(x)^2,x, algorithm="maxima")
```

output
```
integrate((b*cot(x)^2 + a)^(3/2)*tan(x)^2, x)
```

3.30.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 625 vs. $2(66) = 132$.

Time = 18.70 (sec) , antiderivative size = 625, normalized size of antiderivative = 7.81

$$\int \left(a+b\cot^2(x)\right)^{3/2}\tan^2(x)\,dx =$$

$$-\frac{1}{2}\left(\frac{2\sqrt{-a+b}b^2\arctan\left(\frac{\left(\sqrt{-a+b}\cos(x)-\sqrt{-a\cos(x)^2+b\cos(x)^2+a}\right)^2+a-2b}{2\sqrt{ab-b^2}}\right)}{\sqrt{ab-b^2}} - (a-b)\sqrt{-a+b}\log\left(\left(\sqrt{-a+b}\right.\right.\right.$$

$$-\frac{\left(2a\sqrt{-a+b}b^2\arctan\left(\frac{\sqrt{-a+b}\sqrt{b}}{\sqrt{ab-b^2}}\right)-2ab^{\frac{5}{2}}\arctan\left(\frac{\sqrt{-a+b}\sqrt{b}}{\sqrt{ab-b^2}}\right)-2\sqrt{-a+b}b^3\arctan\left(\frac{\sqrt{-a+b}\sqrt{b}}{\sqrt{ab-b^2}}\right)+2b^{\frac{7}{2}}\text{arc}\right.}{}$$

input
```
integrate((a+b*cot(x)^2)^(3/2)*tan(x)^2,x, algorithm="giac")
```

output
```
-1/2*(2*sqrt(-a + b)*b^2*arctan(1/2*((sqrt(-a + b)*cos(x) - sqrt(-a*cos(x)
^2 + b*cos(x)^2 + a))^2 + a - 2*b)/sqrt(a*b - b^2))/sqrt(a*b - b^2) - (a -
 b)*sqrt(-a + b)*log((sqrt(-a + b)*cos(x) - sqrt(-a*cos(x)^2 + b*cos(x)^2
+ a))^2) + 4*a^2*sqrt(-a + b)/((sqrt(-a + b)*cos(x) - sqrt(-a*cos(x)^2 + b
*cos(x)^2 + a))^2 - a))*sgn(sin(x)) - 1/2*(2*a*sqrt(-a + b)*b^2*arctan(sqr
t(-a + b)*sqrt(b)/sqrt(a*b - b^2)) - 2*a*b^(5/2)*arctan(sqrt(-a + b)*sqrt(
b)/sqrt(a*b - b^2)) - 2*sqrt(-a + b)*b^3*arctan(sqrt(-a + b)*sqrt(b)/sqrt(
a*b - b^2)) + 2*b^(7/2)*arctan(sqrt(-a + b)*sqrt(b)/sqrt(a*b - b^2)) + sqr
t(a*b - b^2)*a^2*sqrt(-a + b)*log(-a - 2*sqrt(-a + b)*sqrt(b) + 2*b) - sqr
t(a*b - b^2)*a^2*sqrt(b)*log(-a - 2*sqrt(-a + b)*sqrt(b) + 2*b) - 2*sqrt(a
*b - b^2)*a*sqrt(-a + b)*b*log(-a - 2*sqrt(-a + b)*sqrt(b) + 2*b) + 2*sqrt
(a*b - b^2)*a*b^(3/2)*log(-a - 2*sqrt(-a + b)*sqrt(b) + 2*b) + sqrt(a*b -
b^2)*sqrt(-a + b)*b^2*log(-a - 2*sqrt(-a + b)*sqrt(b) + 2*b) - sqrt(a*b -
b^2)*b^(5/2)*log(-a - 2*sqrt(-a + b)*sqrt(b) + 2*b) + 2*sqrt(a*b - b^2)*a^
2*sqrt(-a + b))*sgn(sin(x))/(sqrt(a*b - b^2)*a + sqrt(a*b - b^2)*sqrt(-a +
 b)*sqrt(b) - sqrt(a*b - b^2)*b)
```

3.30.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+b\cot^2(x)\right)^{3/2}\tan^2(x)\,dx=\int \tan(x)^2\left(b\cot(x)^2+a\right)^{3/2}\,dx$$

input
```
int(tan(x)^2*(a + b*cot(x)^2)^(3/2),x)
```

output
```
int(tan(x)^2*(a + b*cot(x)^2)^(3/2), x)
```

3.31 $\int (a+b\cot^2(c+dx))^{5/2}\,dx$

3.31.1 Optimal result

Integrand size = 16, antiderivative size = 171

$$\int \left(a+b\cot^2(c+dx)\right)^{5/2}\,dx = -\frac{(a-b)^{5/2}\arctan\left(\frac{\sqrt{a-b}\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{d} - \frac{\sqrt{b}(15a^2-20ab+8b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{8d} - \frac{(7a-4b)b\cot(c+dx)\sqrt{a+b\cot^2(c+dx)}}{8d} - \frac{b\cot(c+dx)\left(a+b\cot^2(c+dx)\right)^{3/2}}{4d}$$

output
```
-(a-b)^(5/2)*arctan(cot(d*x+c)*(a-b)^(1/2)/(a+b*cot(d*x+c)^2)^(1/2))/d-1/4
*b*cot(d*x+c)*(a+b*cot(d*x+c)^2)^(3/2)/d-1/8*(15*a^2-20*a*b+8*b^2)*arctanh
(cot(d*x+c)*b^(1/2)/(a+b*cot(d*x+c)^2)^(1/2))*b^(1/2)/d-1/8*(7*a-4*b)*b*co
t(d*x+c)*(a+b*cot(d*x+c)^2)^(1/2)/d
```

3.31.2 Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.99

$$\int \left(a+b\cot^2(c+dx)\right)^{5/2}\,dx = \frac{8(a-b)^{5/2}\arctan\left(\frac{\sqrt{b}+\sqrt{b}\cot^2(c+dx)-\cot(c+dx)\sqrt{a+b\cot^2(c+dx)}}{\sqrt{a-b}}\right) - b\cot(c+dx)\sqrt{a+b\cot^2(c+dx}$$

input
```
Integrate[(a + b*Cot[c + d*x]^2)^(5/2),x]
```

output
```
(8*(a - b)^(5/2)*ArcTan[(Sqrt[b] + Sqrt[b]*Cot[c + d*x]^2 - Cot[c + d*x]*S
qrt[a + b*Cot[c + d*x]^2])/Sqrt[a - b]] - b*Cot[c + d*x]*Sqrt[a + b*Cot[c
+ d*x]^2]*(9*a - 4*b + 2*b*Cot[c + d*x]^2) + Sqrt[b]*(15*a^2 - 20*a*b + 8*
b^2)*Log[-(Sqrt[b]*Cot[c + d*x]) + Sqrt[a + b*Cot[c + d*x]^2]])/(8*d)
```

3.31.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {3042, 4144, 318, 403, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\cot^2(c+dx)\right)^{5/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\tan\left(c+dx+\frac{\pi}{2}\right)^2\right)^{5/2}dx$$

$$\Big\downarrow \text{4144}$$

$$-\frac{\int \frac{\left(b\cot^2(c+dx)+a\right)^{5/2}}{\cot^2(c+dx)+1}d\cot(c+dx)}{d}$$

$$\Big\downarrow \text{318}$$

$$-\frac{\frac{1}{4}\int \frac{\sqrt{b\cot^2(c+dx)+a}\left((7a-4b)b\cot^2(c+dx)+a(4a-b)\right)}{\cot^2(c+dx)+1}d\cot(c+dx)+\frac{1}{4}b\cot(c+dx)\left(a+b\cot^2(c+dx)\right)^{3/2}}{d}$$

$$\Big\downarrow \text{403}$$

$$-\frac{\frac{1}{4}\left(\frac{1}{2}\int \frac{b\left(15a^2-20ba+8b^2\right)\cot^2(c+dx)+a\left(8a^2-9ba+4b^2\right)}{\left(\cot^2(c+dx)+1\right)\sqrt{b\cot^2(c+dx)+a}}d\cot(c+dx)+\frac{1}{2}b(7a-4b)\cot(c+dx)\sqrt{a+b\cot^2(c+dx)}\right)+\frac{1}{4}}{d}$$

$$\Big\downarrow \text{398}$$

$$-\frac{\frac{1}{4}\left(\frac{1}{2}\left(b\left(15a^2-20ab+8b^2\right)\int \frac{1}{\sqrt{b\cot^2(c+dx)+a}}d\cot(c+dx)+8(a-b)^3\int \frac{1}{\left(\cot^2(c+dx)+1\right)\sqrt{b\cot^2(c+dx)+a}}d\cot(c+dx\right.\right.}{d}$$

$$\Big\downarrow \text{224}$$

$$-\frac{\frac{1}{4}\left(\frac{1}{2}\left(b(15a^2-20ab+8b^2)\int \frac{1}{1-\frac{b\cot^2(c+dx)}{b\cot^2(c+dx)+a}}d\frac{\cot(c+dx)}{\sqrt{b\cot^2(c+dx)+a}}+8(a-b)^3\int \frac{1}{(\cot^2(c+dx)+1)\sqrt{b\cot^2(c+dx)+a}}d\cot(c+\right.\right.}{d}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\frac{1}{4}\left(\frac{1}{2}\left(8(a-b)^3\int \frac{1}{(\cot^2(c+dx)+1)\sqrt{b\cot^2(c+dx)+a}}d\cot(c+dx)+\sqrt{b}(15a^2-20ab+8b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)\right)\right.}{d}$$

$$\Big\downarrow \text{291}$$

$$-\frac{\frac{1}{4}\left(\frac{1}{2}\left(8(a-b)^3\int \frac{1}{1-\frac{(b-a)\cot^2(c+dx)}{b\cot^2(c+dx)+a}}d\frac{\cot(c+dx)}{\sqrt{b\cot^2(c+dx)+a}}+\sqrt{b}(15a^2-20ab+8b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)\right)+\frac{1}{2}b(7a\right.}{d}$$

$$\Big\downarrow \text{216}$$

$$-\frac{\frac{1}{4}\left(\frac{1}{2}\left(\sqrt{b}(15a^2-20ab+8b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)+8(a-b)^{5/2}\arctan\left(\frac{\sqrt{a-b}\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)\right)+\frac{1}{2}b(7a-4b)\text{co}\right.}{d}$$

input
```
Int[(a + b*Cot[c + d*x]^2)^(5/2),x]
```

output
```
-(((b*Cot[c + d*x]*(a + b*Cot[c + d*x]^2)^(3/2))/4 + ((8*(a - b)^(5/2)*Arc
Tan[(Sqrt[a - b]*Cot[c + d*x])/Sqrt[a + b*Cot[c + d*x]^2]] + Sqrt[b]*(15*a
^2 - 20*a*b + 8*b^2)*ArcTanh[(Sqrt[b]*Cot[c + d*x])/Sqrt[a + b*Cot[c + d*x
]^2]])/2 + ((7*a - 4*b)*b*Cot[c + d*x]*Sqrt[a + b*Cot[c + d*x]^2])/2)/4)/d
)
```

3.31. $\int \left(a+b\cot^2(c+dx)\right)^{5/2}\,dx$

3.31.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

3.31.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 461 vs. $2(149) = 298$.

Time = 0.18 (sec) , antiderivative size = 462, normalized size of antiderivative = 2.70

method	result
derivativedivides	$-\frac{b^{\frac{5}{2}} \ln\left(\sqrt{b}\, \cot(dx+c)+\sqrt{a+b\cot(dx+c)^{2}}\right)}{d}-\frac{b^{2} \cot(dx+c)^{3} \sqrt{a+b\cot(dx+c)^{2}}}{4d}-\frac{9ba\cot(dx+c)\sqrt{a+b\cot(dx+c)}}{8d}$
default	$-\frac{b^{\frac{5}{2}} \ln\left(\sqrt{b}\, \cot(dx+c)+\sqrt{a+b\cot(dx+c)^{2}}\right)}{d}-\frac{b^{2} \cot(dx+c)^{3} \sqrt{a+b\cot(dx+c)^{2}}}{4d}-\frac{9ba\cot(dx+c)\sqrt{a+b\cot(dx+c)}}{8d}$

input
```
int((a+b*cot(d*x+c)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d*b^(5/2)*ln(b^(1/2)*cot(d*x+c)+(a+b*cot(d*x+c)^2)^(1/2))-1/4/d*b^2*cot
(d*x+c)^3*(a+b*cot(d*x+c)^2)^(1/2)-9/8/d*b*a*cot(d*x+c)*(a+b*cot(d*x+c)^2)
^(1/2)-15/8/d*b^(1/2)*a^2*ln(b^(1/2)*cot(d*x+c)+(a+b*cot(d*x+c)^2)^(1/2))+
1/2/d*b^2*cot(d*x+c)*(a+b*cot(d*x+c)^2)^(1/2)+5/2/d*b^(3/2)*a*ln(b^(1/2)*c
ot(d*x+c)+(a+b*cot(d*x+c)^2)^(1/2))+1/d*b*(b^4*(a-b))^(1/2)/(a-b)*arctan(b
^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*cot(d*x+c)^2)^(1/2)*cot(d*x+c))-3/d*a*(b^4
*(a-b))^(1/2)/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*cot(d*x+c)^2)^
(1/2)*cot(d*x+c))+3/d*a^2/b*(b^4*(a-b))^(1/2)/(a-b)*arctan(b^2*(a-b)/(b^4*
(a-b))^(1/2)/(a+b*cot(d*x+c)^2)^(1/2)*cot(d*x+c))-1/d*a^3*(b^4*(a-b))^(1/2
)/b^2/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*cot(d*x+c)^2)^(1/2)*co
t(d*x+c))
```

3.31.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 373 vs. $2(149) = 298$.

Time = 0.32 (sec) , antiderivative size = 1520, normalized size of antiderivative = 8.89

$$\int \left(a+b\cot^2(c+dx)\right)^{5/2}\,dx=\text{Too large to display}$$

input
```
integrate((a+b*cot(d*x+c)^2)^(5/2),x, algorithm="fricas")
```

output
```
[-1/16*(8*(a^2 - 2*a*b + b^2 - (a^2 - 2*a*b + b^2)*cos(2*d*x + 2*c))*sqrt(
-a + b)*log(-(a - b)*cos(2*d*x + 2*c) + sqrt(-a + b)*sqrt(((a - b)*cos(2*d
*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1))*sin(2*d*x + 2*c) + b)*sin(2*d*x
 + 2*c) + (15*a^2 - 20*a*b + 8*b^2 - (15*a^2 - 20*a*b + 8*b^2)*cos(2*d*x +
 2*c))*sqrt(b)*log(((a - 2*b)*cos(2*d*x + 2*c) + 2*sqrt(b)*sqrt(((a - b)*c
os(2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1))*sin(2*d*x + 2*c) - a - 2*
b)/(cos(2*d*x + 2*c) - 1))*sin(2*d*x + 2*c) - 2*(4*b^2*cos(2*d*x + 2*c) -
3*(3*a*b - 2*b^2)*cos(2*d*x + 2*c)^2 + 9*a*b - 2*b^2)*sqrt(((a - b)*cos(2*
d*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1)))/((d*cos(2*d*x + 2*c) - d)*sin
(2*d*x + 2*c)), 1/16*(16*(a^2 - 2*a*b + b^2 - (a^2 - 2*a*b + b^2)*cos(2*d*
x + 2*c))*sqrt(a - b)*arctan(-sqrt(a - b)*sqrt(((a - b)*cos(2*d*x + 2*c) -
 a - b)/(cos(2*d*x + 2*c) - 1))*sin(2*d*x + 2*c)/((a - b)*cos(2*d*x + 2*c)
 + a - b))*sin(2*d*x + 2*c) - (15*a^2 - 20*a*b + 8*b^2 - (15*a^2 - 20*a*b
+ 8*b^2)*cos(2*d*x + 2*c))*sqrt(b)*log(((a - 2*b)*cos(2*d*x + 2*c) + 2*sqr
t(b)*sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1))*sin(2
*d*x + 2*c) - a - 2*b)/(cos(2*d*x + 2*c) - 1))*sin(2*d*x + 2*c) + 2*(4*b^2
*cos(2*d*x + 2*c) - 3*(3*a*b - 2*b^2)*cos(2*d*x + 2*c)^2 + 9*a*b - 2*b^2)*
sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1)))/((d*cos(2
*d*x + 2*c) - d)*sin(2*d*x + 2*c)), -1/8*((15*a^2 - 20*a*b + 8*b^2 - (15*a
^2 - 20*a*b + 8*b^2)*cos(2*d*x + 2*c))*sqrt(-b)*arctan(sqrt(-b)*sqrt(((...
```

3.31.6 Sympy [F]

$$\int \left(a+b\cot^2(c+dx)\right)^{5/2}\,dx=\int \left(a+b\cot^{2}{\left(c+dx\right)}\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*cot(d*x+c)**2)**(5/2),x)
```

output
```
Integral((a + b*cot(c + d*x)**2)**(5/2), x)
```

3.31.7 Maxima [F]

$$\int \left(a+b\cot^2(c+dx)\right)^{5/2} dx = \int \left(b\cot\left(dx+c\right)^2+a\right)^{\frac{5}{2}} dx$$

input
```
integrate((a+b*cot(d*x+c)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*cot(d*x + c)^2 + a)^(5/2), x)
```

3.31.8 Giac [F(-2)]

Exception generated.

$$\int \left(a+b\cot^2(c+dx)\right)^{5/2} dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*cot(d*x+c)^2)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.31.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+b\cot^2(c+dx)\right)^{5/2} dx = \int \left(b\cot(c+d\,x)^2+a\right)^{5/2} dx$$

input
```
int((a + b*cot(c + d*x)^2)^(5/2),x)
```

output
```
int((a + b*cot(c + d*x)^2)^(5/2), x)
```

3.32 $\int \left(a+b\cot^2(c+dx)\right)^{3/2}\,dx$

3.32.1 Optimal result

Integrand size = 16, antiderivative size = 126

$$\int \left(a+b\cot^2(c+dx)\right)^{3/2}\,dx=-\frac{(a-b)^{3/2}\arctan\left(\frac{\sqrt{a-b}\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{d}-\frac{(3a-2b)\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{2d}-\frac{b\cot(c+dx)\sqrt{a+b\cot^2(c+dx)}}{2d}$$

output
```
-(a-b)^(3/2)*arctan(cot(d*x+c)*(a-b)^(1/2)/(a+b*cot(d*x+c)^2)^(1/2))/d-1/2
*(3*a-2*b)*arctanh(cot(d*x+c)*b^(1/2)/(a+b*cot(d*x+c)^2)^(1/2))*b^(1/2)/d-
1/2*b*cot(d*x+c)*(a+b*cot(d*x+c)^2)^(1/2)/d
```

3.32.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.13

$$\int \left(a+b\cot^2(c+dx)\right)^{3/2}\,dx=\frac{2(a-b)^{3/2}\arctan\left(\frac{\sqrt{b}+\sqrt{b}\cot^2(c+dx)-\cot(c+dx)\sqrt{a+b\cot^2(c+dx)}}{\sqrt{a-b}}\right)-b\cot(c+dx)\sqrt{a+b\cot^2(c+dx)}}{2d}$$

input
```
Integrate[(a + b*Cot[c + d*x]^2)^(3/2),x]
```

output
```
(2*(a - b)^(3/2)*ArcTan[(Sqrt[b] + Sqrt[b]*Cot[c + d*x]^2 - Cot[c + d*x]*S
qrt[a + b*Cot[c + d*x]^2])/Sqrt[a - b]] - b*Cot[c + d*x]*Sqrt[a + b*Cot[c
+ d*x]^2] + (3*a - 2*b)*Sqrt[b]*Log[-(Sqrt[b]*Cot[c + d*x]) + Sqrt[a + b*C
ot[c + d*x]^2]])/(2*d)
```

3.32.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 4144, 318, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\cot^2(c+dx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\tan\left(c+dx+\frac{\pi}{2}\right)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{4144}$$

$$-\frac{\int \frac{\left(b\cot^2(c+dx)+a\right)^{3/2}}{\cot^2(c+dx)+1}d\cot(c+dx)}{d}$$

$$\Big\downarrow \text{318}$$

$$-\frac{\frac{1}{2}\int \frac{(3a-2b)b\cot^2(c+dx)+a(2a-b)}{\left(\cot^2(c+dx)+1\right)\sqrt{b\cot^2(c+dx)+a}}d\cot(c+dx)+\frac{1}{2}b\cot(c+dx)\sqrt{a+b\cot^2(c+dx)}}{d}$$

$$\Big\downarrow \text{398}$$

$$-\frac{\frac{1}{2}\left(2(a-b)^2\int \frac{1}{\left(\cot^2(c+dx)+1\right)\sqrt{b\cot^2(c+dx)+a}}d\cot(c+dx)+b(3a-2b)\int \frac{1}{\sqrt{b\cot^2(c+dx)+a}}d\cot(c+dx)\right)+\frac{1}{2}b\cot(c-}{d}$$

$$\Big\downarrow \text{224}$$

$$-\frac{\frac{1}{2}\left(2(a-b)^2\int \frac{1}{\left(\cot^2(c+dx)+1\right)\sqrt{b\cot^2(c+dx)+a}}d\cot(c+dx)+b(3a-2b)\int \frac{1}{1-\frac{b\cot^2(c+dx)}{b\cot^2(c+dx)+a}}d\frac{\cot(c+dx)}{\sqrt{b\cot^2(c+dx)+a}}\right)+\frac{1}{2}b\cot}{d}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\frac{1}{2}\left(2(a-b)^2\int\frac{1}{(\cot^2(c+dx)+1)\sqrt{b\cot^2(c+dx)+a}}d\cot(c+dx)+\sqrt{b}(3a-2b)\text{arctanh}\left(\frac{\sqrt{b}\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)\right)+\frac{1}{2}b\cot(c+d}{d}$$

$$\Big\downarrow \text{ 291}$$

$$-\frac{\frac{1}{2}\left(2(a-b)^2\int\frac{1}{1-\frac{(b-a)\cot^2(c+dx)}{b\cot^2(c+dx)+a}}d\frac{\cot(c+dx)}{\sqrt{b\cot^2(c+dx)+a}}+\sqrt{b}(3a-2b)\text{arctanh}\left(\frac{\sqrt{b}\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)\right)+\frac{1}{2}b\cot(c+dx)\sqrt{a+}}{d}$$

$$\Big\downarrow \text{ 216}$$

$$-\frac{\frac{1}{2}\left(2(a-b)^{3/2}\arctan\left(\frac{\sqrt{a-b}\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)+\sqrt{b}(3a-2b)\text{arctanh}\left(\frac{\sqrt{b}\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)\right)+\frac{1}{2}b\cot(c+dx)\sqrt{a+b\cot^2(c}}{d}$$

input
```
Int[(a + b*Cot[c + d*x]^2)^(3/2),x]
```

output
```
-(((2*(a - b)^(3/2)*ArcTan[(Sqrt[a - b]*Cot[c + d*x])/Sqrt[a + b*Cot[c + d
*x]^2]] + (3*a - 2*b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Cot[c + d*x])/Sqrt[a + b*Co
t[c + d*x]^2]])/2 + (b*Cot[c + d*x]*Sqrt[a + b*Cot[c + d*x]^2])/2)/d)
```

3.32.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

3.32.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 297 vs. $2(108) = 216$.

Time = 0.04 (sec) , antiderivative size = 298, normalized size of antiderivative = 2.37

method	result
derivativedivides	$\frac{b^{\frac{3}{2}} \ln\left(\sqrt{b}\, \cot(dx+c)+\sqrt{a+b\cot(dx+c)^{2}}\right)}{d}-\frac{b\cot(dx+c)\sqrt{a+b\cot(dx+c)^{2}}}{2d}-\frac{3\sqrt{b}\,a\ln\left(\sqrt{b}\, \cot(dx+c)+\sqrt{a+b\cot(\right.}{2d}$
default	$\frac{b^{\frac{3}{2}} \ln\left(\sqrt{b}\, \cot(dx+c)+\sqrt{a+b\cot(dx+c)^{2}}\right)}{d}-\frac{b\cot(dx+c)\sqrt{a+b\cot(dx+c)^{2}}}{2d}-\frac{3\sqrt{b}\,a\ln\left(\sqrt{b}\, \cot(dx+c)+\sqrt{a+b\cot(\right.}{2d}$

input
```
int((a+b*cot(d*x+c)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*b^(3/2)*ln(b^(1/2)*cot(d*x+c)+(a+b*cot(d*x+c)^2)^(1/2))-1/2*b*cot(d*x+
c)*(a+b*cot(d*x+c)^2)^(1/2)/d-3/2/d*b^(1/2)*a*ln(b^(1/2)*cot(d*x+c)+(a+b*c
ot(d*x+c)^2)^(1/2))-1/d*(b^4*(a-b))^(1/2)/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b
))^(1/2)/(a+b*cot(d*x+c)^2)^(1/2)*cot(d*x+c))+2/d*a/b*(b^4*(a-b))^(1/2)/(a
-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*cot(d*x+c)^2)^(1/2)*cot(d*x+c)
)-1/d*a^2*(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(
a+b*cot(d*x+c)^2)^(1/2)*cot(d*x+c))
```

3.32.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(108) = 216$.

Time = 0.32 (sec) , antiderivative size = 1071, normalized size of antiderivative = 8.50

$$\int \left(a + b\cot^2(c + dx)\right)^{3/2} dx = \text{Too large to display}$$

input
```
integrate((a+b*cot(d*x+c)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*(2*(a - b)*sqrt(-a + b)*log(-(a - b)*cos(2*d*x + 2*c) - sqrt(-a + b)
*sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1))*sin(2*d*x
 + 2*c) + b)*sin(2*d*x + 2*c) + (3*a - 2*b)*sqrt(b)*log(((a - 2*b)*cos(2*d
*x + 2*c) - 2*sqrt(b)*sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x +
 2*c) - 1))*sin(2*d*x + 2*c) - a - 2*b)/(cos(2*d*x + 2*c) - 1))*sin(2*d*x
+ 2*c) + 2*(b*cos(2*d*x + 2*c) + b)*sqrt(((a - b)*cos(2*d*x + 2*c) - a - b
)/(cos(2*d*x + 2*c) - 1)))/(d*sin(2*d*x + 2*c)), 1/2*((3*a - 2*b)*sqrt(-b)
*arctan(sqrt(-b)*sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c)
 - 1))*sin(2*d*x + 2*c)/(b*cos(2*d*x + 2*c) + b))*sin(2*d*x + 2*c) - (a -
b)*sqrt(-a + b)*log(-(a - b)*cos(2*d*x + 2*c) - sqrt(-a + b)*sqrt(((a - b)
*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1))*sin(2*d*x + 2*c) + b)*s
in(2*d*x + 2*c) - (b*cos(2*d*x + 2*c) + b)*sqrt(((a - b)*cos(2*d*x + 2*c)
- a - b)/(cos(2*d*x + 2*c) - 1)))/(d*sin(2*d*x + 2*c)), -1/4*(4*(a - b)^(3
/2)*arctan(-sqrt(a - b)*sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x
 + 2*c) - 1))*sin(2*d*x + 2*c)/((a - b)*cos(2*d*x + 2*c) + a - b))*sin(2*d
*x + 2*c) + (3*a - 2*b)*sqrt(b)*log(((a - 2*b)*cos(2*d*x + 2*c) - 2*sqrt(b
)*sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1))*sin(2*d*
x + 2*c) - a - 2*b)/(cos(2*d*x + 2*c) - 1))*sin(2*d*x + 2*c) + 2*(b*cos(2*
d*x + 2*c) + b)*sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c)
- 1)))/(d*sin(2*d*x + 2*c)), -1/2*(2*(a - b)^(3/2)*arctan(-sqrt(a - b)*...
```

3.32.6 Sympy [F]

$$\int \left(a+b\cot^2(c+dx)\right)^{3/2}\,dx=\int \left(a+b\cot^{2}\left(c+dx\right)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cot(d*x+c)**2)**(3/2),x)
```

output
```
Integral((a + b*cot(c + d*x)**2)**(3/2), x)
```

3.32.7 Maxima [F]

$$\int \left(a+b\cot^2(c+dx)\right)^{3/2}\,dx=\int \left(b\cot\left(dx+c\right)^{2}+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cot(d*x+c)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*cot(d*x + c)^2 + a)^(3/2), x)
```

3.32.8 Giac [F(-2)]

Exception generated.

$$\int \left(a+b\cot^2(c+dx)\right)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+b*cot(d*x+c)^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.32.9 Mupad [F(-1)]

Timed out.

$$\int \left(a + b\cot^2(c + dx)\right)^{3/2} dx = \int \left(b\cot(c + d\,x)^2 + a\right)^{3/2} dx$$

input
```
int((a + b*cot(c + d*x)^2)^(3/2),x)
```

output
```
int((a + b*cot(c + d*x)^2)^(3/2), x)
```

3.33 $\int \sqrt{a+b\cot^2(c+dx)}\,dx$

3.33.1 Optimal result

Integrand size = 16, antiderivative size = 87

$$\int \sqrt{a+b\cot^2(c+dx)}\,dx=-\frac{\sqrt{a-b}\arctan\left(\frac{\sqrt{a-b}\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{d}-\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{d}$$

output
```
-arctan(cot(d*x+c)*(a-b)^(1/2)/(a+b*cot(d*x+c)^2)^(1/2))*(a-b)^(1/2)/d-arc
tanh(cot(d*x+c)*b^(1/2)/(a+b*cot(d*x+c)^2)^(1/2))*b^(1/2)/d
```

3.33.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.23

$$\int \sqrt{a+b\cot^2(c+dx)}\,dx$$
$$=\frac{\sqrt{a-b}\arctan\left(\frac{\sqrt{b}+\sqrt{b}\cot^2(c+dx)-\cot(c+dx)\sqrt{a+b\cot^2(c+dx)}}{\sqrt{a-b}}\right)+\sqrt{b}\log\left(-\sqrt{b}\cot(c+dx)+\sqrt{a+b\cot^2(c+dx)}\right)}{d}$$

input
```
Integrate[Sqrt[a + b*Cot[c + d*x]^2],x]
```

output
```
(Sqrt[a - b]*ArcTan[(Sqrt[b] + Sqrt[b]*Cot[c + d*x]^2 - Cot[c + d*x]*Sqrt[
a + b*Cot[c + d*x]^2])/Sqrt[a - b]] + Sqrt[b]*Log[-(Sqrt[b]*Cot[c + d*x])
+ Sqrt[a + b*Cot[c + d*x]^2]])/d
```

3.33.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3042, 4144, 301, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\cot^2(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a+b\tan\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{4144}$$

$$-\frac{\int \frac{\sqrt{b\cot^2(c+dx)+a}}{\cot^2(c+dx)+1}d\cot(c+dx)}{d}$$

$$\Big\downarrow \text{301}$$

$$-\frac{b\int \frac{1}{\sqrt{b\cot^2(c+dx)+a}}d\cot(c+dx)+(a-b)\int \frac{1}{(\cot^2(c+dx)+1)\sqrt{b\cot^2(c+dx)+a}}d\cot(c+dx)}{d}$$

$$\Big\downarrow \text{224}$$

$$-\frac{(a-b)\int \frac{1}{(\cot^2(c+dx)+1)\sqrt{b\cot^2(c+dx)+a}}d\cot(c+dx)+b\int \frac{1}{1-\frac{b\cot^2(c+dx)}{b\cot^2(c+dx)+a}}d\frac{\cot(c+dx)}{\sqrt{b\cot^2(c+dx)+a}}}{d}$$

$$\Big\downarrow \text{219}$$

$$-\frac{(a-b)\int \frac{1}{(\cot^2(c+dx)+1)\sqrt{b\cot^2(c+dx)+a}}d\cot(c+dx)+\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{d}$$

$$\Big\downarrow \text{291}$$

$$-\frac{(a-b)\int \frac{1}{1-\frac{(b-a)\cot^2(c+dx)}{b\cot^2(c+dx)+a}}d\frac{\cot(c+dx)}{\sqrt{b\cot^2(c+dx)+a}}+\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{d}$$

$$\Big\downarrow \text{216}$$

$$-\frac{\sqrt{a-b}\arctan\left(\frac{\sqrt{a-b}\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)+\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{d}$$

input
```
Int[Sqrt[a + b*Cot[c + d*x]^2],x]
```

output
```
-((Sqrt[a - b]*ArcTan[(Sqrt[a - b]*Cot[c + d*x])/Sqrt[a + b*Cot[c + d*x]^2
]] + Sqrt[b]*ArcTanh[(Sqrt[b]*Cot[c + d*x])/Sqrt[a + b*Cot[c + d*x]^2]])/d
)
```

3.33.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

3.33.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 169 vs. $2(75) = 150$.

Time = 0.05 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.95

method	result
derivativedivides	$-\frac{\sqrt{b}\,\ln\left(\sqrt{b}\,\cot(dx+c)+\sqrt{a+b\cot(dx+c)^2}\right)}{d}+\frac{\sqrt{b^4(a-b)}\,\arctan\left(\frac{b^2(a-b)\cot(dx+c)}{\sqrt{b^4(a-b)}\,\sqrt{a+b\cot(dx+c)^2}}\right)}{db(a-b)}-\frac{a\sqrt{b^4(a-b)}\,\text{arc}}{}$
default	$-\frac{\sqrt{b}\,\ln\left(\sqrt{b}\,\cot(dx+c)+\sqrt{a+b\cot(dx+c)^2}\right)}{d}+\frac{\sqrt{b^4(a-b)}\,\arctan\left(\frac{b^2(a-b)\cot(dx+c)}{\sqrt{b^4(a-b)}\,\sqrt{a+b\cot(dx+c)^2}}\right)}{db(a-b)}-\frac{a\sqrt{b^4(a-b)}\,\text{arc}}{}$

input
```
int((a+b*cot(d*x+c)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d*b^(1/2)*ln(b^(1/2)*cot(d*x+c)+(a+b*cot(d*x+c)^2)^(1/2))+1/d*(b^4*(a-b
))^(1/2)/b/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*cot(d*x+c)^2)^(1/
2)*cot(d*x+c))-1/d*a*(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan(b^2*(a-b)/(b^4*(a-
b))^(1/2)/(a+b*cot(d*x+c)^2)^(1/2)*cot(d*x+c))
```

3.33.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 169 vs. $2(75) = 150$.

Time = 0.30 (sec) , antiderivative size = 703, normalized size of antiderivative = 8.08

$$\int \sqrt{a+b\cot^2(c+dx)}\,dx$$

$$= \left[\frac{\sqrt{-a+b}\log\left(-(a-b)\cos(2\,dx+2\,c)+\sqrt{-a+b}\sqrt{\frac{(a-b)\cos(2\,dx+2\,c)-a-b}{\cos(2\,dx+2\,c)-1}}\sin(2\,dx+2\,c)+b\right)+\sqrt{b}\log\left(\right.}{2\,d}\right.$$

$$-\frac{2\sqrt{a-b}\arctan\left(-\frac{\sqrt{a-b}\sqrt{\frac{(a-b)\cos(2\,dx+2\,c)-a-b}{\cos(2\,dx+2\,c)-1}}\sin(2\,dx+2\,c)}{(a-b)\cos(2\,dx+2\,c)+a-b}\right)-\sqrt{b}\log\left(\frac{(a-2\,b)\cos(2\,dx+2\,c)+2\sqrt{b}\sqrt{\frac{(a-b)\cos(2\,dx+2\,c)-}{\cos(2\,dx+2\,c)-1}}}{\cos(2\,dx+2\,c)-1}\right.}{2\,d}$$

$$-\frac{\sqrt{a-b}\arctan\left(-\frac{\sqrt{a-b}\sqrt{\frac{(a-b)\cos(2\,dx+2\,c)-a-b}{\cos(2\,dx+2\,c)-1}}\sin(2\,dx+2\,c)}{(a-b)\cos(2\,dx+2\,c)+a-b}\right)-\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{(a-b)\cos(2\,dx+2\,c)-a-b}{\cos(2\,dx+2\,c)-1}}\sin(2\,dx+2\,c}{b\cos(2\,dx+2\,c)+b}\right.}{d}$$

input
```
integrate((a+b*cot(d*x+c)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(-a + b)*log(-(a - b)*cos(2*d*x + 2*c) + sqrt(-a + b)*sqrt(((a -
 b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1))*sin(2*d*x + 2*c) + b
) + sqrt(b)*log(((a - 2*b)*cos(2*d*x + 2*c) + 2*sqrt(b)*sqrt(((a - b)*cos(
2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1))*sin(2*d*x + 2*c) - a - 2*b)/
(cos(2*d*x + 2*c) - 1)))/d, -1/2*(2*sqrt(a - b)*arctan(-sqrt(a - b)*sqrt((
(a - b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1))*sin(2*d*x + 2*c)
/((a - b)*cos(2*d*x + 2*c) + a - b)) - sqrt(b)*log(((a - 2*b)*cos(2*d*x +
2*c) + 2*sqrt(b)*sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c)
 - 1))*sin(2*d*x + 2*c) - a - 2*b)/(cos(2*d*x + 2*c) - 1)))/d, 1/2*(2*sqrt
(-b)*arctan(sqrt(-b)*sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x +
2*c) - 1))*sin(2*d*x + 2*c)/(b*cos(2*d*x + 2*c) + b)) + sqrt(-a + b)*log(-
(a - b)*cos(2*d*x + 2*c) + sqrt(-a + b)*sqrt(((a - b)*cos(2*d*x + 2*c) - a
 - b)/(cos(2*d*x + 2*c) - 1))*sin(2*d*x + 2*c) + b))/d, -(sqrt(a - b)*arct
an(-sqrt(a - b)*sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c)
- 1))*sin(2*d*x + 2*c)/((a - b)*cos(2*d*x + 2*c) + a - b)) - sqrt(-b)*arct
an(sqrt(-b)*sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1)
)*sin(2*d*x + 2*c)/(b*cos(2*d*x + 2*c) + b)))/d]
```

3.33. $\int \sqrt{a+b\cot^2(c+dx)}\,dx$

3.33.6 Sympy [F]

$$\int \sqrt{a+b\cot^2(c+dx)}\,dx = \int \sqrt{a+b\cot^2{(c+dx)}}\,dx$$

input
```
integrate((a+b*cot(d*x+c)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*cot(c + d*x)**2), x)
```

3.33.7 Maxima [F(-2)]

Exception generated.

$$\int \sqrt{a+b\cot^2(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*cot(d*x+c)^2)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

3.33.8 Giac [F(-2)]

Exception generated.

$$\int \sqrt{a+b\cot^2(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*cot(d*x+c)^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.33.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\cot^2(c+dx)}\,dx = \int \sqrt{b\cot\left(c+d\,x\right)^2+a}\,dx$$

input
```
int((a + b*cot(c + d*x)^2)^(1/2),x)
```

output
```
int((a + b*cot(c + d*x)^2)^(1/2), x)
```

3.34 $\int \frac{1}{\sqrt{a+b\cot^2(c+dx)}}\,dx$

3.34.1 Optimal result

Integrand size = 16, antiderivative size = 47

$$\int \frac{1}{\sqrt{a+b\cot^2(c+dx)}}\,dx = -\frac{\arctan\left(\frac{\sqrt{a-b}\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{\sqrt{a-b}d}$$

output
```
-arctan(cot(d*x+c)*(a-b)^(1/2)/(a+b*cot(d*x+c)^2)^(1/2))/d/(a-b)^(1/2)
```

3.34.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 111 vs. 2(47) = 94.

Time = 0.48 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.36

$$\int \frac{1}{\sqrt{a+b\cot^2(c+dx)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{-\frac{(a-b)\cot^2(c+dx)}{a}}}{\sqrt{1+\frac{b\cot^2(c+dx)}{a}}}\right)\cot(c+dx)\sqrt{1+\frac{b\cot^2(c+dx)}{a}}}{d\sqrt{-\frac{(a-b)\cot^2(c+dx)}{a}}\sqrt{a+b\cot^2(c+dx)}}$$

input
```
Integrate[1/Sqrt[a + b*Cot[c + d*x]^2],x]
```

output
```
-((ArcTanh[Sqrt[-(((a - b)*Cot[c + d*x]^2)/a)]/Sqrt[1 + (b*Cot[c + d*x]^2)
/a]]*Cot[c + d*x]*Sqrt[1 + (b*Cot[c + d*x]^2)/a])/(d*Sqrt[-(((a - b)*Cot[c
 + d*x]^2)/a)]*Sqrt[a + b*Cot[c + d*x]^2]))
```

3.34.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 4144, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\cot^2(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sqrt{a+b\tan\left(c+dx+\frac{\pi}{2}\right)^2}}dx$$

$$\Big\downarrow \text{ 4144}$$

$$-\frac{\int \frac{1}{(\cot^2(c+dx)+1)\sqrt{b\cot^2(c+dx)+a}}d\cot(c+dx)}{d}$$

$$\Big\downarrow \text{ 291}$$

$$-\frac{\int \frac{1}{1-\frac{(b-a)\cot^2(c+dx)}{b\cot^2(c+dx)+a}}d\frac{\cot(c+dx)}{\sqrt{b\cot^2(c+dx)+a}}}{d}$$

$$\Big\downarrow \text{ 216}$$

$$-\frac{\arctan\left(\frac{\sqrt{a-b}\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{d\sqrt{a-b}}$$

input
```
Int[1/Sqrt[a + b*Cot[c + d*x]^2],x]
```

output
```
-(ArcTan[(Sqrt[a - b]*Cot[c + d*x])/Sqrt[a + b*Cot[c + d*x]^2]]/(Sqrt[a - 
b]*d))
```

3.34.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

3.34.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.45

method	result	size
derivativedivides	$-\frac{\sqrt{b^{4}\left(a-b\right)}\, \arctan\left(\frac{b^{2}\left(a-b\right) \cot\left(dx+c\right)}{\sqrt{b^{4}\left(a-b\right)}\, \sqrt{a+b \cot\left(dx+c\right)^{2}}}\right)}{d\, b^{2}\left(a-b\right)}$	68
default	$-\frac{\sqrt{b^{4}\left(a-b\right)}\, \arctan\left(\frac{b^{2}\left(a-b\right) \cot\left(dx+c\right)}{\sqrt{b^{4}\left(a-b\right)}\, \sqrt{a+b \cot\left(dx+c\right)^{2}}}\right)}{d\, b^{2}\left(a-b\right)}$	68

input
```
int(1/(a+b*cot(d*x+c)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d*(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*c
ot(d*x+c)^2)^(1/2)*cot(d*x+c))
```

3.34.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(41) = 82$.

Time = 0.31 (sec) , antiderivative size = 239, normalized size of antiderivative = 5.09

$$\int \frac{1}{\sqrt{a+b\cot^2(c+dx)}}\,dx$$
$$= \left[-\frac{\sqrt{-a+b}\log\left(-2\left(a^2-2\,ab+b^2\right)\cos\left(2\,dx+2\,c\right)^2-2\left(\left(a-b\right)\cos\left(2\,dx+2\,c\right)-b\right)\sqrt{-a+b}\sqrt{\frac{(a-b)\cos(\cdots}{\cos(2\cdots}}\cdots\right.}{4\left(a-b\right)d}\right.$$
$$\left.-\frac{\arctan\left(-\frac{\sqrt{a-b}\sqrt{\frac{(a-b)\cos(2\,dx+2\,c)-a-b}{\cos(2\,dx+2\,c)-1}}\sin(2\,dx+2\,c)}{(a-b)\cos(2\,dx+2\,c)-b}\right)}{2\sqrt{a-b}d}\right]$$

input
```
integrate(1/(a+b*cot(d*x+c)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/4*sqrt(-a + b)*log(-2*(a^2 - 2*a*b + b^2)*cos(2*d*x + 2*c)^2 - 2*((a -
 b)*cos(2*d*x + 2*c) - b)*sqrt(-a + b)*sqrt(((a - b)*cos(2*d*x + 2*c) - a
- b)/(cos(2*d*x + 2*c) - 1))*sin(2*d*x + 2*c) + a^2 - 2*b^2 + 4*(a*b - b^2
)*cos(2*d*x + 2*c))/((a - b)*d), -1/2*arctan(-sqrt(a - b)*sqrt(((a - b)*co
s(2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1))*sin(2*d*x + 2*c)/((a - b)*
cos(2*d*x + 2*c) - b))/(sqrt(a - b)*d)]
```

3.34.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+b\cot^2(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+b\cot^2\left(c+dx\right)}}\,dx$$

input
```
integrate(1/(a+b*cot(d*x+c)**2)**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*cot(c + d*x)**2), x)
```

3.34.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+b\cot^2(c+dx)}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*cot(d*x+c)^2)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

3.34.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(41) = 82$.

Time = 1.32 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.06

$$\int \frac{1}{\sqrt{a+b\cot^2(c+dx)}}\,dx = \frac{2\arctan\left(-\frac{\sqrt{b}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\sqrt{b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+4a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+b}+\sqrt{b}}{2\sqrt{a-b}}\right)}{\sqrt{a-b}d\text{sgn}\left(\sin\left(dx+c\right)\right)}$$

input
```
integrate(1/(a+b*cot(d*x+c)^2)^(1/2),x, algorithm="giac")
```

output
```
2*arctan(-1/2*(sqrt(b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(b*tan(1/2*d*x + 1/2*c
)^4 + 4*a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c)^2 + b) + sqrt(
b))/sqrt(a - b))/(sqrt(a - b)*d*sgn(sin(d*x + c)))
```

3.34.9 Mupad [B] (verification not implemented)

Time = 13.73 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

$$\int \frac{1}{\sqrt{a+b\cot^2(c+dx)}}\,dx = -\frac{\operatorname{atan}\left(\frac{\cot(c+d\,x)\,\sqrt{a-b}}{\sqrt{b\cot(c+d\,x)^2+a}}\right)}{d\,\sqrt{a-b}}$$

input
```
int(1/(a + b*cot(c + d*x)^2)^(1/2),x)
```

output
```
-atan((cot(c + d*x)*(a - b)^(1/2))/(a + b*cot(c + d*x)^2)^(1/2))/(d*(a - b
)^(1/2))
```

3.35 $\int \frac{1}{(a+b\cot^2(c+dx))^{3/2}}\,dx$

3.35.1 Optimal result

Integrand size = 16, antiderivative size = 85

$$\int \frac{1}{(a+b\cot^2(c+dx))^{3/2}}\,dx=-\frac{\arctan\left(\frac{\sqrt{a-b}\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{(a-b)^{3/2}d}+\frac{b\cot(c+dx)}{a(a-b)d\sqrt{a+b\cot^2(c+dx)}}$$

output
```
-arctan(cot(d*x+c)*(a-b)^(1/2)/(a+b*cot(d*x+c)^2)^(1/2))/(a-b)^(3/2)/d+b*c
ot(d*x+c)/a/(a-b)/d/(a+b*cot(d*x+c)^2)^(1/2)
```

3.35.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 3.72 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.72

$$\int \frac{1}{(a+b\cot^2(c+dx))^{3/2}}\,dx=$$

$$-\frac{\cos^2(c+dx)\cot(c+dx)\left(4(a-b)^2\cos^2(c+dx)\,\text{Hypergeometric2F1}\left(2,2,\frac{7}{2},\frac{(a-b)\cos^2(c+dx)}{a}\right)(b+a\tan^2(\ldots}{15a^3(a-b)d\sqrt{a\ldots}}$$

input
```
Integrate[(a + b*Cot[c + d*x]^2)^(-3/2),x]
```

output
```
-1/15*(Cos[c + d*x]^2*Cot[c + d*x]*(4*(a - b)^2*Cos[c + d*x]^2*Hypergeomet
ric2F1[2, 2, 7/2, ((a - b)*Cos[c + d*x]^2)/a]*(b + a*Tan[c + d*x]^2) - (15
*a*(2*b + 3*a*Tan[c + d*x]^2)*(ArcSin[Sqrt[((a - b)*Cos[c + d*x]^2)/a]]*(b
 + a*Tan[c + d*x]^2) - a*Sec[c + d*x]^2*Sqrt[((a - b)*Cos[c + d*x]^4*(b +
a*Tan[c + d*x]^2))/a^2]))/Sqrt[((a - b)*Cos[c + d*x]^4*(b + a*Tan[c + d*x]
^2))/a^2]))/(a^3*(a - b)*d*Sqrt[a + b*Cot[c + d*x]^2])
```

3.35.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {3042, 4144, 296, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\cot^2(c+dx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a+b\tan\left(c+dx+\frac{\pi}{2}\right)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{4144}$$

$$-\frac{\int \frac{1}{(\cot^2(c+dx)+1)(b\cot^2(c+dx)+a)^{3/2}}d\cot(c+dx)}{d}$$

$$\Big\downarrow \text{296}$$

$$-\frac{\frac{\int \frac{1}{\left(\cot^2(c+dx)+1\right)\sqrt{b\cot^2(c+dx)+a}}d\cot(c+dx)}{a-b}-\frac{b\cot(c+dx)}{a(a-b)\sqrt{a+b\cot^2(c+dx)}}}{d}$$

$$\Big\downarrow \text{291}$$

$$-\frac{\frac{\int \frac{1}{1-\frac{(b-a)\cot^2(c+dx)}{b\cot^2(c+dx)+a}}d\frac{\cot(c+dx)}{\sqrt{b\cot^2(c+dx)+a}}}{a-b}-\frac{b\cot(c+dx)}{a(a-b)\sqrt{a+b\cot^2(c+dx)}}}{d}$$

$$\Big\downarrow \text{216}$$

$$-\frac{\frac{\arctan\left(\frac{\sqrt{a-b}\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{(a-b)^{3/2}}-\frac{b\cot(c+dx)}{a(a-b)\sqrt{a+b\cot^2(c+dx)}}}{d}$$

input
```
Int[(a + b*Cot[c + d*x]^2)^(-3/2),x]
```

output
```
-((ArcTan[(Sqrt[a - b]*Cot[c + d*x])/Sqrt[a + b*Cot[c + d*x]^2]]/(a - b)^(
3/2) - (b*Cot[c + d*x])/(a*(a - b)*Sqrt[a + b*Cot[c + d*x]^2]))/d)
```

3.35.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

3.35.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.20

method	result	size
derivativedivides	$\frac{-\frac{\sqrt{b^4(a-b)}\,\arctan\left(\frac{b^2(a-b)\cot(dx+c)}{\sqrt{b^4(a-b)}\,\sqrt{a+b\cot(dx+c)^2}}\right)}{(a-b)^2b^2}+\frac{b\cot(dx+c)}{(a-b)a\sqrt{a+b\cot(dx+c)^2}}}{d}$	102
default	$\frac{-\frac{\sqrt{b^4(a-b)}\,\arctan\left(\frac{b^2(a-b)\cot(dx+c)}{\sqrt{b^4(a-b)}\,\sqrt{a+b\cot(dx+c)^2}}\right)}{(a-b)^2b^2}+\frac{b\cot(dx+c)}{(a-b)a\sqrt{a+b\cot(dx+c)^2}}}{d}$	102

input
```
int(1/(a+b*cot(d*x+c)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/(a-b)^2*(b^4*(a-b))^(1/2)/b^2*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(
a+b*cot(d*x+c)^2)^(1/2)*cot(d*x+c))+b/(a-b)*cot(d*x+c)/a/(a+b*cot(d*x+c)^2
)^(1/2))
```

3.35.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. 2(77) = 154.

Time = 0.34 (sec) , antiderivative size = 526, normalized size of antiderivative = 6.19

$$\int \frac{1}{\left(a+b\cot^2(c+dx)\right)^{3/2}}\,dx = \left[-\frac{(a^2+ab-(a^2-ab)\cos(2\,dx+2\,c))\sqrt{-a+b}\log\left(-2\,(a^2-2\,ab+b^2)\,\mathrm{co}\right.}{}\right.$$

input
```
integrate(1/(a+b*cot(d*x+c)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*((a^2 + a*b - (a^2 - a*b)*cos(2*d*x + 2*c))*sqrt(-a + b)*log(-2*(a^2
 - 2*a*b + b^2)*cos(2*d*x + 2*c)^2 + 2*((a - b)*cos(2*d*x + 2*c) - b)*sqrt
(-a + b)*sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1))*s
in(2*d*x + 2*c) + a^2 - 2*b^2 + 4*(a*b - b^2)*cos(2*d*x + 2*c)) + 4*(a*b -
 b^2)*sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1))*sin(
2*d*x + 2*c))/((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d*cos(2*d*x + 2*c) - (a
^4 - a^3*b - a^2*b^2 + a*b^3)*d), 1/2*((a^2 + a*b - (a^2 - a*b)*cos(2*d*x
+ 2*c))*sqrt(a - b)*arctan(-sqrt(a - b)*sqrt(((a - b)*cos(2*d*x + 2*c) - a
 - b)/(cos(2*d*x + 2*c) - 1))*sin(2*d*x + 2*c)/((a - b)*cos(2*d*x + 2*c) -
 b)) - 2*(a*b - b^2)*sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x +
2*c) - 1))*sin(2*d*x + 2*c))/((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d*cos(2*
d*x + 2*c) - (a^4 - a^3*b - a^2*b^2 + a*b^3)*d)]
```

3.35.6 Sympy [F]

$$\int \frac{1}{(a+b\cot^2(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(a+b\cot^2\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*cot(d*x+c)**2)**(3/2),x)
```

output
```
Integral((a + b*cot(c + d*x)**2)**(-3/2), x)
```

3.35.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\cot^2(c+dx))^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*cot(d*x+c)^2)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

3.35.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(77) = 154$.

Time = 0.85 (sec) , antiderivative size = 300, normalized size of antiderivative = 3.53

$$\int \frac{1}{(a+b\cot^2(c+dx))^{3/2}}\,dx =$$

$$-\frac{\frac{\frac{\left(a^2b\,\mathrm{sgn}(\sin(dx+c))-2\,ab^2\mathrm{sgn}(\sin(dx+c))+b^3\mathrm{sgn}(\sin(dx+c))\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a^4-3\,a^3b+3\,a^2b^2-ab^3}-\frac{a^2b\,\mathrm{sgn}(\sin(dx+c))-2\,ab^2\mathrm{sgn}(\sin(dx+c))+b^3\mathrm{sgn}(\sin(dx+c))}{a^4-3\,a^3b+3\,a^2b^2-ab^3}}{\sqrt{b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+b}}-\frac{2\arctan\left(-\cdots\right.}{\cdots}}{d}$$

input
```
integrate(1/(a+b*cot(d*x+c)^2)^(3/2),x, algorithm="giac")
```

output
```
-(((a^2*b*sgn(sin(d*x + c)) - 2*a*b^2*sgn(sin(d*x + c)) + b^3*sgn(sin(d*x
+ c)))*tan(1/2*d*x + 1/2*c)^2/(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3) - (a^2*b
*sgn(sin(d*x + c)) - 2*a*b^2*sgn(sin(d*x + c)) + b^3*sgn(sin(d*x + c)))/(a
^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3))/sqrt(b*tan(1/2*d*x + 1/2*c)^4 + 4*a*tan
(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c)^2 + b) - 2*arctan(-1/2*(sqr
t(b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(b*tan(1/2*d*x + 1/2*c)^4 + 4*a*tan(1/2*
d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c)^2 + b) + sqrt(b))/sqrt(a - b))/(
(a*sgn(sin(d*x + c)) - b*sgn(sin(d*x + c)))*sqrt(a - b)))/d
```

3.35.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cot^2(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(b\cot\left(c+d\,x\right)^2+a\right)^{3/2}}\,dx$$

input
```
int(1/(a + b*cot(c + d*x)^2)^(3/2),x)
```

output
```
int(1/(a + b*cot(c + d*x)^2)^(3/2), x)
```

3.36 $\int \frac{1}{\left(a+b\cot^2(c+dx)\right)^{5/2}}\,dx$

3.36.1 Optimal result

Integrand size = 16, antiderivative size = 135

$$\int \frac{1}{\left(a+b\cot^2(c+dx)\right)^{5/2}}\,dx=-\frac{\arctan\left(\frac{\sqrt{a-b}\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{(a-b)^{5/2}d}+\frac{b\cot(c+dx)}{3a(a-b)d\left(a+b\cot^2(c+dx)\right)^{3/2}}+\frac{(5a-2b)b\cot(c+dx)}{3a^2(a-b)^2d\sqrt{a+b\cot^2(c+dx)}}$$

output
```
-arctan(cot(d*x+c)*(a-b)^(1/2)/(a+b*cot(d*x+c)^2)^(1/2))/(a-b)^(5/2)/d+1/3
*b*cot(d*x+c)/a/(a-b)/d/(a+b*cot(d*x+c)^2)^(3/2)+1/3*(5*a-2*b)*b*cot(d*x+c
)/a^2/(a-b)^2/d/(a+b*cot(d*x+c)^2)^(1/2)
```

3.36.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 8.14 (sec) , antiderivative size = 367, normalized size of antiderivative = 2.72

$$\int \frac{1}{\left(a+b\cot^2(c+dx)\right)^{5/2}}\,dx=$$

$$-\frac{\cot^5(c+dx)\left(24(a-b)^3\cos^2(c+dx)\,{}_3F_2\left(2,2,2;1,\frac{9}{2};\frac{(a-b)\cos^2(c+dx)}{a}\right)\left(b+a\tan^2(c+dx)\right)^2+24(a-b)^3\,c\cdots\right.}{\cdots}$$

input
```
Integrate[(a + b*Cot[c + d*x]^2)^(-5/2),x]
```

output
```
-1/315*(Cot[c + d*x]^5*(24*(a - b)^3*Cos[c + d*x]^2*HypergeometricPFQ[{2,
2, 2}, {1, 9/2}, ((a - b)*Cos[c + d*x]^2)/a]*(b + a*Tan[c + d*x]^2)^2 + 24
*(a - b)^3*Cos[c + d*x]^2*Hypergeometric2F1[2, 2, 9/2, ((a - b)*Cos[c + d*
x]^2)/a]*(3*b^2 + 7*a*b*Tan[c + d*x]^2 + 4*a^2*Tan[c + d*x]^4) - (35*a*(8*
b^2 + 20*a*b*Tan[c + d*x]^2 + 15*a^2*Tan[c + d*x]^4)*(-3*ArcSin[Sqrt[((a -
 b)*Cos[c + d*x]^2)/a]]*(b + a*Tan[c + d*x]^2)^2 + a*Sec[c + d*x]^2*Sqrt[(
(a - b)*Cos[c + d*x]^4*(b + a*Tan[c + d*x]^2))/a^2]*(4*b + a*(-1 + 3*Tan[c
 + d*x]^2))))/Sqrt[((a - b)*Cos[c + d*x]^4*(b + a*Tan[c + d*x]^2))/a^2]))/
(a^5*(a - b)^2*d*(1 + Cot[c + d*x]^2)*Sqrt[a + b*Cot[c + d*x]^2]*(1 + (b*C
ot[c + d*x]^2)/a))
```

3.36.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3042, 4144, 316, 402, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\cot^2(c+dx)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a+b\tan\left(c+dx+\frac{\pi}{2}\right)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{4144}$$

$$-\frac{\int \frac{1}{(\cot^2(c+dx)+1)(b\cot^2(c+dx)+a)^{5/2}}d\cot(c+dx)}{d}$$

$$\Big\downarrow \text{316}$$

$$-\frac{\frac{\int \frac{-2b\cot^2(c+dx)+3a-2b}{\left(\cot^2(c+dx)+1\right)\left(b\cot^2(c+dx)+a\right)^{3/2}}d\cot(c+dx)}{3a(a-b)}-\frac{b\cot(c+dx)}{3a(a-b)(a+b\cot^2(c+dx))^{3/2}}}{d}$$

$$\Big\downarrow \text{402}$$

$$-\frac{\frac{\frac{\int \frac{3a^2}{\left(\cot^2(c+dx)+1\right)\sqrt{b\cot^2(c+dx)+a}}d\cot(c+dx)}{a(a-b)}-\frac{b(5a-2b)\cot(c+dx)}{a(a-b)\sqrt{a+b\cot^2(c+dx)}}}{3a(a-b)}-\frac{b\cot(c+dx)}{3a(a-b)(a+b\cot^2(c+dx))^{3/2}}}{d}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\frac{3a\int \frac{1}{\left(\cot^2(c+dx)+1\right)\sqrt{b\cot^2(c+dx)+a}}d\cot(c+dx)}{a-b}-\frac{b(5a-2b)\cot(c+dx)}{a(a-b)\sqrt{a+b\cot^2(c+dx)}}}{3a(a-b)}-\frac{b\cot(c+dx)}{3a(a-b)(a+b\cot^2(c+dx))^{3/2}}}{d}$$

$$\Big\downarrow \text{291}$$

$$-\frac{\frac{\frac{3a\int \frac{1}{1-\frac{(b-a)\cot^2(c+dx)}{b\cot^2(c+dx)+a}}d\frac{\cot(c+dx)}{\sqrt{b\cot^2(c+dx)+a}}}{a-b}-\frac{b(5a-2b)\cot(c+dx)}{a(a-b)\sqrt{a+b\cot^2(c+dx)}}}{3a(a-b)}-\frac{b\cot(c+dx)}{3a(a-b)(a+b\cot^2(c+dx))^{3/2}}}{d}$$

$$\Big\downarrow \text{216}$$

$$-\frac{\frac{\frac{3a\arctan\left(\frac{\sqrt{a-b}\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{(a-b)^{3/2}}-\frac{b(5a-2b)\cot(c+dx)}{a(a-b)\sqrt{a+b\cot^2(c+dx)}}}{3a(a-b)}-\frac{b\cot(c+dx)}{3a(a-b)(a+b\cot^2(c+dx))^{3/2}}}{d}$$

input
```
Int[(a + b*Cot[c + d*x]^2)^(-5/2),x]
```

output
```
-((-1/3*(b*Cot[c + d*x])/(a*(a - b)*(a + b*Cot[c + d*x]^2)^(3/2)) + ((3*a*
ArcTan[(Sqrt[a - b]*Cot[c + d*x])/Sqrt[a + b*Cot[c + d*x]^2]])/(a - b)^(3/
2) - ((5*a - 2*b)*b*Cot[c + d*x])/(a*(a - b)*Sqrt[a + b*Cot[c + d*x]^2]))/
(3*a*(a - b)))/d)
```

3.36.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

3.36. $\int \frac{1}{(a+b\cot^2(c+dx))^{5/2}}\,dx$

3.36.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.20

method	result
derivativedivides	$\frac{-\frac{\sqrt{b^4(a-b)}\,\arctan\left(\frac{b^2(a-b)\cot(dx+c)}{\sqrt{b^4(a-b)}\,\sqrt{a+b\cot(dx+c)^2}}\right)}{(a-b)^3b^2}+\frac{b\left(\frac{\cot(dx+c)}{3a\left(a+b\cot(dx+c)^2\right)^{\frac{3}{2}}}+\frac{2\cot(dx+c)}{3a^2\sqrt{a+b\cot(dx+c)^2}}\right)}{a-b}+\frac{b\cot(dx+c)}{(a-b)^2a\sqrt{a+b\cot(dx+c)^2}}}{d}$
default	$\frac{-\frac{\sqrt{b^4(a-b)}\,\arctan\left(\frac{b^2(a-b)\cot(dx+c)}{\sqrt{b^4(a-b)}\,\sqrt{a+b\cot(dx+c)^2}}\right)}{(a-b)^3b^2}+\frac{b\left(\frac{\cot(dx+c)}{3a\left(a+b\cot(dx+c)^2\right)^{\frac{3}{2}}}+\frac{2\cot(dx+c)}{3a^2\sqrt{a+b\cot(dx+c)^2}}\right)}{a-b}+\frac{b\cot(dx+c)}{(a-b)^2a\sqrt{a+b\cot(dx+c)^2}}}{d}$

input
```
int(1/(a+b*cot(d*x+c)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/(a-b)^3*(b^4*(a-b))^(1/2)/b^2*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(
a+b*cot(d*x+c)^2)^(1/2)*cot(d*x+c))+1/(a-b)*b*(1/3*cot(d*x+c)/a/(a+b*cot(d
*x+c)^2)^(3/2)+2/3/a^2*cot(d*x+c)/(a+b*cot(d*x+c)^2)^(1/2))+1/(a-b)^2*b*co
t(d*x+c)/a/(a+b*cot(d*x+c)^2)^(1/2))
```

3.36.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 423 vs. 2(121) = 242.

Time = 0.35 (sec) , antiderivative size = 898, normalized size of antiderivative = 6.65

$$\int \frac{1}{\left(a+b\cot^2(c+dx)\right)^{5/2}}\,dx = \Bigg[-\frac{3\left(a^4+2\,a^3b+a^2b^2+\left(a^4-2\,a^3b+a^2b^2\right)\cos\left(2\,dx+2\,c\right)^2-2\left(a^4-a^2b^2\right.\cdots}{\cdots}$$

$$-\frac{3\left(a^4+2\,a^3b+a^2b^2+\left(a^4-2\,a^3b+a^2b^2\right)\cos\left(2\,dx+2\,c\right)^2-2\left(a^4-a^2b^2\right)\cos\left(2\,dx+2\,c\right)\right)\sqrt{a-b}\arctan\left(\cdots\right.}{6\left(\left(a^7-5\,a^6b+10\,a^5b^2-10\,a^4b^3+5\,a^3b^4-a^2b^5\right)d\cos\left(2\,dx+2\,c\right)\cdots\right.}$$

input
```
integrate(1/(a+b*cot(d*x+c)^2)^(5/2),x, algorithm="fricas")
```

output
```
[-1/12*(3*(a^4 + 2*a^3*b + a^2*b^2 + (a^4 - 2*a^3*b + a^2*b^2)*cos(2*d*x +
 2*c)^2 - 2*(a^4 - a^2*b^2)*cos(2*d*x + 2*c))*sqrt(-a + b)*log(-2*(a^2 - 2
*a*b + b^2)*cos(2*d*x + 2*c)^2 - 2*((a - b)*cos(2*d*x + 2*c) - b)*sqrt(-a
+ b)*sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1))*sin(2
*d*x + 2*c) + a^2 - 2*b^2 + 4*(a*b - b^2)*cos(2*d*x + 2*c)) - 8*(3*a^3*b -
 2*a^2*b^2 - 2*a*b^3 + b^4 - (3*a^3*b - 7*a^2*b^2 + 5*a*b^3 - b^4)*cos(2*d
*x + 2*c))*sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1))
*sin(2*d*x + 2*c))/((a^7 - 5*a^6*b + 10*a^5*b^2 - 10*a^4*b^3 + 5*a^3*b^4 -
 a^2*b^5)*d*cos(2*d*x + 2*c)^2 - 2*(a^7 - 3*a^6*b + 2*a^5*b^2 + 2*a^4*b^3
- 3*a^3*b^4 + a^2*b^5)*d*cos(2*d*x + 2*c) + (a^7 - a^6*b - 2*a^5*b^2 + 2*a
^4*b^3 + a^3*b^4 - a^2*b^5)*d), -1/6*(3*(a^4 + 2*a^3*b + a^2*b^2 + (a^4 -
2*a^3*b + a^2*b^2)*cos(2*d*x + 2*c)^2 - 2*(a^4 - a^2*b^2)*cos(2*d*x + 2*c)
)*sqrt(a - b)*arctan(-sqrt(a - b)*sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/
(cos(2*d*x + 2*c) - 1))*sin(2*d*x + 2*c)/((a - b)*cos(2*d*x + 2*c) - b)) -
 4*(3*a^3*b - 2*a^2*b^2 - 2*a*b^3 + b^4 - (3*a^3*b - 7*a^2*b^2 + 5*a*b^3 -
 b^4)*cos(2*d*x + 2*c))*sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x
 + 2*c) - 1))*sin(2*d*x + 2*c))/((a^7 - 5*a^6*b + 10*a^5*b^2 - 10*a^4*b^3
+ 5*a^3*b^4 - a^2*b^5)*d*cos(2*d*x + 2*c)^2 - 2*(a^7 - 3*a^6*b + 2*a^5*b^2
 + 2*a^4*b^3 - 3*a^3*b^4 + a^2*b^5)*d*cos(2*d*x + 2*c) + (a^7 - a^6*b - 2*
a^5*b^2 + 2*a^4*b^3 + a^3*b^4 - a^2*b^5)*d)]
```

3.36.6 Sympy [F]

$$\int \frac{1}{\left(a+b\cot^2(c+dx)\right)^{5/2}}\,dx=\int \frac{1}{\left(a+b\cot^2\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*cot(d*x+c)**2)**(5/2),x)
```

output
```
Integral((a + b*cot(c + d*x)**2)**(-5/2), x)
```

3.36.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a+b\cot^2(c+dx)\right)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*cot(d*x+c)^2)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

3.36.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1160 vs. $2(121) = 242$.

Time = 1.14 (sec) , antiderivative size = 1160, normalized size of antiderivative = 8.59

$$\int \frac{1}{\left(a+b\cot^2(c+dx)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*cot(d*x+c)^2)^(5/2),x, algorithm="giac")
```

output
```
-1/3*(((((5*a^9*b^2*sgn(sin(d*x + c)) - 42*a^8*b^3*sgn(sin(d*x + c)) + 156
*a^7*b^4*sgn(sin(d*x + c)) - 336*a^6*b^5*sgn(sin(d*x + c)) + 462*a^5*b^6*s
gn(sin(d*x + c)) - 420*a^4*b^7*sgn(sin(d*x + c)) + 252*a^3*b^8*sgn(sin(d*x
 + c)) - 96*a^2*b^9*sgn(sin(d*x + c)) + 21*a*b^10*sgn(sin(d*x + c)) - 2*b^
11*sgn(sin(d*x + c)))*tan(1/2*d*x + 1/2*c)^2/(a^12 - 10*a^11*b + 45*a^10*b
^2 - 120*a^9*b^3 + 210*a^8*b^4 - 252*a^7*b^5 + 210*a^6*b^6 - 120*a^5*b^7 +
 45*a^4*b^8 - 10*a^3*b^9 + a^2*b^10) + 3*(8*a^10*b*sgn(sin(d*x + c)) - 73*
a^9*b^2*sgn(sin(d*x + c)) + 298*a^8*b^3*sgn(sin(d*x + c)) - 716*a^7*b^4*sg
n(sin(d*x + c)) + 1120*a^6*b^5*sgn(sin(d*x + c)) - 1190*a^5*b^6*sgn(sin(d*
x + c)) + 868*a^4*b^7*sgn(sin(d*x + c)) - 428*a^3*b^8*sgn(sin(d*x + c)) +
136*a^2*b^9*sgn(sin(d*x + c)) - 25*a*b^10*sgn(sin(d*x + c)) + 2*b^11*sgn(s
in(d*x + c)))/(a^12 - 10*a^11*b + 45*a^10*b^2 - 120*a^9*b^3 + 210*a^8*b^4
- 252*a^7*b^5 + 210*a^6*b^6 - 120*a^5*b^7 + 45*a^4*b^8 - 10*a^3*b^9 + a^2*
b^10))*tan(1/2*d*x + 1/2*c)^2 - 3*(8*a^10*b*sgn(sin(d*x + c)) - 73*a^9*b^2
*sgn(sin(d*x + c)) + 298*a^8*b^3*sgn(sin(d*x + c)) - 716*a^7*b^4*sgn(sin(d
*x + c)) + 1120*a^6*b^5*sgn(sin(d*x + c)) - 1190*a^5*b^6*sgn(sin(d*x + c))
 + 868*a^4*b^7*sgn(sin(d*x + c)) - 428*a^3*b^8*sgn(sin(d*x + c)) + 136*a^2
*b^9*sgn(sin(d*x + c)) - 25*a*b^10*sgn(sin(d*x + c)) + 2*b^11*sgn(sin(d*x
+ c)))/(a^12 - 10*a^11*b + 45*a^10*b^2 - 120*a^9*b^3 + 210*a^8*b^4 - 252*a
^7*b^5 + 210*a^6*b^6 - 120*a^5*b^7 + 45*a^4*b^8 - 10*a^3*b^9 + a^2*b^10...
```

3.36.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cot^2(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(b\cot\left(c+d\,x\right)^2+a\right)^{5/2}}\,dx$$

input
```
int(1/(a + b*cot(c + d*x)^2)^(5/2),x)
```

output
```
int(1/(a + b*cot(c + d*x)^2)^(5/2), x)
```

3.37 $\int \frac{1}{\left(a+b\cot^2(c+dx)\right)^{7/2}}\,dx$

3.37.1 Optimal result

Integrand size = 16, antiderivative size = 190

$$\int \frac{1}{\left(a+b\cot^2(c+dx)\right)^{7/2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{a-b}\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{(a-b)^{7/2}d} + \frac{b\cot(c+dx)}{5a(a-b)d\left(a+b\cot^2(c+dx)\right)^{5/2}} + \frac{(9a-4b)b\cot(c+dx)}{15a^2(a-b)^2d\left(a+b\cot^2(c+dx)\right)^{3/2}} + \frac{b(33a^2-26ab+8b^2)\cot(c+dx)}{15a^3(a-b)^3d\sqrt{a+b\cot^2(c+dx)}}$$

output
```
-arctan(cot(d*x+c)*(a-b)^(1/2)/(a+b*cot(d*x+c)^2)^(1/2))/(a-b)^(7/2)/d+1/5
*b*cot(d*x+c)/a/(a-b)/d/(a+b*cot(d*x+c)^2)^(5/2)+1/15*(9*a-4*b)*b*cot(d*x+
c)/a^2/(a-b)^2/d/(a+b*cot(d*x+c)^2)^(3/2)+1/15*b*(33*a^2-26*a*b+8*b^2)*cot
(d*x+c)/a^3/(a-b)^3/d/(a+b*cot(d*x+c)^2)^(1/2)
```

3.37.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 15.97 (sec) , antiderivative size = 2553, normalized size of antiderivative = 13.44

$$\int \frac{1}{\left(a+b\cot^2(c+dx)\right)^{7/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*Cot[c + d*x]^2)^(-7/2),x]
```

output
```
-1/4725*(Cot[c + d*x]*(-33075*ArcSin[Sqrt[((a - b)*Cos[c + d*x]^2)/a]] + (
99225*(a - b)*ArcSin[Sqrt[((a - b)*Cos[c + d*x]^2)/a]]*Cos[c + d*x]^2)/a -
 (99225*(a - b)^2*ArcSin[Sqrt[((a - b)*Cos[c + d*x]^2)/a]]*Cos[c + d*x]^4)
/a^2 + (33075*(a - b)^3*ArcSin[Sqrt[((a - b)*Cos[c + d*x]^2)/a]]*Cos[c + d
*x]^6)/a^3 - (66150*b*ArcSin[Sqrt[((a - b)*Cos[c + d*x]^2)/a]]*Cot[c + d*x
]^2)/a + (198450*(a - b)*b*ArcSin[Sqrt[((a - b)*Cos[c + d*x]^2)/a]]*Cos[c
+ d*x]^2*Cot[c + d*x]^2)/a^2 + (66150*(a - b)^3*b*ArcSin[Sqrt[((a - b)*Cos
[c + d*x]^2)/a]]*Cos[c + d*x]^6*Cot[c + d*x]^2)/a^4 - (52920*b^2*ArcSin[Sq
rt[((a - b)*Cos[c + d*x]^2)/a]]*Cot[c + d*x]^4)/a^2 + (158760*(a - b)*b^2*
ArcSin[Sqrt[((a - b)*Cos[c + d*x]^2)/a]]*Cos[c + d*x]^2*Cot[c + d*x]^4)/a^
3 - (158760*(a - b)^2*b^2*ArcSin[Sqrt[((a - b)*Cos[c + d*x]^2)/a]]*Cos[c +
 d*x]^4*Cot[c + d*x]^4)/a^4 + (52920*(a - b)^3*b^2*ArcSin[Sqrt[((a - b)*Co
s[c + d*x]^2)/a]]*Cos[c + d*x]^6*Cot[c + d*x]^4)/a^5 - (15120*b^3*ArcSin[S
qrt[((a - b)*Cos[c + d*x]^2)/a]]*Cot[c + d*x]^6)/a^3 + (45360*(a - b)*b^3*
ArcSin[Sqrt[((a - b)*Cos[c + d*x]^2)/a]]*Cos[c + d*x]^2*Cot[c + d*x]^6)/a^
4 - (45360*(a - b)^2*b^3*ArcSin[Sqrt[((a - b)*Cos[c + d*x]^2)/a]]*Cos[c +
d*x]^4*Cot[c + d*x]^6)/a^5 + (15120*(a - b)^3*b^3*ArcSin[Sqrt[((a - b)*Cos
[c + d*x]^2)/a]]*Cos[c + d*x]^6*Cot[c + d*x]^6)/a^6 - 77175*(((a - b)*Cos[
c + d*x]^2)/a)^(3/2)*Sqrt[(Cos[c + d*x]^2*(b + a*Tan[c + d*x]^2))/a] + 507
15*(((a - b)*Cos[c + d*x]^2)/a)^(5/2)*Sqrt[(Cos[c + d*x]^2*(b + a*Tan[c...
```

3.37.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4144, 316, 402, 402, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\cot^2(c+dx)\right)^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a+b\tan\left(c+dx+\frac{\pi}{2}\right)^2\right)^{7/2}}dx$$

$$\Big\downarrow \text{4144}$$

$$-\frac{\int \frac{1}{(\cot^2(c+dx)+1)(b\cot^2(c+dx)+a)^{7/2}}d\cot(c+dx)}{d}$$

$$\Big\downarrow \text{316}$$

$$-\frac{\frac{\int \frac{-4b\cot^2(c+dx)+5a-4b}{\left(\cot^2(c+dx)+1\right)\left(b\cot^2(c+dx)+a\right)^{5/2}}d\cot(c+dx)}{5a(a-b)}-\frac{b\cot(c+dx)}{5a(a-b)(a+b\cot^2(c+dx))^{5/2}}}{d}$$

$$\Big\downarrow \text{402}$$

$$-\frac{\frac{\frac{\int \frac{15a^2-18ba+8b^2-2(9a-4b)b\cot^2(c+dx)}{\left(\cot^2(c+dx)+1\right)\left(b\cot^2(c+dx)+a\right)^{3/2}}d\cot(c+dx)}{3a(a-b)}-\frac{b(9a-4b)\cot(c+dx)}{3a(a-b)\left(a+b\cot^2(c+dx)\right)^{3/2}}}{5a(a-b)}-\frac{b\cot(c+dx)}{5a(a-b)(a+b\cot^2(c+dx))^{5/2}}}{d}$$

$$\Big\downarrow \text{402}$$

$$-\frac{\frac{\frac{\frac{\int \frac{15a^3}{\left(\cot^2(c+dx)+1\right)\sqrt{b\cot^2(c+dx)+a}}d\cot(c+dx)}{a(a-b)}-\frac{b\left(33a^2-26ab+8b^2\right)\cot(c+dx)}{a(a-b)\sqrt{a+b\cot^2(c+dx)}}}{3a(a-b)}-\frac{b(9a-4b)\cot(c+dx)}{3a(a-b)\left(a+b\cot^2(c+dx)\right)^{3/2}}}{5a(a-b)}-\frac{b\cot(c+dx)}{5a(a-b)(a+b\cot^2(c+dx))^{5/2}}}{d}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\frac{\frac{15a^2\int \frac{1}{\left(\cot^2(c+dx)+1\right)\sqrt{b\cot^2(c+dx)+a}}d\cot(c+dx)}{a-b}-\frac{b\left(33a^2-26ab+8b^2\right)\cot(c+dx)}{a(a-b)\sqrt{a+b\cot^2(c+dx)}}}{3a(a-b)}-\frac{b(9a-4b)\cot(c+dx)}{3a(a-b)\left(a+b\cot^2(c+dx)\right)^{3/2}}}{5a(a-b)}-\frac{b\cot(c+dx)}{5a(a-b)(a+b\cot^2(c+dx))^{5/2}}}{d}$$

$$\Big\downarrow \text{291}$$

$$-\frac{\frac{\frac{\frac{15a^2\int \frac{1}{1-\frac{(b-a)\cot^2(c+dx)}{b\cot^2(c+dx)+a}}d\frac{\cot(c+dx)}{\sqrt{b\cot^2(c+dx)+a}}}{a-b}-\frac{b\left(33a^2-26ab+8b^2\right)\cot(c+dx)}{a(a-b)\sqrt{a+b\cot^2(c+dx)}}}{3a(a-b)}-\frac{b(9a-4b)\cot(c+dx)}{3a(a-b)\left(a+b\cot^2(c+dx)\right)^{3/2}}}{5a(a-b)}-\frac{b\cot(c+dx)}{5a(a-b)(a+b\cot^2(c+dx))^{5/2}}}{d}$$

$$\Big\downarrow \text{216}$$

$$-\frac{\frac{\frac{\frac{15a^2\arctan\left(\frac{\sqrt{a-b}\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{(a-b)^{3/2}}-\frac{b\left(33a^2-26ab+8b^2\right)\cot(c+dx)}{a(a-b)\sqrt{a+b\cot^2(c+dx)}}}{3a(a-b)}-\frac{b(9a-4b)\cot(c+dx)}{3a(a-b)\left(a+b\cot^2(c+dx)\right)^{3/2}}}{5a(a-b)}-\frac{b\cot(c+dx)}{5a(a-b)(a+b\cot^2(c+dx))^{5/2}}}{d}$$

3.37. $\int \frac{1}{(a+b\cot^2(c+dx))^{7/2}}\,dx$

input
```
Int[(a + b*Cot[c + d*x]^2)^(-7/2),x]
```

output
```
-((-1/5*(b*Cot[c + d*x])/(a*(a - b)*(a + b*Cot[c + d*x]^2)^(5/2)) + (-1/3*
((9*a - 4*b)*b*Cot[c + d*x])/(a*(a - b)*(a + b*Cot[c + d*x]^2)^(3/2)) + ((
15*a^2*ArcTan[(Sqrt[a - b]*Cot[c + d*x])/Sqrt[a + b*Cot[c + d*x]^2]])/(a -
 b)^(3/2) - (b*(33*a^2 - 26*a*b + 8*b^2)*Cot[c + d*x])/(a*(a - b)*Sqrt[a +
 b*Cot[c + d*x]^2]))/(3*a*(a - b)))/(5*a*(a - b)))/d)
```

3.37.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

3.37.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.33

method	result
derivativedivides	$\frac{\frac{b\left(\frac{\cot(dx+c)}{5a\left(a+b\cot(dx+c)^2\right)^{\frac{5}{2}}}+\frac{\frac{4\cot(dx+c)}{15a\left(a+b\cot(dx+c)^2\right)^{\frac{3}{2}}}+\frac{8\cot(dx+c)}{15a^2\sqrt{a+b\cot(dx+c)^2}}}{a}\right)}{a-b}-\frac{\sqrt{b^4(a-b)}\,\arctan\left(\frac{b^2(a-b)\cot(dx+c)}{\sqrt{b^4(a-b)}\sqrt{a+b\cot(dx+c)^2}}\right)}{(a-b)^4b^2}}{d}$
default	$\frac{\frac{b\left(\frac{\cot(dx+c)}{5a\left(a+b\cot(dx+c)^2\right)^{\frac{5}{2}}}+\frac{\frac{4\cot(dx+c)}{15a\left(a+b\cot(dx+c)^2\right)^{\frac{3}{2}}}+\frac{8\cot(dx+c)}{15a^2\sqrt{a+b\cot(dx+c)^2}}}{a}\right)}{a-b}-\frac{\sqrt{b^4(a-b)}\,\arctan\left(\frac{b^2(a-b)\cot(dx+c)}{\sqrt{b^4(a-b)}\sqrt{a+b\cot(dx+c)^2}}\right)}{(a-b)^4b^2}}{d}$

input
```
int(1/(a+b*cot(d*x+c)^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/(a-b)*b*(1/5*cot(d*x+c)/a/(a+b*cot(d*x+c)^2)^(5/2)+4/5/a*(1/3*cot(d
*x+c)/a/(a+b*cot(d*x+c)^2)^(3/2)+2/3/a^2*cot(d*x+c)/(a+b*cot(d*x+c)^2)^(1/
2)))-1/(a-b)^4*(b^4*(a-b))^(1/2)/b^2*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a
+b*cot(d*x+c)^2)^(1/2)*cot(d*x+c))+1/(a-b)^2*b*(1/3*cot(d*x+c)/a/(a+b*cot(
d*x+c)^2)^(3/2)+2/3/a^2*cot(d*x+c)/(a+b*cot(d*x+c)^2)^(1/2))+b/(a-b)^3*cot
(d*x+c)/a/(a+b*cot(d*x+c)^2)^(1/2))
```

3.37.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 700 vs. $2(172) = 344$.

Time = 0.42 (sec) , antiderivative size = 1452, normalized size of antiderivative = 7.64

$$\int \frac{1}{(a+b\cot^2(c+dx))^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*cot(d*x+c)^2)^(7/2),x, algorithm="fricas")
```

output
```
[-1/60*(15*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3 - (a^6 - 3*a^5*b + 3*a^4*b
^2 - a^3*b^3)*cos(2*d*x + 2*c)^3 + 3*(a^6 - a^5*b - a^4*b^2 + a^3*b^3)*cos
(2*d*x + 2*c)^2 - 3*(a^6 + a^5*b - a^4*b^2 - a^3*b^3)*cos(2*d*x + 2*c))*sq
rt(-a + b)*log(-2*(a^2 - 2*a*b + b^2)*cos(2*d*x + 2*c)^2 + 2*((a - b)*cos(
2*d*x + 2*c) - b)*sqrt(-a + b)*sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(co
s(2*d*x + 2*c) - 1))*sin(2*d*x + 2*c) + a^2 - 2*b^2 + 4*(a*b - b^2)*cos(2*
d*x + 2*c)) + 4*(45*a^5*b - 15*a^4*b^2 - 47*a^3*b^3 + 11*a^2*b^4 + 14*a*b^
5 - 8*b^6 + (45*a^5*b - 165*a^4*b^2 + 233*a^3*b^3 - 159*a^2*b^4 + 54*a*b^5
 - 8*b^6)*cos(2*d*x + 2*c)^2 - 2*(45*a^5*b - 90*a^4*b^2 + 27*a^3*b^3 + 44*
a^2*b^4 - 34*a*b^5 + 8*b^6)*cos(2*d*x + 2*c))*sqrt(((a - b)*cos(2*d*x + 2*
c) - a - b)/(cos(2*d*x + 2*c) - 1))*sin(2*d*x + 2*c))/((a^10 - 7*a^9*b + 2
1*a^8*b^2 - 35*a^7*b^3 + 35*a^6*b^4 - 21*a^5*b^5 + 7*a^4*b^6 - a^3*b^7)*d*
cos(2*d*x + 2*c)^3 - 3*(a^10 - 5*a^9*b + 9*a^8*b^2 - 5*a^7*b^3 - 5*a^6*b^4
 + 9*a^5*b^5 - 5*a^4*b^6 + a^3*b^7)*d*cos(2*d*x + 2*c)^2 + 3*(a^10 - 3*a^9
*b + a^8*b^2 + 5*a^7*b^3 - 5*a^6*b^4 - a^5*b^5 + 3*a^4*b^6 - a^3*b^7)*d*co
s(2*d*x + 2*c) - (a^10 - a^9*b - 3*a^8*b^2 + 3*a^7*b^3 + 3*a^6*b^4 - 3*a^5
*b^5 - a^4*b^6 + a^3*b^7)*d), 1/30*(15*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^
3 - (a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*cos(2*d*x + 2*c)^3 + 3*(a^6 - a^
5*b - a^4*b^2 + a^3*b^3)*cos(2*d*x + 2*c)^2 - 3*(a^6 + a^5*b - a^4*b^2 - a
^3*b^3)*cos(2*d*x + 2*c))*sqrt(a - b)*arctan(-sqrt(a - b)*sqrt(((a - b)...
```

3.37.6 Sympy [F]

$$\int \frac{1}{(a+b\cot^2(c+dx))^{7/2}}\,dx = \int \frac{1}{(a+b\cot^2{(c+dx)})^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a+b*cot(d*x+c)**2)**(7/2),x)
```

output
```
Integral((a + b*cot(c + d*x)**2)**(-7/2), x)
```

3.37.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a+b\cot^2(c+dx)\right)^{7/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*cot(d*x+c)^2)^(7/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

3.37.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3249 vs. $2(172) = 344$.

Time = 2.09 (sec) , antiderivative size = 3249, normalized size of antiderivative = 17.10

$$\int \frac{1}{\left(a+b\cot^2(c+dx)\right)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*cot(d*x+c)^2)^(7/2),x, algorithm="giac")
```

output
```
1/15*(30*arctan(-1/2*(sqrt(b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(b*tan(1/2*d*x
+ 1/2*c)^4 + 4*a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c)^2 + b)
+ sqrt(b))/sqrt(a - b))/((a^3*sgn(sin(d*x + c)) - 3*a^2*b*sgn(sin(d*x + c)
) + 3*a*b^2*sgn(sin(d*x + c)) - b^3*sgn(sin(d*x + c)))*sqrt(a - b)) - ((((
((33*a^20*b^3*sgn(sin(d*x + c)) - 620*a^19*b^4*sgn(sin(d*x + c)) + 5525*a^
18*b^5*sgn(sin(d*x + c)) - 31050*a^17*b^6*sgn(sin(d*x + c)) + 123420*a^16*
b^7*sgn(sin(d*x + c)) - 368832*a^15*b^8*sgn(sin(d*x + c)) + 859860*a^14*b^
9*sgn(sin(d*x + c)) - 1601400*a^13*b^10*sgn(sin(d*x + c)) + 2419950*a^12*b
^11*sgn(sin(d*x + c)) - 2996760*a^11*b^12*sgn(sin(d*x + c)) + 3058198*a^10
*b^13*sgn(sin(d*x + c)) - 2576860*a^9*b^14*sgn(sin(d*x + c)) + 1790100*a^8
*b^15*sgn(sin(d*x + c)) - 1020000*a^7*b^16*sgn(sin(d*x + c)) + 472260*a^6*
b^17*sgn(sin(d*x + c)) - 175032*a^5*b^18*sgn(sin(d*x + c)) + 50745*a^4*b^1
9*sgn(sin(d*x + c)) - 11100*a^3*b^20*sgn(sin(d*x + c)) + 1725*a^2*b^21*sgn
(sin(d*x + c)) - 170*a*b^22*sgn(sin(d*x + c)) + 8*b^23*sgn(sin(d*x + c)))*
tan(1/2*d*x + 1/2*c)^2/(a^24 - 21*a^23*b + 210*a^22*b^2 - 1330*a^21*b^3 +
5985*a^20*b^4 - 20349*a^19*b^5 + 54264*a^18*b^6 - 116280*a^17*b^7 + 203490
*a^16*b^8 - 293930*a^15*b^9 + 352716*a^14*b^10 - 352716*a^13*b^11 + 293930
*a^12*b^12 - 203490*a^11*b^13 + 116280*a^10*b^14 - 54264*a^9*b^15 + 20349*
a^8*b^16 - 5985*a^7*b^17 + 1330*a^6*b^18 - 210*a^5*b^19 + 21*a^4*b^20 - a^
3*b^21) + 5*(60*a^21*b^2*sgn(sin(d*x + c)) - 1165*a^20*b^3*sgn(sin(d*x ...
```

3.37.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cot^2(c+dx))^{7/2}}\,dx = \int \frac{1}{\left(b\cot\left(c+d\,x\right)^2+a\right)^{7/2}}\,dx$$

input
```
int(1/(a + b*cot(c + d*x)^2)^(7/2),x)
```

output
```
int(1/(a + b*cot(c + d*x)^2)^(7/2), x)
```

3.38 $\int \left(1-\cot^2(x)\right)^{3/2} dx$

3.38.1 Optimal result

Integrand size = 12, antiderivative size = 54

$$\int \left(1-\cot^2(x)\right)^{3/2} dx = \frac{5}{2}\arcsin(\cot(x)) - 2\sqrt{2}\arctan\left(\frac{\sqrt{2}\cot(x)}{\sqrt{1-\cot^2(x)}}\right)+\frac{1}{2}\cot(x)\sqrt{1-\cot^2(x)}$$

output
```
5/2*arcsin(cot(x))-2*arctan(cot(x)*2^(1/2)/(1-cot(x)^2)^(1/2))*2^(1/2)+1/2
*cot(x)*(1-cot(x)^2)^(1/2)
```

3.38.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 123 vs. 2(54) = 108.

Time = 0.51 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.28

$$\int \left(1-\cot^2(x)\right)^{3/2} dx = \frac{1}{2}(1 -\cot^2(x))^{3/2}\sec^2(2x)\left(\arctan\left(\frac{\cos(x)}{\sqrt{-\cos(2x)}}\right)\sqrt{-\cos(2x)}\sin^3(x)+4\text{arctanh}\left(\frac{\cos(x)}{\sqrt{\cos(2x)}}\right)\sqrt{\cos(2x)}\sin^3$$

input
```
Integrate[(1 - Cot[x]^2)^(3/2),x]
```

output
```
((1 - Cot[x]^2)^(3/2)*Sec[2*x]^2*(ArcTan[Cos[x]/Sqrt[-Cos[2*x]]]*Sqrt[-Cos
[2*x]]*Sin[x]^3 + 4*ArcTanh[Cos[x]/Sqrt[Cos[2*x]]]*Sqrt[Cos[2*x]]*Sin[x]^3
 - 4*Sqrt[2]*Sqrt[Cos[2*x]]*Log[Sqrt[2]*Cos[x] + Sqrt[Cos[2*x]]]*Sin[x]^3
- Sin[4*x]/4))/2
```

3.38.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {3042, 4144, 318, 398, 223, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(1-\cot^2(x)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(1-\tan\left(x+\frac{\pi}{2}\right)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{4144}$$

$$-\int \frac{\left(1-\cot^2(x)\right)^{3/2}}{\cot^2(x)+1}d\cot(x)$$

$$\Big\downarrow \text{318}$$

$$\frac{1}{2}\cot(x)\sqrt{1-\cot^2(x)}-\frac{1}{2}\int \frac{3-5\cot^2(x)}{\sqrt{1-\cot^2(x)}\left(\cot^2(x)+1\right)}d\cot(x)$$

$$\Big\downarrow \text{398}$$

$$\frac{1}{2}\left(5\int \frac{1}{\sqrt{1-\cot^2(x)}}d\cot(x)-8\int \frac{1}{\sqrt{1-\cot^2(x)}\left(\cot^2(x)+1\right)}d\cot(x)\right)+\frac{1}{2}\sqrt{1-\cot^2(x)}\cot(x)$$

$$\Big\downarrow \text{223}$$

$$\frac{1}{2}\left(5\arcsin(\cot(x))-8\int \frac{1}{\sqrt{1-\cot^2(x)}\left(\cot^2(x)+1\right)}d\cot(x)\right)+\frac{1}{2}\sqrt{1-\cot^2(x)}\cot(x)$$

$$\Big\downarrow \text{291}$$

$$\frac{1}{2}\left(5\arcsin(\cot(x))-8\int \frac{1}{\frac{2\cot^2(x)}{1-\cot^2(x)}+1}d\frac{\cot(x)}{\sqrt{1-\cot^2(x)}}\right)+\frac{1}{2}\sqrt{1-\cot^2(x)}\cot(x)$$

$$\Big\downarrow 216$$

$$\frac{1}{2}\left(5\arcsin(\cot(x))-4\sqrt{2}\arctan\left(\frac{\sqrt{2}\cot(x)}{\sqrt{1-\cot^2(x)}}\right)\right)+\frac{1}{2}\cot(x)\sqrt{1-\cot^2(x)}$$

input
```
Int[(1 - Cot[x]^2)^(3/2),x]
```

output
```
(5*ArcSin[Cot[x]] - 4*Sqrt[2]*ArcTan[(Sqrt[2]*Cot[x])/Sqrt[1 - Cot[x]^2]])
/2 + (Cot[x]*Sqrt[1 - Cot[x]^2])/2
```

3.38.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

3.38.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$\frac{5\arcsin(\cot(x))}{2}+\frac{\cot(x)\sqrt{1-\cot(x)^2}}{2}+2\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{1-\cot(x)^2}\,\cot(x)}{-1+\cot(x)^2}\right)$	51
default	$\frac{5\arcsin(\cot(x))}{2}+\frac{\cot(x)\sqrt{1-\cot(x)^2}}{2}+2\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{1-\cot(x)^2}\,\cot(x)}{-1+\cot(x)^2}\right)$	51

input
```
int((1-cot(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
5/2*arcsin(cot(x))+1/2*cot(x)*(1-cot(x)^2)^(1/2)+2*2^(1/2)*arctan(2^(1/2)*
(1-cot(x)^2)^(1/2)/(-1+cot(x)^2)*cot(x))
```

3.38.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. 2(42) = 84.

Time = 0.31 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.04

$$\int (1-\cot^2(x))^{3/2}\,dx = \frac{4\sqrt{2}\arctan\left(\frac{\sqrt{\frac{\cos(2x)}{\cos(2x)-1}}\sin(2x)}{\cos(2x)+1}\right)\sin(2x)+\sqrt{2}\sqrt{\frac{\cos(2x)}{\cos(2x)-1}}(\cos(2x)+1)-5\arctan\left(\frac{\sqrt{2}\sqrt{}\cdots}{\cdots}\right.}{2\sin(2x)}$$

input
```
integrate((1-cot(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(4*sqrt(2)*arctan(sqrt(cos(2*x)/(cos(2*x) - 1))*sin(2*x)/(cos(2*x) + 1
))*sin(2*x) + sqrt(2)*sqrt(cos(2*x)/(cos(2*x) - 1))*(cos(2*x) + 1) - 5*arc
tan(sqrt(2)*sqrt(cos(2*x)/(cos(2*x) - 1))*sin(2*x)/(cos(2*x) + 1))*sin(2*x
))/sin(2*x)
```

3.38.6 Sympy [F]

$$\int \left(1-\cot^2(x)\right)^{3/2}\,dx=\int \left(1-\cot^2\left(x\right)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((1-cot(x)**2)**(3/2),x)
```

output
```
Integral((1 - cot(x)**2)**(3/2), x)
```

3.38.7 Maxima [F]

$$\int \left(1-\cot^2(x)\right)^{3/2}\,dx=\int \left(-\cot\left(x\right)^2+1\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((1-cot(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-cot(x)^2 + 1)^(3/2), x)
```

3.38.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 257 vs. 2(42) = 84.

Time = 0.33 (sec) , antiderivative size = 257, normalized size of antiderivative = 4.76

$$\int (1 -\cot^2(x))^{3/2}\,dx=\frac{1}{4}\left(5\,\pi\mathrm{sgn}(\cos\left(x\right))-4\,\sqrt{2}\left(\pi\mathrm{sgn}(\cos\left(x\right))+2\,\arctan\left(-\frac{\left(\frac{\left(\sqrt{2}\sqrt{-2\,\cos\left(x\right)^2+1}-\sqrt{2}\right)^2}{\cos\left(x\right)^2}-4\right)}{4\left(\sqrt{2}\sqrt{-2\,\cos\left(x\right)^2+1}-\right.}\right.\right.\right.$$

input
```
integrate((1-cot(x)^2)^(3/2),x, algorithm="giac")
```

output
```
1/4*(5*pi*sgn(cos(x)) - 4*sqrt(2)*(pi*sgn(cos(x)) + 2*arctan(-1/4*((sqrt(2
)*sqrt(-2*cos(x)^2 + 1) - sqrt(2))^2/cos(x)^2 - 4)*cos(x)/(sqrt(2)*sqrt(-2
*cos(x)^2 + 1) - sqrt(2)))) + 4*sqrt(2)*((sqrt(2)*sqrt(-2*cos(x)^2 + 1) -
sqrt(2))/cos(x) - 4*cos(x)/(sqrt(2)*sqrt(-2*cos(x)^2 + 1) - sqrt(2)))/(((s
qrt(2)*sqrt(-2*cos(x)^2 + 1) - sqrt(2))/cos(x) - 4*cos(x)/(sqrt(2)*sqrt(-2
*cos(x)^2 + 1) - sqrt(2)))^2 + 8) + 10*arctan(-1/4*sqrt(2)*((sqrt(2)*sqrt(
-2*cos(x)^2 + 1) - sqrt(2))^2/cos(x)^2 - 4)*cos(x)/(sqrt(2)*sqrt(-2*cos(x)
^2 + 1) - sqrt(2))))*sgn(sin(x))
```

3.38.9 Mupad [B] (verification not implemented)

Time = 14.22 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.93

$$\int \left(1-\cot^2(x)\right)^{3/2} dx = \frac{5\,\mathrm{asin}(\cot(x))}{2} + \frac{\cot(x)\,\sqrt{1-\cot{(x)}^2}}{2} - \sqrt{2}\,\ln\left(\frac{\frac{\sqrt{2}\,(-1+\cot(x)\,1\mathrm{i})\,1\mathrm{i}}{2} - \sqrt{1-\cot{(x)}^2}\,1\mathrm{i}}{\cot{(x)}-\mathrm{i}}\right)\,1\mathrm{i} + \sqrt{2}\,\ln\left(\frac{\frac{\sqrt{2}\,(1+\cot(x)\,1\mathrm{i})\,1\mathrm{i}}{2} + \sqrt{1-\cot{(x)}^2}\,1\mathrm{i}}{\cot{(x)}+1\mathrm{i}}\right)\,1\mathrm{i}$$

input
```
int((1 - cot(x)^2)^(3/2),x)
```

output
```
(5*asin(cot(x)))/2 + (cot(x)*(1 - cot(x)^2)^(1/2))/2 - 2^(1/2)*log(((2^(1/
2)*(cot(x)*1i - 1)*1i)/2 - (1 - cot(x)^2)^(1/2)*1i)/(cot(x) - 1i))*1i + 2^
(1/2)*log(((2^(1/2)*(cot(x)*1i + 1)*1i)/2 + (1 - cot(x)^2)^(1/2)*1i)/(cot(
x) + 1i))*1i
```

3.39 $\int \sqrt{1-\cot^2(x)}\,dx$

3.39.1 Optimal result

Integrand size = 12, antiderivative size = 32

$$\int \sqrt{1-\cot^2(x)}\,dx = \arcsin(\cot(x)) - \sqrt{2}\arctan\left(\frac{\sqrt{2}\cot(x)}{\sqrt{1-\cot^2(x)}}\right)$$

output
```
arcsin(cot(x))-arctan(cot(x)*2^(1/2)/(1-cot(x)^2)^(1/2))*2^(1/2)
```

3.39.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.94

$$\int \sqrt{1-\cot^2(x)}\,dx = \frac{\sqrt{1-\cot^2(x)}\left(-\operatorname{arctanh}\left(\frac{\cos(x)}{\sqrt{\cos(2x)}}\right)+\sqrt{2}\log\left(\sqrt{2}\cos(x)+\sqrt{\cos(2x)}\right)\right)\sin(x)}{\sqrt{\cos(2x)}}$$

input
```
Integrate[Sqrt[1 - Cot[x]^2],x]
```

output
```
(Sqrt[1 - Cot[x]^2]*(-ArcTanh[Cos[x]/Sqrt[Cos[2*x]]] + Sqrt[2]*Log[Sqrt[2]
*Cos[x] + Sqrt[Cos[2*x]]])*Sin[x])/Sqrt[Cos[2*x]]
```

3.39.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 4144, 301, 223, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{1-\cot^2(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{1-\tan\left(x+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{4144}$$

$$-\int \frac{\sqrt{1-\cot^2(x)}}{\cot^2(x)+1}d\cot(x)$$

$$\Big\downarrow \text{301}$$

$$\int \frac{1}{\sqrt{1-\cot^2(x)}}d\cot(x)-2\int \frac{1}{\sqrt{1-\cot^2(x)}\,(\cot^2(x)+1)}d\cot(x)$$

$$\Big\downarrow \text{223}$$

$$\arcsin(\cot(x))-2\int \frac{1}{\sqrt{1-\cot^2(x)}\,(\cot^2(x)+1)}d\cot(x)$$

$$\Big\downarrow \text{291}$$

$$\arcsin(\cot(x))-2\int \frac{1}{\frac{2\cot^2(x)}{1-\cot^2(x)}+1}d\frac{\cot(x)}{\sqrt{1-\cot^2(x)}}$$

$$\Big\downarrow \text{216}$$

$$\arcsin(\cot(x))-\sqrt{2}\arctan\left(\frac{\sqrt{2}\cot(x)}{\sqrt{1-\cot^2(x)}}\right)$$

input
```
Int[Sqrt[1 - Cot[x]^2],x]
```

output
```
ArcSin[Cot[x]] - Sqrt[2]*ArcTan[(Sqrt[2]*Cot[x])/Sqrt[1 - Cot[x]^2]]
```

3.39.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

3.39.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$\arcsin\left(\cot\left(x\right)\right)+\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{1-\cot\left(x\right)^{2}}\,\cot\left(x\right)}{-1+\cot\left(x\right)^{2}}\right)$	34
default	$\arcsin\left(\cot\left(x\right)\right)+\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{1-\cot\left(x\right)^{2}}\,\cot\left(x\right)}{-1+\cot\left(x\right)^{2}}\right)$	34

input
```
int((1-cot(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arcsin(cot(x))+2^(1/2)*arctan(2^(1/2)*(1-cot(x)^2)^(1/2)/(-1+cot(x)^2)*cot
(x))
```

3.39.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(26) = 52.

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.12

$$\int \sqrt{1-\cot^2(x)}\,dx = \sqrt{2}\arctan\left(\frac{\sqrt{\frac{\cos(2x)}{\cos(2x)-1}}\sin(2x)}{\cos(2x)+1}\right)-\arctan\left(\frac{\sqrt{2}\sqrt{\frac{\cos(2x)}{\cos(2x)-1}}\sin(2x)}{\cos(2x)+1}\right)$$

input
```
integrate((1-cot(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(2)*arctan(sqrt(cos(2*x)/(cos(2*x) - 1))*sin(2*x)/(cos(2*x) + 1)) - ar
ctan(sqrt(2)*sqrt(cos(2*x)/(cos(2*x) - 1))*sin(2*x)/(cos(2*x) + 1))
```

3.39.6 Sympy [F]

$$\int \sqrt{1-\cot^2(x)}\,dx = \int \sqrt{1-\cot^2(x)}\,dx$$

input
```
integrate((1-cot(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(1 - cot(x)**2), x)
```

3.39.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.85 (sec) , antiderivative size = 507, normalized size of antiderivative = 15.84

$$\int \sqrt{1-\cot^2(x)}\,dx =$$

$$-\frac{1}{2}\sqrt{2}\left(\sqrt{2}\arctan\left(\frac{\left(\left|2\,e^{(2i\,x)}-2\right|^4+16\,\cos\left(2\,x\right)^4+16\,\sin\left(2\,x\right)^4+8\,\left(\cos\left(2\,x\right)^2-\sin\left(2\,x\right)^2+2\,\cos\left(2\right.\right.\right.}{}\right.\right.$$

input
```
integrate((1-cot(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*sqrt(2)*(sqrt(2)*arctan2(((abs(2*e^(2*I*x) - 2)^4 + 16*cos(2*x)^4 + 1
6*sin(2*x)^4 + 8*(cos(2*x)^2 - sin(2*x)^2 + 2*cos(2*x) + 1)*abs(2*e^(2*I*x
) - 2)^2 + 64*cos(2*x)^3 + 32*(cos(2*x)^2 + 2*cos(2*x) + 1)*sin(2*x)^2 + 9
6*cos(2*x)^2 + 64*cos(2*x) + 16)^(1/4)*sin(1/2*arctan2(8*(cos(2*x) + 1)*si
n(2*x)/abs(2*e^(2*I*x) - 2)^2, (abs(2*e^(2*I*x) - 2)^2 + 4*cos(2*x)^2 - 4*
sin(2*x)^2 + 8*cos(2*x) + 4)/abs(2*e^(2*I*x) - 2)^2)) + 2*sin(2*x))/abs(2*
e^(2*I*x) - 2), ((abs(2*e^(2*I*x) - 2)^4 + 16*cos(2*x)^4 + 16*sin(2*x)^4 +
 8*(cos(2*x)^2 - sin(2*x)^2 + 2*cos(2*x) + 1)*abs(2*e^(2*I*x) - 2)^2 + 64*
cos(2*x)^3 + 32*(cos(2*x)^2 + 2*cos(2*x) + 1)*sin(2*x)^2 + 96*cos(2*x)^2 +
 64*cos(2*x) + 16)^(1/4)*cos(1/2*arctan2(8*(cos(2*x) + 1)*sin(2*x)/abs(2*e
^(2*I*x) - 2)^2, (abs(2*e^(2*I*x) - 2)^2 + 4*cos(2*x)^2 - 4*sin(2*x)^2 + 8
*cos(2*x) + 4)/abs(2*e^(2*I*x) - 2)^2)) + 2*cos(2*x) + 2)/abs(2*e^(2*I*x)
- 2)) - arctan2((cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*sin(1/2*a
rctan2(sin(4*x), cos(4*x) + 1)) + sin(2*x), (cos(4*x)^2 + sin(4*x)^2 + 2*c
os(4*x) + 1)^(1/4)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1)) + cos(2*x)))
```

3.39.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 170, normalized size of antiderivative = 5.31

$$\int \sqrt{1-\cot^2(x)}\,dx = -\frac{1}{2}\left(\pi-\sqrt{2}\pi-2\sqrt{2}\arctan\left(-\frac{1}{2}i\sqrt{2}\right)+2\arctan(-i)\right)\mathrm{sgn}(\sin(x)) + \frac{1}{2}\left(\pi\mathrm{sgn}(\cos(x))-\sqrt{2}\left(\pi\mathrm{sgn}(\cos(x))+2\arctan\left(-\frac{\left(\frac{\left(\sqrt{2}\sqrt{-2\cos(x)^2+1}-\sqrt{2}\right)^2}{\cos(x)^2}-4\right)\cos(x)}{4\left(\sqrt{2}\sqrt{-2\cos(x)^2+1}-\sqrt{2}\right)}\right)\right)+2\text{ a}$$

input
```
integrate((1-cot(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*(pi - sqrt(2)*pi - 2*sqrt(2)*arctan(-1/2*I*sqrt(2)) + 2*arctan(-I))*s
gn(sin(x)) + 1/2*(pi*sgn(cos(x)) - sqrt(2)*(pi*sgn(cos(x)) + 2*arctan(-1/4
*((sqrt(2)*sqrt(-2*cos(x)^2 + 1) - sqrt(2))^2/cos(x)^2 - 4)*cos(x)/(sqrt(2
)*sqrt(-2*cos(x)^2 + 1) - sqrt(2)))) + 2*arctan(-1/4*sqrt(2)*((sqrt(2)*sqr
t(-2*cos(x)^2 + 1) - sqrt(2))^2/cos(x)^2 - 4)*cos(x)/(sqrt(2)*sqrt(-2*cos(
x)^2 + 1) - sqrt(2))))*sgn(sin(x))
```

3.39.9 Mupad [B] (verification not implemented)

Time = 14.19 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.75

$$\int \sqrt{1-\cot^2(x)}\,dx = \mathrm{asin}(\cot(x)) - \frac{\sqrt{2}\ln\left(\frac{\frac{\sqrt{2}\,(-1+\cot(x)\,1\mathrm{i})\,1\mathrm{i}}{2}-\sqrt{1-\cot(x)^2}\,1\mathrm{i}}{\cot(x)-\mathrm{i}}\right)1\mathrm{i}}{2} + \frac{\sqrt{2}\ln\left(\frac{\frac{\sqrt{2}\,(1+\cot(x)\,1\mathrm{i})\,1\mathrm{i}}{2}+\sqrt{1-\cot(x)^2}\,1\mathrm{i}}{\cot(x)+1\mathrm{i}}\right)1\mathrm{i}}{2}$$

input
```
int((1 - cot(x)^2)^(1/2),x)
```

output
```
asin(cot(x)) - (2^(1/2)*log(((2^(1/2)*(cot(x)*1i - 1)*1i)/2 - (1 - cot(x)^
2)^(1/2)*1i)/(cot(x) - 1i))*1i)/2 + (2^(1/2)*log(((2^(1/2)*(cot(x)*1i + 1)
*1i)/2 + (1 - cot(x)^2)^(1/2)*1i)/(cot(x) + 1i))*1i)/2
```

3.40 $\int \frac{1}{\sqrt{1-\cot^2(x)}}\,dx$

3.40.1 Optimal result

Integrand size = 12, antiderivative size = 28

$$\int \frac{1}{\sqrt{1-\cot^2(x)}}\,dx = -\frac{\arctan\left(\frac{\sqrt{2}\cot(x)}{\sqrt{1-\cot^2(x)}}\right)}{\sqrt{2}}$$

output
```
-1/2*arctan(cot(x)*2^(1/2)/(1-cot(x)^2)^(1/2))*2^(1/2)
```

3.40.2 Mathematica [A] (warning: unable to verify)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{\sqrt{1-\cot^2(x)}}\,dx = -\frac{\arcsin\left(\frac{\sqrt{2}\cot(x)}{\sqrt{1+\cot^2(x)}}\right)}{\sqrt{2}}$$

input
```
Integrate[1/Sqrt[1 - Cot[x]^2],x]
```

output
```
-(ArcSin[(Sqrt[2]*Cot[x])/Sqrt[1 + Cot[x]^2]]/Sqrt[2])
```

3.40.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4144, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{1-\cot^2(x)}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sqrt{1-\tan\left(x+\frac{\pi}{2}\right)^2}}dx$$

$$\Big\downarrow \; 4144$$

$$-\int \frac{1}{\sqrt{1-\cot^2(x)}\left(\cot^2(x)+1\right)}d\cot(x)$$

$$\Big\downarrow \; 291$$

$$-\int \frac{1}{\frac{2\cot^2(x)}{1-\cot^2(x)}+1}d\frac{\cot(x)}{\sqrt{1-\cot^2(x)}}$$

$$\Big\downarrow \; 216$$

$$-\frac{\arctan\left(\frac{\sqrt{2}\cot(x)}{\sqrt{1-\cot^2(x)}}\right)}{\sqrt{2}}$$

input
```
Int[1/Sqrt[1 - Cot[x]^2],x]
```

output
```
-(ArcTan[(Sqrt[2]*Cot[x])/Sqrt[1 - Cot[x]^2]]/Sqrt[2])
```

3.40.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

3.40.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

method	result	size
derivativedivides	$\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{1-\cot(x)^{2}}\,\cot(x)}{-1+\cot(x)^{2}}\right)}{2}$	31
default	$\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{1-\cot(x)^{2}}\,\cot(x)}{-1+\cot(x)^{2}}\right)}{2}$	31

input
```
int(1/(1-cot(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*2^(1/2)*arctan(2^(1/2)*(1-cot(x)^2)^(1/2)/(-1+cot(x)^2)*cot(x))
```

3.40.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(22) = 44$.

Time = 0.31 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.00

$$\int \frac{1}{\sqrt{1-\cot^2(x)}}\,dx = \frac{1}{4}\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{2}\cos(2x)+\sqrt{2}\right)\sqrt{\frac{\cos(2x)}{\cos(2x)-1}}\sin(2x)}{4\left(\cos(2x)^2+\cos(2x)\right)}\right)$$

input
```
integrate(1/(1-cot(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*arctan(1/4*sqrt(2)*(2*sqrt(2)*cos(2*x) + sqrt(2))*sqrt(cos(2*x
)/(cos(2*x) - 1))*sin(2*x)/(cos(2*x)^2 + cos(2*x)))
```

3.40.6 Sympy [F]

$$\int \frac{1}{\sqrt{1-\cot^2(x)}}\,dx = \int \frac{1}{\sqrt{1-\cot^2(x)}}\,dx$$

input
```
integrate(1/(1-cot(x)**2)**(1/2),x)
```

output
```
Integral(1/sqrt(1 - cot(x)**2), x)
```

3.40.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(22) = 44$.

Time = 0.48 (sec) , antiderivative size = 90, normalized size of antiderivative = 3.21

$$\int \frac{1}{\sqrt{1-\cot^2(x)}}\,dx = \frac{1}{4}\sqrt{2}\arctan\left(\left(\cos(4x)^2+\sin(4x)^2+2\cos(4x)+1\right)^{\frac{1}{4}}\sin\left(\frac{1}{2}\arctan(\sin(4x),\cos(4x)+1)\right)+\sin(2x),\left(\cos(4x)^2+\sin(4x)^2+2\cos(4x)+1\right)^{\frac{1}{4}}\cos\left(\frac{1}{2}\arctan(\sin(4x),\cos(4x)+1)\right)+\cos(2x)\right)$$

input
```
integrate(1/(1-cot(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/4*sqrt(2)*arctan2((cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*sin(1
/2*arctan2(sin(4*x), cos(4*x) + 1)) + sin(2*x), (cos(4*x)^2 + sin(4*x)^2 +
 2*cos(4*x) + 1)^(1/4)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1)) + cos(2*x)
)
```

3.40.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.21

$$\int \frac{1}{\sqrt{1-\cot^2(x)}}\,dx = -\frac{1}{2}i\sqrt{2}\log\left(i\sqrt{2}+i\right)\operatorname{sgn}(\sin(x)) - \frac{\sqrt{2}\arcsin\left(\sqrt{2}\cos(x)\right)}{2\operatorname{sgn}(\sin(x))}$$

input
```
integrate(1/(1-cot(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*I*sqrt(2)*log(I*sqrt(2) + I)*sgn(sin(x)) - 1/2*sqrt(2)*arcsin(sqrt(2)
*cos(x))/sgn(sin(x))
```

3.40.9 Mupad [B] (verification not implemented)

Time = 13.18 (sec) , antiderivative size = 85, normalized size of antiderivative = 3.04

$$\int \frac{1}{\sqrt{1-\cot^2(x)}}\,dx = -\frac{\sqrt{2}\ln\left(\frac{\frac{\sqrt{2}\,(-1+\cot(x)\,1\mathrm{i})\,1\mathrm{i}}{2}-\sqrt{1-\cot(x)^2}\,1\mathrm{i}}{\cot(x)-\mathrm{i}}\right)\,1\mathrm{i}}{4} + \frac{\sqrt{2}\ln\left(\frac{\frac{\sqrt{2}\,(1+\cot(x)\,1\mathrm{i})\,1\mathrm{i}}{2}+\sqrt{1-\cot(x)^2}\,1\mathrm{i}}{\cot(x)+1\mathrm{i}}\right)\,1\mathrm{i}}{4}$$

input
```
int(1/(1 - cot(x)^2)^(1/2),x)
```

output
```
(2^(1/2)*log(((2^(1/2)*(cot(x)*1i + 1)*1i)/2 + (1 - cot(x)^2)^(1/2)*1i)/(c
ot(x) + 1i))*1i)/4 - (2^(1/2)*log(((2^(1/2)*(cot(x)*1i - 1)*1i)/2 - (1 - c
ot(x)^2)^(1/2)*1i)/(cot(x) - 1i))*1i)/4
```

3.41 $\int (-1+\cot^2(x))^{3/2}\,dx$

3.41.1 Optimal result

Integrand size = 10, antiderivative size = 61

$$\int (-1+\cot^2(x))^{3/2}\,dx = \frac{5}{2}\text{arctanh}\left(\frac{\cot(x)}{\sqrt{-1+\cot^2(x)}}\right) - 2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\cot(x)}{\sqrt{-1+\cot^2(x)}}\right) - \frac{1}{2}\cot(x)\sqrt{-1+\cot^2(x)}$$

output
```
5/2*arctanh(cot(x)/(-1+cot(x)^2)^(1/2))-2*arctanh(cot(x)*2^(1/2)/(-1+cot(x
)^2)^(1/2))*2^(1/2)-1/2*cot(x)*(-1+cot(x)^2)^(1/2)
```

3.41.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.98

$$\int (-1+\cot^2(x))^{3/2}\,dx = \frac{1}{2}(-1 +\cot^2(x))^{3/2}\sec^2(2x)\left(\arctan\left(\frac{\cos(x)}{\sqrt{-\cos(2x)}}\right)\sqrt{-\cos(2x)}\sin^3(x)+4\text{arctanh}\left(\frac{\cos(x)}{\sqrt{\cos(2x)}}\right)\sqrt{\cos(2x)}\sin^3($$

input
```
Integrate[(-1 + Cot[x]^2)^(3/2),x]
```

output
```
((-1 + Cot[x]^2)^(3/2)*Sec[2*x]^2*(ArcTan[Cos[x]/Sqrt[-Cos[2*x]]]*Sqrt[-Co
s[2*x]]*Sin[x]^3 + 4*ArcTanh[Cos[x]/Sqrt[Cos[2*x]]]*Sqrt[Cos[2*x]]*Sin[x]^
3 - 4*Sqrt[2]*Sqrt[Cos[2*x]]*Log[Sqrt[2]*Cos[x] + Sqrt[Cos[2*x]]]*Sin[x]^3
 - Sin[4*x]/4))/2
```

3.41.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {3042, 4144, 318, 398, 224, 219, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\cot^2(x)-1\right)^{3/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(\tan\left(x+\frac{\pi}{2}\right)^2-1\right)^{3/2} dx$$

$$\Big\downarrow \text{4144}$$

$$-\int \frac{\left(\cot^2(x)-1\right)^{3/2}}{\cot^2(x)+1}d\cot(x)$$

$$\Big\downarrow \text{318}$$

$$-\frac{1}{2}\int \frac{3-5\cot^2(x)}{\sqrt{\cot^2(x)-1}\left(\cot^2(x)+1\right)}d\cot(x)-\frac{1}{2}\sqrt{\cot^2(x)-1}\cot(x)$$

$$\Big\downarrow \text{398}$$

$$\frac{1}{2}\left(5\int \frac{1}{\sqrt{\cot^2(x)-1}}d\cot(x)-8\int \frac{1}{\sqrt{\cot^2(x)-1}\left(\cot^2(x)+1\right)}d\cot(x)\right)-\frac{1}{2}\cot(x)\sqrt{\cot^2(x)-1}$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{2}\left(5\int \frac{1}{1-\frac{\cot^2(x)}{\cot^2(x)-1}}d\frac{\cot(x)}{\sqrt{\cot^2(x)-1}}-8\int \frac{1}{\sqrt{\cot^2(x)-1}\left(\cot^2(x)+1\right)}d\cot(x)\right)-\frac{1}{2}\cot(x)\sqrt{\cot^2(x)-1}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(5\text{arctanh}\left(\frac{\cot(x)}{\sqrt{\cot^2(x)-1}}\right)-8\int \frac{1}{\sqrt{\cot^2(x)-1}\,(\cot^2(x)+1)}d\cot(x)\right)-\frac{1}{2}\cot(x)\sqrt{\cot^2(x)-1}$$

$\Big\downarrow$ 291

$$\frac{1}{2}\left(5\text{arctanh}\left(\frac{\cot(x)}{\sqrt{\cot^2(x)-1}}\right)-8\int \frac{1}{1-\frac{2\cot^2(x)}{\cot^2(x)-1}}d\frac{\cot(x)}{\sqrt{\cot^2(x)-1}}\right)-\frac{1}{2}\cot(x)\sqrt{\cot^2(x)-1}$$

$\Big\downarrow$ 219

$$\frac{1}{2}\left(5\text{arctanh}\left(\frac{\cot(x)}{\sqrt{\cot^2(x)-1}}\right)-4\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\cot(x)}{\sqrt{\cot^2(x)-1}}\right)\right)-\frac{1}{2}\cot(x)\sqrt{\cot^2(x)-1}$$

input
```
Int[(-1 + Cot[x]^2)^(3/2),x]
```

output
```
(5*ArcTanh[Cot[x]/Sqrt[-1 + Cot[x]^2]] - 4*Sqrt[2]*ArcTanh[(Sqrt[2]*Cot[x]
)/Sqrt[-1 + Cot[x]^2]])/2 - (Cot[x]*Sqrt[-1 + Cot[x]^2])/2
```

3.41.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

3.41.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$\frac{5\ln\left(\cot(x)+\sqrt{-1+\cot(x)^2}\right)}{2}-\frac{\cot(x)\sqrt{-1+\cot(x)^2}}{2}-2\,\mathrm{arctanh}\left(\frac{\cot(x)\sqrt{2}}{\sqrt{-1+\cot(x)^2}}\right)\sqrt{2}$	48
default	$\frac{5\ln\left(\cot(x)+\sqrt{-1+\cot(x)^2}\right)}{2}-\frac{\cot(x)\sqrt{-1+\cot(x)^2}}{2}-2\,\mathrm{arctanh}\left(\frac{\cot(x)\sqrt{2}}{\sqrt{-1+\cot(x)^2}}\right)\sqrt{2}$	48

input
```
int((-1+cot(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
5/2*ln(cot(x)+(-1+cot(x)^2)^(1/2))-1/2*cot(x)*(-1+cot(x)^2)^(1/2)-2*arctan
h(cot(x)*2^(1/2)/(-1+cot(x)^2)^(1/2))*2^(1/2)
```

3.41.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(47) = 94$.

Time = 0.28 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.79

$$\int (-1+\cot^2(x))^{3/2}\,dx = \frac{4\sqrt{2}\log\left(2\sqrt{-\frac{\cos(2x)}{\cos(2x)-1}}\sin(2x) - 2\cos(2x) - 1\right)\sin(2x) - 2\sqrt{2}\sqrt{-\frac{\cos(2x)}{\cos(2x)-1}}(\cos(2x}{}$$

input
```
integrate((-1+cot(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/4*(4*sqrt(2)*log(2*sqrt(-cos(2*x)/(cos(2*x) - 1))*sin(2*x) - 2*cos(2*x)
- 1)*sin(2*x) - 2*sqrt(2)*sqrt(-cos(2*x)/(cos(2*x) - 1))*(cos(2*x) + 1) +
5*log((sqrt(2)*sqrt(-cos(2*x)/(cos(2*x) - 1))*sin(2*x) + cos(2*x) + 1)/(co
s(2*x) + 1))*sin(2*x) - 5*log((sqrt(2)*sqrt(-cos(2*x)/(cos(2*x) - 1))*sin(
2*x) - cos(2*x) - 1)/(cos(2*x) + 1))*sin(2*x))/sin(2*x)
```

3.41.6 Sympy [F]

$$\int (-1+\cot^2(x))^{3/2}\,dx = \int \left(\cot^2(x) - 1\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((-1+cot(x)**2)**(3/2),x)
```

output
```
Integral((cot(x)**2 - 1)**(3/2), x)
```

3.41.7 Maxima [F]

$$\int \left(-1+\cot^2(x)\right)^{3/2}\,dx=\int \left(\cot\left(x\right)^2-1\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((-1+cot(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((cot(x)^2 - 1)^(3/2), x)
```

3.41.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. 2(47) = 94.

Time = 0.52 (sec) , antiderivative size = 179, normalized size of antiderivative = 2.93

$$\int (-1+\cot^2(x))^{3/2}\,dx=\frac{1}{4}\left(4\sqrt{2}\log\left(\left(\sqrt{2}\cos\left(x\right)-\sqrt{2\cos\left(x\right)^2-1}\right)^2\right)-\frac{4\sqrt{2}\left(3\left(\sqrt{2}\cos\left(x\right)\right.\right.}{\left(\sqrt{2}\cos\left(x\right)-\sqrt{2\cos\left(x\right)^2-1}\right)^4\ldots}\right.$$

input
```
integrate((-1+cot(x)^2)^(3/2),x, algorithm="giac")
```

output
```
1/4*(4*sqrt(2)*log((sqrt(2)*cos(x) - sqrt(2*cos(x)^2 - 1))^2) - 4*sqrt(2)*
(3*(sqrt(2)*cos(x) - sqrt(2*cos(x)^2 - 1))^2 - 1)/((sqrt(2)*cos(x) - sqrt(
2*cos(x)^2 - 1))^4 - 6*(sqrt(2)*cos(x) - sqrt(2*cos(x)^2 - 1))^2 + 1) + 5*
log(abs(2*(sqrt(2)*cos(x) - sqrt(2*cos(x)^2 - 1))^2 - 4*sqrt(2) - 6)/abs(2
*(sqrt(2)*cos(x) - sqrt(2*cos(x)^2 - 1))^2 + 4*sqrt(2) - 6)))*sgn(sin(x))
```

3.41.9 Mupad [F(-1)]

Timed out.

$$\int \left(-1+\cot^2(x)\right)^{3/2} \, dx = \int \left(\cot(x)^2-1\right)^{3/2} \, dx$$

input
```
int((cot(x)^2 - 1)^(3/2),x)
```

output
```
int((cot(x)^2 - 1)^(3/2), x)
```

3.42 $\int \sqrt{-1+\cot^2(x)}\,dx$

3.42.1 Optimal result

Integrand size = 10, antiderivative size = 42

$$\int \sqrt{-1+\cot^2(x)}\,dx=-\text{arctanh}\left(\frac{\cot(x)}{\sqrt{-1+\cot^2(x)}}\right)+\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\cot(x)}{\sqrt{-1+\cot^2(x)}}\right)$$

output
```
-arctanh(cot(x)/(-1+cot(x)^2)^(1/2))+arctanh(cot(x)*2^(1/2)/(-1+cot(x)^2)^
(1/2))*2^(1/2)
```

3.42.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.43

$$\int \sqrt{-1+\cot^2(x)}\,dx = \frac{\sqrt{-1+\cot^2(x)}\left(-\text{arctanh}\left(\frac{\cos(x)}{\sqrt{\cos(2x)}}\right)+\sqrt{2}\log\left(\sqrt{2}\cos(x)+\sqrt{\cos(2x)}\right)\right)\sin(x)}{\sqrt{\cos(2x)}}$$

input
```
Integrate[Sqrt[-1 + Cot[x]^2],x]
```

output
```
(Sqrt[-1 + Cot[x]^2]*(-ArcTanh[Cos[x]/Sqrt[Cos[2*x]]] + Sqrt[2]*Log[Sqrt[2
]*Cos[x] + Sqrt[Cos[2*x]]])*Sin[x])/Sqrt[Cos[2*x]]
```

3.42.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {3042, 4144, 301, 224, 219, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cot^2(x)-1}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\tan\left(x+\frac{\pi}{2}\right)^2-1}dx$$

$$\Big\downarrow \text{4144}$$

$$-\int \frac{\sqrt{\cot^2(x)-1}}{\cot^2(x)+1}d\cot(x)$$

$$\Big\downarrow \text{301}$$

$$2\int \frac{1}{\sqrt{\cot^2(x)-1}\,(\cot^2(x)+1)}d\cot(x)-\int \frac{1}{\sqrt{\cot^2(x)-1}}d\cot(x)$$

$$\Big\downarrow \text{224}$$

$$2\int \frac{1}{\sqrt{\cot^2(x)-1}\,(\cot^2(x)+1)}d\cot(x)-\int \frac{1}{1-\frac{\cot^2(x)}{\cot^2(x)-1}}d\frac{\cot(x)}{\sqrt{\cot^2(x)-1}}$$

$$\Big\downarrow \text{219}$$

$$2\int \frac{1}{\sqrt{\cot^2(x)-1}\,(\cot^2(x)+1)}d\cot(x)-\text{arctanh}\left(\frac{\cot(x)}{\sqrt{\cot^2(x)-1}}\right)$$

$$\Big\downarrow \text{291}$$

$$2\int \frac{1}{1-\frac{2\cot^2(x)}{\cot^2(x)-1}}d\frac{\cot(x)}{\sqrt{\cot^2(x)-1}}-\text{arctanh}\left(\frac{\cot(x)}{\sqrt{\cot^2(x)-1}}\right)$$

$$\Big\downarrow \text{219}$$

$$\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\cot(x)}{\sqrt{\cot^2(x)-1}}\right)-\text{arctanh}\left(\frac{\cot(x)}{\sqrt{\cot^2(x)-1}}\right)$$

input
```
Int[Sqrt[-1 + Cot[x]^2],x]
```

output
```
-ArcTanh[Cot[x]/Sqrt[-1 + Cot[x]^2]] + Sqrt[2]*ArcTanh[(Sqrt[2]*Cot[x])/Sq
rt[-1 + Cot[x]^2]]
```

3.42.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

3.42.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$-\ln\left(\cot\left(x\right)+\sqrt{-1+\cot\left(x\right)^{2}}\right)+\operatorname{arctanh}\left(\frac{\cot\left(x\right)\sqrt{2}}{\sqrt{-1+\cot\left(x\right)^{2}}}\right)\sqrt{2}$	35
default	$-\ln\left(\cot\left(x\right)+\sqrt{-1+\cot\left(x\right)^{2}}\right)+\operatorname{arctanh}\left(\frac{\cot\left(x\right)\sqrt{2}}{\sqrt{-1+\cot\left(x\right)^{2}}}\right)\sqrt{2}$	35

input
```
int((-1+cot(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-ln(cot(x)+(-1+cot(x)^2)^(1/2))+arctanh(cot(x)*2^(1/2)/(-1+cot(x)^2)^(1/2)
)*2^(1/2)
```

3.42.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 123 vs. 2(34) = 68.

Time = 0.29 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.93

$$\int\sqrt{-1+\cot^2(x)}\,dx=\frac{1}{2}\sqrt{2}\log\left(-2\sqrt{-\frac{\cos(2x)}{\cos(2x)-1}}\sin(2x)-2\cos(2x)-1\right)-\frac{1}{2}\log\left(\frac{\sqrt{2}\sqrt{-\frac{\cos(2x)}{\cos(2x)-1}}\sin(2x)+\cos(2x)+1}{\cos(2x)+1}\right)+\frac{1}{2}\log\left(\frac{\sqrt{2}\sqrt{-\frac{\cos(2x)}{\cos(2x)-1}}\sin(2x)-\cos(2x)-1}{\cos(2x)+1}\right)$$

input
```
integrate((-1+cot(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*log(-2*sqrt(-cos(2*x)/(cos(2*x) - 1))*sin(2*x) - 2*cos(2*x) -
1) - 1/2*log((sqrt(2)*sqrt(-cos(2*x)/(cos(2*x) - 1))*sin(2*x) + cos(2*x) +
 1)/(cos(2*x) + 1)) + 1/2*log((sqrt(2)*sqrt(-cos(2*x)/(cos(2*x) - 1))*sin(
2*x) - cos(2*x) - 1)/(cos(2*x) + 1))
```

3.42.6 Sympy [F]

$$\int \sqrt{-1+\cot^2(x)}\,dx = \int \sqrt{\cot^2(x)-1}\,dx$$

input
```
integrate((-1+cot(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(cot(x)**2 - 1), x)
```

3.42.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 941, normalized size of antiderivative = 22.40

$$\int \sqrt{-1+\cot^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate((-1+cot(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(2)*arcsinh(1) + 1/4*sqrt(2)*log(cos(2*x)^2 + sin(2*x)^2 + sqrt(co
s(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)*(cos(1/2*arctan2(sin(4*x), cos(4*x
) + 1))^2 + sin(1/2*arctan2(sin(4*x), cos(4*x) + 1))^2) + 2*(cos(4*x)^2 +
sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*(cos(2*x)*cos(1/2*arctan2(sin(4*x), cos
(4*x) + 1)) + sin(2*x)*sin(1/2*arctan2(sin(4*x), cos(4*x) + 1)))) - 1/2*lo
g((sqrt(abs(2*e^(2*I*x) - 2)^4 + 16*cos(2*x)^4 + 16*sin(2*x)^4 + 8*(cos(2*
x)^2 - sin(2*x)^2 + 2*cos(2*x) + 1)*abs(2*e^(2*I*x) - 2)^2 + 64*cos(2*x)^3
 + 32*(cos(2*x)^2 + 2*cos(2*x) + 1)*sin(2*x)^2 + 96*cos(2*x)^2 + 64*cos(2*
x) + 16)*cos(1/2*arctan2(8*(cos(2*x) + 1)*sin(2*x)/abs(2*e^(2*I*x) - 2)^2,
 (abs(2*e^(2*I*x) - 2)^2 + 4*cos(2*x)^2 - 4*sin(2*x)^2 + 8*cos(2*x) + 4)/a
bs(2*e^(2*I*x) - 2)^2))^2 + sqrt(abs(2*e^(2*I*x) - 2)^4 + 16*cos(2*x)^4 +
16*sin(2*x)^4 + 8*(cos(2*x)^2 - sin(2*x)^2 + 2*cos(2*x) + 1)*abs(2*e^(2*I*
x) - 2)^2 + 64*cos(2*x)^3 + 32*(cos(2*x)^2 + 2*cos(2*x) + 1)*sin(2*x)^2 +
96*cos(2*x)^2 + 64*cos(2*x) + 16)*sin(1/2*arctan2(8*(cos(2*x) + 1)*sin(2*x
)/abs(2*e^(2*I*x) - 2)^2, (abs(2*e^(2*I*x) - 2)^2 + 4*cos(2*x)^2 - 4*sin(2
*x)^2 + 8*cos(2*x) + 4)/abs(2*e^(2*I*x) - 2)^2))^2 + 4*(abs(2*e^(2*I*x) -
2)^4 + 16*cos(2*x)^4 + 16*sin(2*x)^4 + 8*(cos(2*x)^2 - sin(2*x)^2 + 2*cos(
2*x) + 1)*abs(2*e^(2*I*x) - 2)^2 + 64*cos(2*x)^3 + 32*(cos(2*x)^2 + 2*cos(
2*x) + 1)*sin(2*x)^2 + 96*cos(2*x)^2 + 64*cos(2*x) + 16)^(1/4)*(cos(2*x) +
 1)*cos(1/2*arctan2(8*(cos(2*x) + 1)*sin(2*x)/abs(2*e^(2*I*x) - 2)^2, (...
```

3.42.8 Giac [F(-1)]

Timed out.

$$\int \sqrt{-1+\cot^2(x)}\,dx = \text{Timed out}$$

input
```
integrate((-1+cot(x)^2)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.42.9 Mupad [B] (verification not implemented)

Time = 13.92 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.81

$$\int \sqrt{-1+\cot^2(x)}\,dx = \sqrt{2}\,\text{atanh}\left(\frac{\sqrt{2}\cot(x)}{\sqrt{\cot(x)^2-1}}\right) - \ln\left(\cot(x)+\sqrt{\cot(x)^2-1}\right)$$

input
```
int((cot(x)^2 - 1)^(1/2),x)
```

output
```
2^(1/2)*atanh((2^(1/2)*cot(x))/(cot(x)^2 - 1)^(1/2)) - log(cot(x) + (cot(x
)^2 - 1)^(1/2))
```

3.43 $\int \frac{1}{\sqrt{-1+\cot^2(x)}}\,dx$

3.43.1 Optimal result

Integrand size = 10, antiderivative size = 26

$$\int \frac{1}{\sqrt{-1+\cot^2(x)}}\,dx=-\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\cot(x)}{\sqrt{-1+\cot^2(x)}}\right)}{\sqrt{2}}$$

output
```
-1/2*arctanh(cot(x)*2^(1/2)/(-1+cot(x)^2)^(1/2))*2^(1/2)
```

3.43.2 Mathematica [A] (warning: unable to verify)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.85

$$\int \frac{1}{\sqrt{-1+\cot^2(x)}}\,dx=-\frac{\arcsin\left(\frac{\sqrt{2}\cot(x)}{\sqrt{1+\cot^2(x)}}\right)\sqrt{1-\cot^2(x)}}{\sqrt{2}\sqrt{-1+\cot^2(x)}}$$

input
```
Integrate[1/Sqrt[-1 + Cot[x]^2],x]
```

output
```
-((ArcSin[(Sqrt[2]*Cot[x])/Sqrt[1 + Cot[x]^2]]*Sqrt[1 - Cot[x]^2])/(Sqrt[2
]*Sqrt[-1 + Cot[x]^2]))
```

3.43.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 4144, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\cot^2(x)-1}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\tan\left(x+\frac{\pi}{2}\right)^2-1}}dx$$

$$\Big\downarrow \text{4144}$$

$$-\int \frac{1}{\sqrt{\cot^2(x)-1}\left(\cot^2(x)+1\right)}d\cot(x)$$

$$\Big\downarrow \text{291}$$

$$-\int \frac{1}{1-\frac{2\cot^2(x)}{\cot^2(x)-1}}d\frac{\cot(x)}{\sqrt{\cot^2(x)-1}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\cot(x)}{\sqrt{\cot^2(x)-1}}\right)}{\sqrt{2}}$$

input
```
Int[1/Sqrt[-1 + Cot[x]^2],x]
```

output
```
-(ArcTanh[(Sqrt[2]*Cot[x])/Sqrt[-1 + Cot[x]^2]]/Sqrt[2])
```

3.43.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

3.43.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$-\frac{\operatorname{arctanh}\left(\frac{\cot(x)\sqrt{2}}{\sqrt{-1+\cot(x)^2}}\right)\sqrt{2}}{2}$	21
default	$-\frac{\operatorname{arctanh}\left(\frac{\cot(x)\sqrt{2}}{\sqrt{-1+\cot(x)^2}}\right)\sqrt{2}}{2}$	21

input
```
int(1/(-1+cot(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*arctanh(cot(x)*2^(1/2)/(-1+cot(x)^2)^(1/2))*2^(1/2)
```

3.43.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(20) = 40.

Time = 0.30 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.31

$$\int \frac{1}{\sqrt{-1+\cot^2(x)}}\,dx = \frac{1}{8}\sqrt{2}\log\left(2\sqrt{2}\left(2\sqrt{2}\cos(2x)+\sqrt{2}\right)\sqrt{-\frac{\cos(2x)}{\cos(2x)-1}}\sin(2x) - 8\cos(2x)^2 - 8\cos(2x) - 1\right)$$

input
```
integrate(1/(-1+cot(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/8*sqrt(2)*log(2*sqrt(2)*(2*sqrt(2)*cos(2*x) + sqrt(2))*sqrt(-cos(2*x)/(c
os(2*x) - 1))*sin(2*x) - 8*cos(2*x)^2 - 8*cos(2*x) - 1)
```

3.43.6 Sympy [F]

$$\int \frac{1}{\sqrt{-1+\cot^2(x)}}\,dx = \int \frac{1}{\sqrt{\cot^2(x)-1}}\,dx$$

input
```
integrate(1/(-1+cot(x)**2)**(1/2),x)
```

output
```
Integral(1/sqrt(cot(x)**2 - 1), x)
```

3.43.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(20) = 40.

Time = 0.53 (sec) , antiderivative size = 143, normalized size of antiderivative = 5.50

$$\int \frac{1}{\sqrt{-1+\cot^2(x)}}\,dx =$$

$$-\frac{1}{8}\sqrt{2}\left(2\,\mathrm{arsinh}(1) + \log\left(\cos(2x)^2 + \sin(2x)^2 + \sqrt{\cos(4x)^2 + \sin(4x)^2 + 2\cos(4x) + 1}\left(\cos\left(\frac{1}{2}\right.\right.\right.\right. \text{a}$$

input
```
integrate(1/(-1+cot(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/8*sqrt(2)*(2*arcsinh(1) + log(cos(2*x)^2 + sin(2*x)^2 + sqrt(cos(4*x)^2
 + sin(4*x)^2 + 2*cos(4*x) + 1)*(cos(1/2*arctan2(sin(4*x), cos(4*x) + 1))^
2 + sin(1/2*arctan2(sin(4*x), cos(4*x) + 1))^2) + 2*(cos(4*x)^2 + sin(4*x)
^2 + 2*cos(4*x) + 1)^(1/4)*(cos(2*x)*cos(1/2*arctan2(sin(4*x), cos(4*x) +
1)) + sin(2*x)*sin(1/2*arctan2(sin(4*x), cos(4*x) + 1)))))
```

3.43.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. 2(20) = 40.

Time = 0.30 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.73

$$\int \frac{1}{\sqrt{-1+\cot^2(x)}}\,dx = -\frac{1}{2}\sqrt{2}\log\left(\sqrt{2}-1\right)\operatorname{sgn}(\sin(x)) + \frac{\sqrt{2}\log\left(\left|-\sqrt{2}\cos(x)+\sqrt{2\cos(x)^2-1}\right|\right)}{2\operatorname{sgn}(\sin(x))}$$

input
```
integrate(1/(-1+cot(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*log(sqrt(2) - 1)*sgn(sin(x)) + 1/2*sqrt(2)*log(abs(-sqrt(2)*c
os(x) + sqrt(2*cos(x)^2 - 1)))/sgn(sin(x))
```

3.43.9 Mupad [B] (verification not implemented)

Time = 13.91 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{1}{\sqrt{-1+\cot^2(x)}}\,dx = -\frac{\sqrt{2}\operatorname{atanh}\left(\frac{\sqrt{2}\cot(x)}{\sqrt{\cot(x)^2-1}}\right)}{2}$$

input
```
int(1/(cot(x)^2 - 1)^(1/2),x)
```

output
```
-(2^(1/2)*atanh((2^(1/2)*cot(x))/(cot(x)^2 - 1)^(1/2)))/2
```

3.44 $\int \frac{\cot^3(x)}{\sqrt{a+b\cot^2(x)}}\,dx$

3.44.1 Optimal result

Integrand size = 17, antiderivative size = 52

$$\int \frac{\cot^3(x)}{\sqrt{a+b\cot^2(x)}}\,dx=-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}-\frac{\sqrt{a+b\cot^2(x)}}{b}$$

output
```
-arctanh((a+b*cot(x)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(1/2)-(a+b*cot(x)^2)^(1/2
)/b
```

3.44.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int \frac{\cot^3(x)}{\sqrt{a+b\cot^2(x)}}\,dx=-\frac{\frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}+\sqrt{a+b\cot^2(x)}}{b}$$

input
```
Integrate[Cot[x]^3/Sqrt[a + b*Cot[x]^2],x]
```

output
```
-(((b*ArcTanh[Sqrt[a + b*Cot[x]^2]/Sqrt[a - b]])/Sqrt[a - b] + Sqrt[a + b*
Cot[x]^2])/b)
```

3.44.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {3042, 25, 4153, 25, 354, 90, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(x)}{\sqrt{a+b\cot^2(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\tan\left(x+\frac{\pi}{2}\right)^3}{\sqrt{a+b\tan\left(x+\frac{\pi}{2}\right)^2}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\tan\left(x+\frac{\pi}{2}\right)^3}{\sqrt{b\tan\left(x+\frac{\pi}{2}\right)^2+a}}dx$$

$$\Big\downarrow \text{4153}$$

$$\int -\frac{\cot^3(x)}{\left(\cot^2(x)+1\right)\sqrt{a+b\cot^2(x)}}d\cot(x)$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\cot^3(x)}{\left(\cot^2(x)+1\right)\sqrt{b\cot^2(x)+a}}d\cot(x)$$

$$\Big\downarrow \text{354}$$

$$-\frac{1}{2}\int \frac{\cot^2(x)}{\left(\cot^2(x)+1\right)\sqrt{b\cot^2(x)+a}}d\cot^2(x)$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{2}\left(\int \frac{1}{\left(\cot^2(x)+1\right)\sqrt{b\cot^2(x)+a}}d\cot^2(x)-\frac{2\sqrt{a+b\cot^2(x)}}{b}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{2\int \frac{1}{\frac{\cot^4(x)}{b}-\frac{a}{b}+1}d\sqrt{b\cot^2(x)+a}}{b}-\frac{2\sqrt{a+b\cot^2(x)}}{b}\right)$$

$$\downarrow \textit{221}$$

$$\frac{1}{2}\left(-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}-\frac{2\sqrt{a+b\cot^2(x)}}{b}\right)$$

input
```
Int[Cot[x]^3/Sqrt[a + b*Cot[x]^2],x]
```

output
```
((-2*ArcTanh[Sqrt[a + b*Cot[x]^2]/Sqrt[a - b]])/Sqrt[a - b] - (2*Sqrt[a +
b*Cot[x]^2])/b)/2
```

3.44.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.44.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$-\frac{\sqrt{a+b\cot(x)^2}}{b}+\frac{\arctan\left(\frac{\sqrt{a+b\cot(x)^2}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}$	44
default	$-\frac{\sqrt{a+b\cot(x)^2}}{b}+\frac{\arctan\left(\frac{\sqrt{a+b\cot(x)^2}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}$	44

input
```
int(cot(x)^3/(a+b*cot(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(a+b*cot(x)^2)^(1/2)/b+1/(-a+b)^(1/2)*arctan((a+b*cot(x)^2)^(1/2)/(-a+b)^
(1/2))
```

3.44.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(44) = 88$.

Time = 0.35 (sec) , antiderivative size = 284, normalized size of antiderivative = 5.46

$$\int \frac{\cot^3(x)}{\sqrt{a+b\cot^2(x)}}\,dx$$
$$= \left[\frac{\sqrt{a-b}\,b\log\left(-2\left(a^2-2ab+b^2\right)\cos(2x)^2-2a^2+b^2+2\left((a-b)\cos(2x)^2-(2a-b)\cos(2x)+a\right)\sqrt{}\right.}{4\left(ab-b^2\right)}\right.$$
$$\left. -\frac{\sqrt{-a+b}\,b\arctan\left(-\frac{\sqrt{-a+b}\sqrt{\frac{(a-b)\cos(2x)-a-b}{\cos(2x)-1}}(\cos(2x)-1)}{(a-b)\cos(2x)-a}\right)+2(a-b)\sqrt{\frac{(a-b)\cos(2x)-a-b}{\cos(2x)-1}}}{2\left(ab-b^2\right)}\right]$$

input
```
integrate(cot(x)^3/(a+b*cot(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(a - b)*b*log(-2*(a^2 - 2*a*b + b^2)*cos(2*x)^2 - 2*a^2 + b^2 +
2*((a - b)*cos(2*x)^2 - (2*a - b)*cos(2*x) + a)*sqrt(a - b)*sqrt(((a - b)*
cos(2*x) - a - b)/(cos(2*x) - 1)) + 4*(a^2 - a*b)*cos(2*x)) - 4*(a - b)*sq
rt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1)))/(a*b - b^2), -1/2*(sqrt(-a
+ b)*b*arctan(-sqrt(-a + b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1)
)*(cos(2*x) - 1)/((a - b)*cos(2*x) - a)) + 2*(a - b)*sqrt(((a - b)*cos(2*x
) - a - b)/(cos(2*x) - 1)))/(a*b - b^2)]
```

3.44.6 Sympy [F]

$$\int \frac{\cot^3(x)}{\sqrt{a+b\cot^2(x)}}\,dx = \int \frac{\cot^3(x)}{\sqrt{a+b\cot^2(x)}}\,dx$$

input
```
integrate(cot(x)**3/(a+b*cot(x)**2)**(1/2),x)
```

output
```
Integral(cot(x)**3/sqrt(a + b*cot(x)**2), x)
```

3.44.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cot^3(x)}{\sqrt{a+b\cot^2(x)}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cot(x)^3/(a+b*cot(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for m
ore detail
```

3.44.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(44) = 88$.

Time = 0.36 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.85

$$\int \frac{\cot^3(x)}{\sqrt{a+b\cot^2(x)}}\,dx = \frac{\frac{\log\left(\left(\sqrt{a-b}\sin(x)-\sqrt{a\sin(x)^2-b\sin(x)^2+b}\right)^2\right)}{\sqrt{a-b}} + \frac{4\sqrt{a-b}}{\left(\sqrt{a-b}\sin(x)-\sqrt{a\sin(x)^2-b\sin(x)^2+b}\right)^2-b}}{2\,\mathrm{sgn}\left(\sin(x)\right)}$$

input
```
integrate(cot(x)^3/(a+b*cot(x)^2)^(1/2),x, algorithm="giac")
```

output
```
1/2*(log((sqrt(a - b)*sin(x) - sqrt(a*sin(x)^2 - b*sin(x)^2 + b))^2)/sqrt(
a - b) + 4*sqrt(a - b)/((sqrt(a - b)*sin(x) - sqrt(a*sin(x)^2 - b*sin(x)^2
 + b))^2 - b))/sgn(sin(x))
```

3.44.9 Mupad [B] (verification not implemented)

Time = 14.61 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

$$\int \frac{\cot^3(x)}{\sqrt{a+b\cot^2(x)}}\,dx = -\frac{\sqrt{b\cot(x)^2+a}}{b} - \frac{\operatorname{atanh}\left(\frac{\sqrt{b\cot(x)^2+a}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

input
```
int(cot(x)^3/(a + b*cot(x)^2)^(1/2),x)
```

output
```
- (a + b*cot(x)^2)^(1/2)/b - atanh((a + b*cot(x)^2)^(1/2)/(a - b)^(1/2))/(
a - b)^(1/2)
```

3.45 $\int \frac{\cot^2(x)}{\sqrt{a+b\cot^2(x)}}\,dx$

3.45.1 Optimal result

Integrand size = 17, antiderivative size = 64

$$\int \frac{\cot^2(x)}{\sqrt{a+b\cot^2(x)}}\,dx=\frac{\arctan\left(\frac{\sqrt{a-b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{\sqrt{a-b}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{\sqrt{b}}$$

output
```
arctan(cot(x)*(a-b)^(1/2)/(a+b*cot(x)^2)^(1/2))/(a-b)^(1/2)-arctanh(cot(x)
*b^(1/2)/(a+b*cot(x)^2)^(1/2))/b^(1/2)
```

3.45.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 158 vs. 2(64) = 128.

Time = 0.35 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.47

$$\int \frac{\cot^2(x)}{\sqrt{a+b\cot^2(x)}}\,dx$$
$$=\frac{\left(-\sqrt{-b}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{a-b}\cos(x)}{\sqrt{-a-b+(a-b)\cos(2x)}}\right)+\sqrt{a-b}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{-b}\cos(x)}{\sqrt{-a-b+(a-b)\cos(2x)}}\right)\right)\sqrt{(a+b+(-a+b)\cos(2x}}{\sqrt{a-b}\sqrt{-b}\sqrt{-a-b+(a-b)\cos(2x)}}$$

input
```
Integrate[Cot[x]^2/Sqrt[a + b*Cot[x]^2],x]
```

output
```
((-(Sqrt[-b]*ArcTanh[(Sqrt[2]*Sqrt[a - b]*Cos[x])/Sqrt[-a - b + (a - b)*Co
s[2*x]]]) + Sqrt[a - b]*ArcTanh[(Sqrt[2]*Sqrt[-b]*Cos[x])/Sqrt[-a - b + (a
 - b)*Cos[2*x]]])*Sqrt[(a + b + (-a + b)*Cos[2*x])*Csc[x]^2]*Sin[x])/(Sqrt
[a - b]*Sqrt[-b]*Sqrt[-a - b + (a - b)*Cos[2*x]])
```

3.45.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {3042, 4153, 385, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(x)}{\sqrt{a+b\cot^2(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan\left(x+\frac{\pi}{2}\right)^2}{\sqrt{a+b\tan\left(x+\frac{\pi}{2}\right)^2}}dx$$

$$\Big\downarrow \text{4153}$$

$$-\int \frac{\cot^2(x)}{\left(\cot^2(x)+1\right)\sqrt{b\cot^2(x)+a}}d\cot(x)$$

$$\Big\downarrow \text{385}$$

$$\int \frac{1}{\left(\cot^2(x)+1\right)\sqrt{b\cot^2(x)+a}}d\cot(x)-\int \frac{1}{\sqrt{b\cot^2(x)+a}}d\cot(x)$$

$$\Big\downarrow \text{224}$$

$$\int \frac{1}{\left(\cot^2(x)+1\right)\sqrt{b\cot^2(x)+a}}d\cot(x)-\int \frac{1}{1-\frac{b\cot^2(x)}{b\cot^2(x)+a}}d\frac{\cot(x)}{\sqrt{b\cot^2(x)+a}}$$

$$\Big\downarrow \text{219}$$

$$\int \frac{1}{\left(\cot^2(x)+1\right)\sqrt{b\cot^2(x)+a}}d\cot(x)-\frac{\text{arctanh}\left(\frac{\sqrt{b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{\sqrt{b}}$$

$$\Big\downarrow \text{291}$$

$$\int \frac{1}{1-\frac{(b-a)\cot^2(x)}{b\cot^2(x)+a}}d\frac{\cot(x)}{\sqrt{b\cot^2(x)+a}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{\sqrt{b}}$$

$$\Big\downarrow \text{216}$$

$$\frac{\arctan\left(\frac{\sqrt{a-b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{\sqrt{a-b}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{\sqrt{b}}$$

input
```
Int[Cot[x]^2/Sqrt[a + b*Cot[x]^2],x]
```

output
```
ArcTan[(Sqrt[a - b]*Cot[x])/Sqrt[a + b*Cot[x]^2]]/Sqrt[a - b] - ArcTanh[(S
qrt[b]*Cot[x])/Sqrt[a + b*Cot[x]^2]]/Sqrt[b]
```

3.45.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 385
```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2),
x_Symbol] :> Simp[e^2/b   Int[(e*x)^(m - 2)*(c + d*x^2)^q, x], x] - Simp[a*
(e^2/b)   Int[(e*x)^(m - 2)*((c + d*x^2)^q/(a + b*x^2)), x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LeQ[2, m, 3] && IntBinomial
Q[a, b, c, d, e, m, 2, -1, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.45.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.25

method	result	size
derivativedivides	$-\frac{\ln\left(\sqrt{b}\,\cot(x)+\sqrt{a+b\cot(x)^2}\right)}{\sqrt{b}}+\frac{\sqrt{b^4(a-b)}\,\arctan\left(\frac{b^2(a-b)\cot(x)}{\sqrt{b^4(a-b)}\,\sqrt{a+b\cot(x)^2}}\right)}{b^2(a-b)}$	80
default	$-\frac{\ln\left(\sqrt{b}\,\cot(x)+\sqrt{a+b\cot(x)^2}\right)}{\sqrt{b}}+\frac{\sqrt{b^4(a-b)}\,\arctan\left(\frac{b^2(a-b)\cot(x)}{\sqrt{b^4(a-b)}\,\sqrt{a+b\cot(x)^2}}\right)}{b^2(a-b)}$	80

input
```
int(cot(x)^2/(a+b*cot(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-ln(b^(1/2)*cot(x)+(a+b*cot(x)^2)^(1/2))/b^(1/2)+(b^4*(a-b))^(1/2)/b^2/(a-
b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*cot(x)^2)^(1/2)*cot(x))
```

3.45.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(52) = 104.

Time = 0.32 (sec) , antiderivative size = 588, normalized size of antiderivative = 9.19

$$\int \frac{\cot^2(x)}{\sqrt{a+b\cot^2(x)}}\,dx$$
$$= \left[-\frac{\sqrt{-a+b}b\log\left(-(a-b)\cos(2x)+\sqrt{-a+b}\sqrt{\frac{(a-b)\cos(2x)-a-b}{\cos(2x)-1}}\sin(2x)+b\right)-(a-b)\sqrt{b}\log\left(\frac{(a-2b)\,c}{}\right.}{2\,(ab-b^2)}\right.$$

input
```
integrate(cot(x)^2/(a+b*cot(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*(sqrt(-a + b)*b*log(-(a - b)*cos(2*x) + sqrt(-a + b)*sqrt(((a - b)*c
os(2*x) - a - b)/(cos(2*x) - 1))*sin(2*x) + b) - (a - b)*sqrt(b)*log(((a -
 2*b)*cos(2*x) + 2*sqrt(b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))
*sin(2*x) - a - 2*b)/(cos(2*x) - 1)))/(a*b - b^2), 1/2*(2*(a - b)*sqrt(-b)
*arctan(sqrt(-b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))*sin(2*x)/
(b*cos(2*x) + b)) - sqrt(-a + b)*b*log(-(a - b)*cos(2*x) + sqrt(-a + b)*sq
rt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))*sin(2*x) + b))/(a*b - b^2),
1/2*(2*sqrt(a - b)*b*arctan(-sqrt(a - b)*sqrt(((a - b)*cos(2*x) - a - b)/(
cos(2*x) - 1))*sin(2*x)/((a - b)*cos(2*x) + a - b)) + (a - b)*sqrt(b)*log(
((a - 2*b)*cos(2*x) + 2*sqrt(b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x)
- 1))*sin(2*x) - a - 2*b)/(cos(2*x) - 1)))/(a*b - b^2), (sqrt(a - b)*b*arc
tan(-sqrt(a - b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))*sin(2*x)/
((a - b)*cos(2*x) + a - b)) + (a - b)*sqrt(-b)*arctan(sqrt(-b)*sqrt(((a -
b)*cos(2*x) - a - b)/(cos(2*x) - 1))*sin(2*x)/(b*cos(2*x) + b)))/(a*b - b^
2)]
```

3.45.6 Sympy [F]

$$\int \frac{\cot^2(x)}{\sqrt{a+b\cot^2(x)}}\,dx = \int \frac{\cot^2{(x)}}{\sqrt{a+b\cot^2{(x)}}}\,dx$$

input
```
integrate(cot(x)**2/(a+b*cot(x)**2)**(1/2),x)
```

output
```
Integral(cot(x)**2/sqrt(a + b*cot(x)**2), x)
```

3.45.7 Maxima [F]

$$\int \frac{\cot^2(x)}{\sqrt{a+b\cot^2(x)}}\,dx = \int \frac{\cot{(x)}^2}{\sqrt{b\cot{(x)}^2+a}}\,dx$$

input
```
integrate(cot(x)^2/(a+b*cot(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(cot(x)^2/sqrt(b*cot(x)^2 + a), x)
```

3.45.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(52) = 104$.

Time = 0.54 (sec) , antiderivative size = 229, normalized size of antiderivative = 3.58

$$\int \frac{\cot^2(x)}{\sqrt{a+b\cot^2(x)}}\,dx$$
$$= \frac{\left(2\,a\arctan\left(\frac{\sqrt{-a+b}\sqrt{b}}{\sqrt{ab-b^2}}\right) - 2\,b\arctan\left(\frac{\sqrt{-a+b}\sqrt{b}}{\sqrt{ab-b^2}}\right) + \sqrt{ab-b^2}\log\left(-a-2\sqrt{-a+b}\sqrt{b}+2\,b\right)\right)\mathrm{sgn}(\sin(x))}{2\sqrt{ab-b^2}\sqrt{-a+b}}$$
$$-\frac{\frac{2\sqrt{-a+b}\arctan\left(\frac{\left(\sqrt{-a+b}\cos(x)-\sqrt{-a\cos(x)^2+b\cos(x)^2+a}\right)^2+a-2\,b}{2\sqrt{ab-b^2}}\right)}{\sqrt{ab-b^2}} + \frac{\log\left(\left(\sqrt{-a+b}\cos(x)-\sqrt{-a\cos(x)^2+b\cos(x)^2+a}\right)^2\right)}{\sqrt{-a+b}}}{2\,\mathrm{sgn}(\sin(x))}$$

input
```
integrate(cot(x)^2/(a+b*cot(x)^2)^(1/2),x, algorithm="giac")
```

output
```
1/2*(2*a*arctan(sqrt(-a + b)*sqrt(b)/sqrt(a*b - b^2)) - 2*b*arctan(sqrt(-a
 + b)*sqrt(b)/sqrt(a*b - b^2)) + sqrt(a*b - b^2)*log(-a - 2*sqrt(-a + b)*s
qrt(b) + 2*b))*sgn(sin(x))/(sqrt(a*b - b^2)*sqrt(-a + b)) - 1/2*(2*sqrt(-a
 + b)*arctan(1/2*((sqrt(-a + b)*cos(x) - sqrt(-a*cos(x)^2 + b*cos(x)^2 + a
))^2 + a - 2*b)/sqrt(a*b - b^2))/sqrt(a*b - b^2) + log((sqrt(-a + b)*cos(x
) - sqrt(-a*cos(x)^2 + b*cos(x)^2 + a))^2)/sqrt(-a + b))/sgn(sin(x))
```

3.45.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^2(x)}{\sqrt{a+b\cot^2(x)}}\,dx = \int \frac{\cot(x)^2}{\sqrt{b\cot(x)^2+a}}\,dx$$

input
```
int(cot(x)^2/(a + b*cot(x)^2)^(1/2),x)
```

output
```
int(cot(x)^2/(a + b*cot(x)^2)^(1/2), x)
```

3.46 $\int \frac{\cot(x)}{\sqrt{a+b\cot^2(x)}}\,dx$

3.46.1 Optimal result

Integrand size = 15, antiderivative size = 33

$$\int \frac{\cot(x)}{\sqrt{a+b\cot^2(x)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

output
```
arctanh((a+b*cot(x)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(1/2)
```

3.46.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{\cot(x)}{\sqrt{a+b\cot^2(x)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

input
```
Integrate[Cot[x]/Sqrt[a + b*Cot[x]^2],x]
```

output
```
ArcTanh[Sqrt[a + b*Cot[x]^2]/Sqrt[a - b]]/Sqrt[a - b]
```

3.46.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {3042, 25, 4153, 25, 353, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(x)}{\sqrt{a+b\cot^2(x)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{\tan\left(x+\frac{\pi}{2}\right)}{\sqrt{a+b\tan\left(x+\frac{\pi}{2}\right)^2}}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{\tan\left(x+\frac{\pi}{2}\right)}{\sqrt{b\tan\left(x+\frac{\pi}{2}\right)^2+a}}dx$$

$$\Big\downarrow \text{ 4153}$$

$$\int -\frac{\cot(x)}{\left(\cot^2(x)+1\right)\sqrt{a+b\cot^2(x)}}d\cot(x)$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{\cot(x)}{\left(\cot^2(x)+1\right)\sqrt{b\cot^2(x)+a}}d\cot(x)$$

$$\Big\downarrow \text{ 353}$$

$$-\frac{1}{2}\int \frac{1}{\left(\cot^2(x)+1\right)\sqrt{b\cot^2(x)+a}}d\cot^2(x)$$

$$\Big\downarrow \text{ 73}$$

$$-\frac{\int \frac{1}{\frac{\cot^4(x)}{b}-\frac{a}{b}+1}d\sqrt{b\cot^2(x)+a}}{b}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

input
```
Int[Cot[x]/Sqrt[a + b*Cot[x]^2],x]
```

output
```
ArcTanh[Sqrt[a + b*Cot[x]^2]/Sqrt[a - b]]/Sqrt[a - b]
```

3.46.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.46.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$-\frac{\arctan\left(\frac{\sqrt{a+b\cot(x)^{2}}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}$	29
default	$-\frac{\arctan\left(\frac{\sqrt{a+b\cot(x)^{2}}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}$	29

input
```
int(cot(x)/(a+b*cot(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/(-a+b)^(1/2)*arctan((a+b*cot(x)^2)^(1/2)/(-a+b)^(1/2))
```

3.46.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(27) = 54.

Time = 0.29 (sec) , antiderivative size = 127, normalized size of antiderivative = 3.85

$$\int \frac{\cot(x)}{\sqrt{a+b\cot^2(x)}}\,dx$$
$$= \left[\frac{\log\left(-\sqrt{a-b}\sqrt{\frac{(a-b)\cos(2x)-a-b}{\cos(2x)-1}}(\cos(2x)-1)-(a-b)\cos(2x)+a\right)}{2\sqrt{a-b}}, \frac{\sqrt{-a+b}\arctan\left(-\frac{\sqrt{-a+b}\sqrt{\frac{(a-b)\ldots}{\mathrm{co}\ldots}}}{a-b\ldots}\right.}{a-b}\right.$$

input
```
integrate(cot(x)/(a+b*cot(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*log(-sqrt(a - b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))*(cos
(2*x) - 1) - (a - b)*cos(2*x) + a)/sqrt(a - b), sqrt(-a + b)*arctan(-sqrt(
-a + b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))/(a - b))/(a - b)]
```

3.46.6 Sympy [F]

$$\int \frac{\cot(x)}{\sqrt{a+b\cot^2(x)}}\,dx = \int \frac{\cot(x)}{\sqrt{a+b\cot^2(x)}}\,dx$$

input
```
integrate(cot(x)/(a+b*cot(x)**2)**(1/2),x)
```

output
```
Integral(cot(x)/sqrt(a + b*cot(x)**2), x)
```

3.46.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cot(x)}{\sqrt{a+b\cot^2(x)}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cot(x)/(a+b*cot(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for m
ore detail
```

3.46.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(27) = 54$.

Time = 0.33 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.85

$$\int \frac{\cot(x)}{\sqrt{a+b\cot^2(x)}}\,dx = \frac{\log(|b|)\operatorname{sgn}(\sin(x))}{2\sqrt{a-b}} - \frac{\log\left(\left|-\sqrt{a-b}\sin(x)+\sqrt{a\sin(x)^2-b\sin(x)^2+b}\right|\right)}{\sqrt{a-b}\operatorname{sgn}(\sin(x))}$$

input
```
integrate(cot(x)/(a+b*cot(x)^2)^(1/2),x, algorithm="giac")
```

output
```
1/2*log(abs(b))*sgn(sin(x))/sqrt(a - b) - log(abs(-sqrt(a - b)*sin(x) + sq
rt(a*sin(x)^2 - b*sin(x)^2 + b)))/(sqrt(a - b)*sgn(sin(x)))
```

3.46.9 Mupad [B] (verification not implemented)

Time = 13.51 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \frac{\cot(x)}{\sqrt{a+b\cot^2(x)}}\,dx = \frac{\operatorname{atanh}\left(\frac{\sqrt{b\cot(x)^2+a}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

input
```
int(cot(x)/(a + b*cot(x)^2)^(1/2),x)
```

output
```
atanh((a + b*cot(x)^2)^(1/2)/(a - b)^(1/2))/(a - b)^(1/2)
```

3.47 $\int \frac{\tan(x)}{\sqrt{a+b\cot^2(x)}}\,dx$

3.47.1 Optimal result

Integrand size = 15, antiderivative size = 60

$$\int \frac{\tan(x)}{\sqrt{a+b\cot^2(x)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

output
```
arctanh((a+b*cot(x)^2)^(1/2)/a^(1/2))/a^(1/2)-arctanh((a+b*cot(x)^2)^(1/2)
/(a-b)^(1/2))/(a-b)^(1/2)
```

3.47.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int \frac{\tan(x)}{\sqrt{a+b\cot^2(x)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

input
```
Integrate[Tan[x]/Sqrt[a + b*Cot[x]^2],x]
```

output
```
ArcTanh[Sqrt[a + b*Cot[x]^2]/Sqrt[a]]/Sqrt[a] - ArcTanh[Sqrt[a + b*Cot[x]^
2]/Sqrt[a - b]]/Sqrt[a - b]
```

3.47.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {3042, 25, 4153, 25, 354, 97, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(x)}{\sqrt{a+b\cot^2(x)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{1}{\tan\left(x+\frac{\pi}{2}\right)\sqrt{a+b\tan\left(x+\frac{\pi}{2}\right)^2}}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{1}{\tan\left(x+\frac{\pi}{2}\right)\sqrt{b\tan\left(x+\frac{\pi}{2}\right)^2+a}}dx$$

$$\Big\downarrow \text{ 4153}$$

$$\int -\frac{\tan(x)}{\left(\cot^2(x)+1\right)\sqrt{a+b\cot^2(x)}}d\cot(x)$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{\tan(x)}{\left(\cot^2(x)+1\right)\sqrt{b\cot^2(x)+a}}d\cot(x)$$

$$\Big\downarrow \text{ 354}$$

$$-\frac{1}{2}\int \frac{\tan(x)}{\left(\cot^2(x)+1\right)\sqrt{b\cot^2(x)+a}}d\cot^2(x)$$

$$\Big\downarrow \text{ 97}$$

$$\frac{1}{2}\left(\int \frac{1}{\left(\cot^2(x)+1\right)\sqrt{b\cot^2(x)+a}}d\cot^2(x)-\int \frac{\tan(x)}{\sqrt{b\cot^2(x)+a}}d\cot^2(x)\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{2}\left(\frac{2\int \frac{1}{\frac{\cot^4(x)}{b}-\frac{a}{b}+1}d\sqrt{b\cot^2(x)+a}}{b}-\frac{2\int \frac{1}{\frac{\cot^4(x)}{b}-\frac{a}{b}}d\sqrt{b\cot^2(x)+a}}{b}\right)$$

$$\Big\downarrow \text{ 221}$$

$$\frac{1}{2}\left(\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}\right)$$

input
```
Int[Tan[x]/Sqrt[a + b*Cot[x]^2],x]
```

output
```
((2*ArcTanh[Sqrt[a + b*Cot[x]^2]/Sqrt[a]])/Sqrt[a] - (2*ArcTanh[Sqrt[a + b
*Cot[x]^2]/Sqrt[a - b]])/Sqrt[a - b])/2
```

3.47.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 97
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[b/(b*c - a*d)   Int[(e + f*x)^p/(a + b*x), x], x] - Simp[d/(b*c
 - a*d)   Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p},
 x] &&  !IntegerQ[p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.47.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(48) = 96$.

Time = 0.80 (sec) , antiderivative size = 149, normalized size of antiderivative = 2.48

method	result
default	$\frac{\sqrt{4}\left(\arctan\left(\frac{\sqrt{-\frac{a\cos(x)^2-\cos(x)^2b-a}{(\cos(x)+1)^2}}\,(\cot(x)+\csc(x))}{\sqrt{-a+b}}\right)\sqrt{a}+\operatorname{arctanh}\left(\frac{\sqrt{-\frac{a\cos(x)^2-\cos(x)^2b-a}{(\cos(x)+1)^2}}\,(\cot(x)+\csc(x))}{\sqrt{a}}\right)\sqrt{-a+b}\right)\sqrt{-\frac{a\cos(x)^2-\cos(x)^2b-a}{(\cos(x)+1)^2}}\,(\cot(x)+\csc(x))}{2\sqrt{-a+b}\,\sqrt{a}\,\sqrt{a+b\cot(x)^2}}$

input
```
int(tan(x)/(a+b*cot(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*4^(1/2)/(-a+b)^(1/2)/a^(1/2)*(arctan(1/(-a+b)^(1/2)*(-(a*cos(x)^2-cos(
x)^2*b-a)/(cos(x)+1)^2)^(1/2)*(cot(x)+csc(x)))*a^(1/2)+arctanh(1/a^(1/2)*(
-(a*cos(x)^2-cos(x)^2*b-a)/(cos(x)+1)^2)^(1/2)*(cot(x)+csc(x)))*(-a+b)^(1/
2))*(-(a*cos(x)^2-cos(x)^2*b-a)/(cos(x)+1)^2)^(1/2)/(a+b*cot(x)^2)^(1/2)*(
cot(x)+csc(x))
```

3.47.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 419, normalized size of antiderivative = 6.98

$$\int \frac{\tan(x)}{\sqrt{a+b\cot^2(x)}}\,dx$$

$$= \left[\frac{(a-b)\sqrt{a}\log\left(2\,a\tan\left(x\right)^{2} + 2\,\sqrt{a}\sqrt{\frac{a\tan(x)^2+b}{\tan(x)^2}}\tan\left(x\right)^{2} + b\right) + \sqrt{a-b}a\log\left(\frac{(2\,a-b)\tan(x)^2-2\,\sqrt{a-b}\sqrt{\frac{a\tan(x)^2}{\tan(x)}}}{\tan(x)^2+1}\right.}{2\,(a^2-ab)}\right.$$

$$-\frac{2\,a\sqrt{-a+b}\arctan\left(-\frac{\sqrt{-a+b}\sqrt{\frac{a\tan(x)^2+b}{\tan(x)^2}}}{a-b}\right) - (a-b)\sqrt{a}\log\left(2\,a\tan\left(x\right)^{2} + 2\,\sqrt{a}\sqrt{\frac{a\tan(x)^2+b}{\tan(x)^2}}\tan\left(x\right)^{2} + \right.}{2\,(a^2-ab)}$$

$$-\frac{2\,\sqrt{-a}(a-b)\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{a\tan(x)^2+b}{\tan(x)^2}}}{a}\right) - \sqrt{a-b}a\log\left(\frac{(2\,a-b)\tan(x)^2-2\,\sqrt{a-b}\sqrt{\frac{a\tan(x)^2+b}{\tan(x)^2}}\tan(x)^2+b}{\tan(x)^2+1}\right)}{2\,(a^2-ab)},$$

$$\left. -\frac{\sqrt{-a}(a-b)\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{a\tan(x)^2+b}{\tan(x)^2}}}{a}\right) + a\sqrt{-a+b}\arctan\left(-\frac{\sqrt{-a+b}\sqrt{\frac{a\tan(x)^2+b}{\tan(x)^2}}}{a-b}\right)}{a^2-ab}\right]$$

input
```
integrate(tan(x)/(a+b*cot(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*((a - b)*sqrt(a)*log(2*a*tan(x)^2 + 2*sqrt(a)*sqrt((a*tan(x)^2 + b)/t
an(x)^2)*tan(x)^2 + b) + sqrt(a - b)*a*log(((2*a - b)*tan(x)^2 - 2*sqrt(a
- b)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x)^2 + b)/(tan(x)^2 + 1)))/(a^2 -
 a*b), -1/2*(2*a*sqrt(-a + b)*arctan(-sqrt(-a + b)*sqrt((a*tan(x)^2 + b)/t
an(x)^2)/(a - b)) - (a - b)*sqrt(a)*log(2*a*tan(x)^2 + 2*sqrt(a)*sqrt((a*t
an(x)^2 + b)/tan(x)^2)*tan(x)^2 + b))/(a^2 - a*b), -1/2*(2*sqrt(-a)*(a - b
)*arctan(sqrt(-a)*sqrt((a*tan(x)^2 + b)/tan(x)^2)/a) - sqrt(a - b)*a*log((
(2*a - b)*tan(x)^2 - 2*sqrt(a - b)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x)^
2 + b)/(tan(x)^2 + 1)))/(a^2 - a*b), -(sqrt(-a)*(a - b)*arctan(sqrt(-a)*sq
rt((a*tan(x)^2 + b)/tan(x)^2)/a) + a*sqrt(-a + b)*arctan(-sqrt(-a + b)*sqr
t((a*tan(x)^2 + b)/tan(x)^2)/(a - b)))/(a^2 - a*b)]
```

3.47.6 Sympy [F]

$$\int \frac{\tan(x)}{\sqrt{a+b\cot^2(x)}}\,dx = \int \frac{\tan{(x)}}{\sqrt{a+b\cot^2{(x)}}}\,dx$$

input
```
integrate(tan(x)/(a+b*cot(x)**2)**(1/2),x)
```

output
```
Integral(tan(x)/sqrt(a + b*cot(x)**2), x)
```

3.47.7 Maxima [F]

$$\int \frac{\tan(x)}{\sqrt{a+b\cot^2(x)}}\,dx = \int \frac{\tan{(x)}}{\sqrt{b\cot{(x)}^2+a}}\,dx$$

input
```
integrate(tan(x)/(a+b*cot(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(tan(x)/sqrt(b*cot(x)^2 + a), x)
```

3.47.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. $2(48) = 96$.

Time = 0.37 (sec) , antiderivative size = 203, normalized size of antiderivative = 3.38

$$\int \frac{\tan(x)}{\sqrt{a+b\cot^2(x)}}\,dx = -\frac{\left(2\,a\arctan\left(-\frac{a-b}{\sqrt{-a^2+ab}}\right)-2\,b\arctan\left(-\frac{a-b}{\sqrt{-a^2+ab}}\right)+\sqrt{-a^2+ab}\log{(b)}\right)\operatorname{sgn}(\sin{(x)})}{2\sqrt{-a^2+ab}\sqrt{a-b}} + \frac{\frac{2\sqrt{a-b}\arctan\left(\frac{\left(\sqrt{a-b}\sin(x)-\sqrt{a\sin(x)^2-b\sin(x)^2+b}\right)^2-2\,a+b}{2\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}}+\frac{\log\left(\left(\sqrt{a-b}\sin(x)-\sqrt{a\sin(x)^2-b\sin(x)^2+b}\right)^2\right)}{\sqrt{a-b}}}{2\operatorname{sgn}{(\sin{(x)})}}$$

input
```
integrate(tan(x)/(a+b*cot(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*(2*a*arctan(-(a - b)/sqrt(-a^2 + a*b)) - 2*b*arctan(-(a - b)/sqrt(-a^
2 + a*b)) + sqrt(-a^2 + a*b)*log(b))*sgn(sin(x))/(sqrt(-a^2 + a*b)*sqrt(a
- b)) + 1/2*(2*sqrt(a - b)*arctan(1/2*((sqrt(a - b)*sin(x) - sqrt(a*sin(x)
^2 - b*sin(x)^2 + b))^2 - 2*a + b)/sqrt(-a^2 + a*b))/sqrt(-a^2 + a*b) + lo
g((sqrt(a - b)*sin(x) - sqrt(a*sin(x)^2 - b*sin(x)^2 + b))^2)/sqrt(a - b))
/sgn(sin(x))
```

3.47.9 Mupad [B] (verification not implemented)

Time = 13.10 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.55

$$\int \frac{\tan(x)}{\sqrt{a+b\cot^2(x)}}\,dx = \frac{\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{\tan(x)^2}}}{\sqrt{a-b}}+\frac{2\sqrt{a-b}\,\sqrt{a+\frac{b}{\tan(x)^2}}}{b}-\frac{2\,a\,\sqrt{a+\frac{b}{\tan(x)^2}}}{b\,\sqrt{a-b}}\right)}{\sqrt{a-b}} + \frac{\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{\tan(x)^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
int(tan(x)/(a + b*cot(x)^2)^(1/2),x)
```

output
```
atanh((a + b/tan(x)^2)^(1/2)/(a - b)^(1/2) + (2*(a - b)^(1/2)*(a + b/tan(x
)^2)^(1/2))/b - (2*a*(a + b/tan(x)^2)^(1/2))/(b*(a - b)^(1/2)))/(a - b)^(1
/2) + atanh((a + b/tan(x)^2)^(1/2)/a^(1/2))/a^(1/2)
```

3.48 $\int \frac{\tan^2(x)}{\sqrt{a+b\cot^2(x)}}\,dx$

3.48.1 Optimal result

Integrand size = 17, antiderivative size = 54

$$\int \frac{\tan^2(x)}{\sqrt{a+b\cot^2(x)}}\,dx = \frac{\arctan\left(\frac{\sqrt{a-b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{\sqrt{a-b}} + \frac{\sqrt{a+b\cot^2(x)}\tan(x)}{a}$$

output

```
arctan(cot(x)*(a-b)^(1/2)/(a+b*cot(x)^2)^(1/2))/(a-b)^(1/2)+(a+b*cot(x)^2)
^(1/2)*tan(x)/a
```

3.48.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.85 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.48

$$\int \frac{\tan^2(x)}{\sqrt{a+b\cot^2(x)}}\,dx = \frac{\left(1+\frac{b\cot^2(x)}{a}\right)\sin^2(x)\left(\frac{4(a-b)\cos^2(x)\left(a+b\cot^2(x)\right)\text{Hypergeometric2F1}\left(2,2,\frac{5}{2},\frac{(a-b)\cos^2(x)}{a}\right)}{3a^2} + \frac{\arcsin\left(\sqrt{\frac{(a-b)\cos^2(x)}{a}}\right)\left(a+2b\cot\right.}{a\sqrt{\frac{(a-b)\cos^2(x)\left(a+b\cot^2(x)\right)\sin^2}{a^2}}}\right.}{\sqrt{a+b\cot^2(x)}}$$

input

```
Integrate[Tan[x]^2/Sqrt[a + b*Cot[x]^2],x]
```

output
```
((1 + (b*Cot[x]^2)/a)*Sin[x]^2*((4*(a - b)*Cos[x]^2*(a + b*Cot[x]^2)*Hyper
geometric2F1[2, 2, 5/2, ((a - b)*Cos[x]^2)/a])/(3*a^2) + (ArcSin[Sqrt[((a
- b)*Cos[x]^2)/a]]*(a + 2*b*Cot[x]^2))/(a*Sqrt[((a - b)*Cos[x]^2*(a + b*Co
t[x]^2)*Sin[x]^2)/a^2]))*Tan[x])/Sqrt[a + b*Cot[x]^2]
```

3.48.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {3042, 4153, 382, 25, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(x)}{\sqrt{a+b\cot^2(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan\left(x+\frac{\pi}{2}\right)^2\sqrt{a+b\tan\left(x+\frac{\pi}{2}\right)^2}}dx$$

$$\Big\downarrow \text{4153}$$

$$-\int \frac{\tan^2(x)}{\left(\cot^2(x)+1\right)\sqrt{b\cot^2(x)+a}}d\cot(x)$$

$$\Big\downarrow \text{382}$$

$$\frac{\tan(x)\sqrt{a+b\cot^2(x)}}{a}-\frac{\int -\frac{a}{\left(\cot^2(x)+1\right)\sqrt{b\cot^2(x)+a}}d\cot(x)}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{a}{\left(\cot^2(x)+1\right)\sqrt{b\cot^2(x)+a}}d\cot(x)}{a}+\frac{\tan(x)\sqrt{a+b\cot^2(x)}}{a}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{1}{\left(\cot^2(x)+1\right)\sqrt{b\cot^2(x)+a}}d\cot(x)+\frac{\tan(x)\sqrt{a+b\cot^2(x)}}{a}$$

$$\Big\downarrow \text{291}$$

$$\int \frac{1}{1-\frac{(b-a)\cot^2(x)}{b\cot^2(x)+a}}d\frac{\cot(x)}{\sqrt{b\cot^2(x)+a}}+\frac{\tan(x)\sqrt{a+b\cot^2(x)}}{a}$$

$$\downarrow 216$$

$$\frac{\arctan\left(\frac{\sqrt{a-b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{\sqrt{a-b}}+\frac{\tan(x)\sqrt{a+b\cot^2(x)}}{a}$$

input
```
Int[Tan[x]^2/Sqrt[a + b*Cot[x]^2],x]
```

output
```
ArcTan[(Sqrt[a - b]*Cot[x])/Sqrt[a + b*Cot[x]^2]]/Sqrt[a - b] + (Sqrt[a +
b*Cot[x]^2]*Tan[x])/a
```

3.48.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.48.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(46) = 92$.

Time = 0.91 (sec) , antiderivative size = 283, normalized size of antiderivative = 5.24

method	result
default	$\frac{\sqrt{4}\left(\sqrt{-\frac{a\cos(x)^2-\cos(x)^2b-a}{(\cos(x)+1)^2}}\ \ln\left(4\cos(x)\sqrt{-a+b}\ \sqrt{-\frac{a\cos(x)^2-\cos(x)^2b-a}{(\cos(x)+1)^2}}-4\cos(x)a+4b\cos(x)+4\sqrt{-a+b}\ \sqrt{-\frac{a\cos(x)^2-\cos(x)^2b-}{(\cos(x)+1)^2}}\right.\right.}{}$

input
```
int(tan(x)^2/(a+b*cot(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*4^(1/2)/a/(-a+b)^(1/2)/(a+b*cot(x)^2)^(1/2)*((-(a*cos(x)^2-cos(x)^2*b-
a)/(cos(x)+1)^2)^(1/2)*ln(4*cos(x)*(-a+b)^(1/2)*(-(a*cos(x)^2-cos(x)^2*b-a
)/(cos(x)+1)^2)^(1/2)-4*cos(x)*a+4*b*cos(x)+4*(-a+b)^(1/2)*(-(a*cos(x)^2-c
os(x)^2*b-a)/(cos(x)+1)^2)^(1/2))*a*cot(x)+(-a+b)^(1/2)*a*tan(x)+(-a+b)^(1
/2)*b*cot(x)+(-(a*cos(x)^2-cos(x)^2*b-a)/(cos(x)+1)^2)^(1/2)*ln(4*cos(x)*(
-a+b)^(1/2)*(-(a*cos(x)^2-cos(x)^2*b-a)/(cos(x)+1)^2)^(1/2)-4*cos(x)*a+4*b
*cos(x)+4*(-a+b)^(1/2)*(-(a*cos(x)^2-cos(x)^2*b-a)/(cos(x)+1)^2)^(1/2))*a*
csc(x))
```

3.48.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 229, normalized size of antiderivative = 4.24

$$\int \frac{\tan^2(x)}{\sqrt{a+b\cot^2(x)}}\,dx$$

$$=\left[-\frac{a\sqrt{-a+b}\log\left(-\frac{a^2\tan(x)^4-2\left(3a^2-4ab\right)\tan(x)^2+a^2-8ab+8b^2+4\left(a\tan(x)^3-(a-2b)\tan(x)\right)\sqrt{-a+b}\sqrt{\frac{a\tan(x)^2+b}{\tan(x)^2}}}{\tan(x)^4+2\tan(x)^2+1}\right)-4\,(a}{4\left(a^2-ab\right)}\right.$$

input
```
integrate(tan(x)^2/(a+b*cot(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/4*(a*sqrt(-a + b)*log(-(a^2*tan(x)^4 - 2*(3*a^2 - 4*a*b)*tan(x)^2 + a^
2 - 8*a*b + 8*b^2 + 4*(a*tan(x)^3 - (a - 2*b)*tan(x))*sqrt(-a + b)*sqrt((a
*tan(x)^2 + b)/tan(x)^2))/(tan(x)^4 + 2*tan(x)^2 + 1)) - 4*(a - b)*sqrt((a
*tan(x)^2 + b)/tan(x)^2)*tan(x))/(a^2 - a*b), 1/2*(sqrt(a - b)*a*arctan(2*
sqrt(a - b)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x)/(a*tan(x)^2 - a + 2*b))
 + 2*(a - b)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x))/(a^2 - a*b)]
```

3.48.6 Sympy [F]

$$\int \frac{\tan^2(x)}{\sqrt{a+b\cot^2(x)}}\,dx = \int \frac{\tan^2{(x)}}{\sqrt{a+b\cot^2{(x)}}}\,dx$$

input
```
integrate(tan(x)**2/(a+b*cot(x)**2)**(1/2),x)
```

output
```
Integral(tan(x)**2/sqrt(a + b*cot(x)**2), x)
```

3.48.7 Maxima [F]

$$\int \frac{\tan^2(x)}{\sqrt{a+b\cot^2(x)}}\,dx = \int \frac{\tan{(x)}^2}{\sqrt{b\cot{(x)}^2+a}}\,dx$$

input
```
integrate(tan(x)^2/(a+b*cot(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(tan(x)^2/sqrt(b*cot(x)^2 + a), x)
```

3.48.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(46) = 92$.

Time = 0.34 (sec) , antiderivative size = 216, normalized size of antiderivative = 4.00

$$\int \frac{\tan^2(x)}{\sqrt{a+b\cot^2(x)}}\,dx$$
$$= \frac{\left(a\log\left(-a-2\sqrt{-a+b}\sqrt{b}+2\,b\right)+\sqrt{-a+b}\sqrt{b}\log\left(-a-2\sqrt{-a+b}\sqrt{b}+2\,b\right)-b\log\left(-a-2\sqrt{-a+\right.\right.}{2\left(a\sqrt{-a+b}-a\sqrt{b}-\sqrt{-a+b}b+b^{\frac{3}{2}}\right)}$$
$$-\frac{\frac{\log\left(\left(\sqrt{-a+b}\cos(x)-\sqrt{-a\cos(x)^2+b\cos(x)^2+a}\right)^2\right)}{\sqrt{-a+b}}+\frac{4\sqrt{-a+b}}{\left(\sqrt{-a+b}\cos(x)-\sqrt{-a\cos(x)^2+b\cos(x)^2+a}\right)^2-a}}{2\,\mathrm{sgn}\left(\sin\left(x\right)\right)}$$

input
```
integrate(tan(x)^2/(a+b*cot(x)^2)^(1/2),x, algorithm="giac")
```

output
```
1/2*(a*log(-a - 2*sqrt(-a + b)*sqrt(b) + 2*b) + sqrt(-a + b)*sqrt(b)*log(-
a - 2*sqrt(-a + b)*sqrt(b) + 2*b) - b*log(-a - 2*sqrt(-a + b)*sqrt(b) + 2*
b) + 2*a - 2*b)*sgn(sin(x))/(a*sqrt(-a + b) - a*sqrt(b) - sqrt(-a + b)*b +
 b^(3/2)) - 1/2*(log((sqrt(-a + b)*cos(x) - sqrt(-a*cos(x)^2 + b*cos(x)^2
+ a))^2)/sqrt(-a + b) + 4*sqrt(-a + b)/((sqrt(-a + b)*cos(x) - sqrt(-a*cos
(x)^2 + b*cos(x)^2 + a))^2 - a))/sgn(sin(x))
```

3.48.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^2(x)}{\sqrt{a+b\cot^2(x)}}\,dx = \int \frac{\tan(x)^2}{\sqrt{b\cot\left(x\right)^2+a}}\,dx$$

input
```
int(tan(x)^2/(a + b*cot(x)^2)^(1/2),x)
```

output
```
int(tan(x)^2/(a + b*cot(x)^2)^(1/2), x)
```

3.49 $\int \frac{\cot^3(x)}{(a+b\cot^2(x))^{3/2}}\,dx$

3.49.1 Optimal result

Integrand size = 17, antiderivative size = 59

$$\int \frac{\cot^3(x)}{(a+b\cot^2(x))^{3/2}}\,dx=-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}}+\frac{a}{(a-b)b\sqrt{a+b\cot^2(x)}}$$

output
```
-arctanh((a+b*cot(x)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(3/2)+a/(a-b)/b/(a+b*cot(
x)^2)^(1/2)
```

3.49.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00

$$\int \frac{\cot^3(x)}{(a+b\cot^2(x))^{3/2}}\,dx=-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}}+\frac{a}{(a-b)b\sqrt{a+b\cot^2(x)}}$$

input
```
Integrate[Cot[x]^3/(a + b*Cot[x]^2)^(3/2),x]
```

output
```
-(ArcTanh[Sqrt[a + b*Cot[x]^2]/Sqrt[a - b]]/(a - b)^(3/2)) + a/((a - b)*b*
Sqrt[a + b*Cot[x]^2])
```

3.49.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {3042, 25, 4153, 25, 354, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(x)}{\left(a+b\cot^2(x)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\tan\left(x+\frac{\pi}{2}\right)^3}{\left(a+b\tan\left(x+\frac{\pi}{2}\right)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\tan\left(x+\frac{\pi}{2}\right)^3}{\left(b\tan\left(x+\frac{\pi}{2}\right)^2+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4153}$$

$$\int -\frac{\cot^3(x)}{\left(\cot^2(x)+1\right)\left(a+b\cot^2(x)\right)^{3/2}}d\cot(x)$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\cot^3(x)}{\left(\cot^2(x)+1\right)\left(b\cot^2(x)+a\right)^{3/2}}d\cot(x)$$

$$\Big\downarrow \text{354}$$

$$-\frac{1}{2}\int \frac{\cot^2(x)}{\left(\cot^2(x)+1\right)\left(b\cot^2(x)+a\right)^{3/2}}d\cot^2(x)$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{2}\left(\frac{\int \frac{1}{\left(\cot^2(x)+1\right)\sqrt{b\cot^2(x)+a}}d\cot^2(x)}{a-b}+\frac{2a}{b(a-b)\sqrt{a+b\cot^2(x)}}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{2\int \frac{1}{\frac{\cot^4(x)}{b}-\frac{a}{b}+1}d\sqrt{b\cot^2(x)+a}}{b(a-b)}+\frac{2a}{b(a-b)\sqrt{a+b\cot^2(x)}}\right)$$

$$\downarrow 221$$

$$\frac{1}{2}\left(\frac{2a}{b(a-b)\sqrt{a+b\cot^2(x)}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}}\right)$$

input
```
Int[Cot[x]^3/(a + b*Cot[x]^2)^(3/2),x]
```

output
```
((-2*ArcTanh[Sqrt[a + b*Cot[x]^2]/Sqrt[a - b]])/(a - b)^(3/2) + (2*a)/((a
- b)*b*Sqrt[a + b*Cot[x]^2]))/2
```

3.49.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.49.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.15

method	result	size
derivativedivides	$\frac{1}{b\sqrt{a+b\cot(x)^2}}+\frac{1}{(a-b)\sqrt{a+b\cot(x)^2}}+\frac{\arctan\left(\frac{\sqrt{a+b\cot(x)^2}}{\sqrt{-a+b}}\right)}{(a-b)\sqrt{-a+b}}$	68
default	$\frac{1}{b\sqrt{a+b\cot(x)^2}}+\frac{1}{(a-b)\sqrt{a+b\cot(x)^2}}+\frac{\arctan\left(\frac{\sqrt{a+b\cot(x)^2}}{\sqrt{-a+b}}\right)}{(a-b)\sqrt{-a+b}}$	68

input
```
int(cot(x)^3/(a+b*cot(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/b/(a+b*cot(x)^2)^(1/2)+1/(a-b)/(a+b*cot(x)^2)^(1/2)+1/(a-b)/(-a+b)^(1/2)
*arctan((a+b*cot(x)^2)^(1/2)/(-a+b)^(1/2))
```

3.49.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 186 vs. 2(51) = 102.

Time = 0.31 (sec) , antiderivative size = 385, normalized size of antiderivative = 6.53

$$\int \frac{\cot^3(x)}{(a+b\cot^2(x))^{3/2}}\,dx = \left[-\frac{(ab+b^2-(ab-b^2)\cos(2x))\sqrt{a-b}\log\left(-\sqrt{a-b}\sqrt{\frac{(a-b)\cos(2x)-a-b}{\cos(2x)-1}}(\cos(2x)\right.}{2\left(a^3b-a^2b^2-ab^3+b^4-(a\right.}\right.$$

$$-\frac{(ab+b^2-(ab-b^2)\cos(2x))\sqrt{-a+b}\arctan\left(-\frac{\sqrt{-a+b}\sqrt{\frac{(a-b)\cos(2x)-a-b}{\cos(2x)-1}}}{a-b}\right)-(a^2-ab-(a^2-ab)\cos(2x))}{a^3b-a^2b^2-ab^3+b^4-(a^3b-3a^2b^2+3ab^3-b^4)\cos(2x)}$$

input
```
integrate(cot(x)^3/(a+b*cot(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/2*((a*b + b^2 - (a*b - b^2)*cos(2*x))*sqrt(a - b)*log(-sqrt(a - b)*sqr
t(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))*(cos(2*x) - 1) - (a - b)*cos(
2*x) + a) - 2*(a^2 - a*b - (a^2 - a*b)*cos(2*x))*sqrt(((a - b)*cos(2*x) -
a - b)/(cos(2*x) - 1)))/(a^3*b - a^2*b^2 - a*b^3 + b^4 - (a^3*b - 3*a^2*b^
2 + 3*a*b^3 - b^4)*cos(2*x)), -((a*b + b^2 - (a*b - b^2)*cos(2*x))*sqrt(-a
 + b)*arctan(-sqrt(-a + b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))
/(a - b)) - (a^2 - a*b - (a^2 - a*b)*cos(2*x))*sqrt(((a - b)*cos(2*x) - a
- b)/(cos(2*x) - 1)))/(a^3*b - a^2*b^2 - a*b^3 + b^4 - (a^3*b - 3*a^2*b^2
+ 3*a*b^3 - b^4)*cos(2*x))]
```

3.49.6 Sympy [F]

$$\int \frac{\cot^3(x)}{(a+b\cot^2(x))^{3/2}}\,dx = \int \frac{\cot^3(x)}{(a+b\cot^2(x))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(x)**3/(a+b*cot(x)**2)**(3/2),x)
```

output
```
Integral(cot(x)**3/(a + b*cot(x)**2)**(3/2), x)
```

3.49.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cot^3(x)}{(a+b\cot^2(x))^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cot(x)^3/(a+b*cot(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for m
ore detail
```

3.49.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. 2(51) = 102.

Time = 0.32 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.85

$$\int \frac{\cot^3(x)}{(a+b\cot^2(x))^{3/2}}\,dx = -\frac{\log(|b|)\operatorname{sgn}(\sin(x))}{2\left(\sqrt{a-b}a-\sqrt{a-b}b\right)} + \frac{\frac{a\sin(x)}{\sqrt{a\sin(x)^2-b\sin(x)^2+b}(ab-b^2)} + \frac{\log\left(\left|-\sqrt{a-b}\sin(x)+\sqrt{a\sin(x)^2-b\sin(x)^2+b}\right|\right)}{(a-b)^{\frac{3}{2}}}}{\operatorname{sgn}(\sin(x))}$$

input
```
integrate(cot(x)^3/(a+b*cot(x)^2)^(3/2),x, algorithm="giac")
```

output
```
-1/2*log(abs(b))*sgn(sin(x))/(sqrt(a - b)*a - sqrt(a - b)*b) + (a*sin(x)/(
sqrt(a*sin(x)^2 - b*sin(x)^2 + b)*(a*b - b^2)) + log(abs(-sqrt(a - b)*sin(
x) + sqrt(a*sin(x)^2 - b*sin(x)^2 + b)))/(a - b)^(3/2))/sgn(sin(x))
```

3.49.9 Mupad [B] (verification not implemented)

Time = 14.40 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.88

$$\int \frac{\cot^3(x)}{(a+b\cot^2(x))^{3/2}}\,dx = \frac{a}{(a\,b-b^2)\,\sqrt{b\cot{(x)}^2+a}} - \frac{\operatorname{atanh}\left(\frac{\sqrt{b\cot(x)^2+a}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}}$$

input
```
int(cot(x)^3/(a + b*cot(x)^2)^(3/2),x)
```

output
```
a/((a*b - b^2)*(a + b*cot(x)^2)^(1/2)) - atanh((a + b*cot(x)^2)^(1/2)/(a -
 b)^(1/2))/(a - b)^(3/2)
```

3.50 $\int \frac{\cot^2(x)}{(a+b\cot^2(x))^{3/2}}\,dx$

3.50.1 Optimal result

Integrand size = 17, antiderivative size = 59

$$\int \frac{\cot^2(x)}{(a+b\cot^2(x))^{3/2}}\,dx=\frac{\arctan\left(\frac{\sqrt{a-b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{(a-b)^{3/2}}-\frac{\cot(x)}{(a-b)\sqrt{a+b\cot^2(x)}}$$

output
```
arctan(cot(x)*(a-b)^(1/2)/(a+b*cot(x)^2)^(1/2))/(a-b)^(3/2)-cot(x)/(a-b)/(
a+b*cot(x)^2)^(1/2)
```

3.50.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 137 vs. 2(59) = 118.

Time = 0.81 (sec) , antiderivative size = 137, normalized size of antiderivative = 2.32

$$\int \frac{\cot^2(x)}{(a+b\cot^2(x))^{3/2}}\,dx=\frac{(-a+b)\cot(x)\sqrt{1+\frac{b\cot^2(x)}{a}}+\frac{1}{2}\operatorname{arctanh}\left(\frac{\sqrt{-\frac{(a-b)\cot^2(x)}{a}}}{\sqrt{1+\frac{b\cot^2(x)}{a}}}\right)(-a-b+(a-b)\cos(2}{(a-b)^2\sqrt{a+b\cot^2(x)}\sqrt{1+\frac{b\cot^2(x)}{a}}}$$

input
```
Integrate[Cot[x]^2/(a + b*Cot[x]^2)^(3/2),x]
```

output
```
((-a + b)*Cot[x]*Sqrt[1 + (b*Cot[x]^2)/a] + (ArcTanh[Sqrt[-(((a - b)*Cot[x
]^2)/a)]/Sqrt[1 + (b*Cot[x]^2)/a]]*(-a - b + (a - b)*Cos[2*x])*Sqrt[-(((a
- b)*Cot[x]^2)/a)]*Csc[x]*Sec[x])/2)/((a - b)^2*Sqrt[a + b*Cot[x]^2]*Sqrt[
1 + (b*Cot[x]^2)/a])
```

3.50.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3042, 4153, 373, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(x)}{\left(a+b\cot^2(x)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan\left(x+\frac{\pi}{2}\right)^2}{\left(a+b\tan\left(x+\frac{\pi}{2}\right)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{4153}$$

$$-\int \frac{\cot^2(x)}{\left(\cot^2(x)+1\right)\left(b\cot^2(x)+a\right)^{3/2}}d\cot(x)$$

$$\Big\downarrow \text{373}$$

$$\frac{\int \frac{1}{\left(\cot^2(x)+1\right)\sqrt{b\cot^2(x)+a}}d\cot(x)}{a-b}-\frac{\cot(x)}{(a-b)\sqrt{a+b\cot^2(x)}}$$

$$\Big\downarrow \text{291}$$

$$\frac{\int \frac{1}{1-\frac{(b-a)\cot^2(x)}{b\cot^2(x)+a}}d\frac{\cot(x)}{\sqrt{b\cot^2(x)+a}}}{a-b}-\frac{\cot(x)}{(a-b)\sqrt{a+b\cot^2(x)}}$$

$$\Big\downarrow \text{216}$$

$$\frac{\arctan\left(\frac{\sqrt{a-b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{(a-b)^{3/2}}-\frac{\cot(x)}{(a-b)\sqrt{a+b\cot^2(x)}}$$

input
```
Int[Cot[x]^2/(a + b*Cot[x]^2)^(3/2),x]
```

output
```
ArcTan[(Sqrt[a - b]*Cot[x])/Sqrt[a + b*Cot[x]^2]]/(a - b)^(3/2) - Cot[x]/(
(a - b)*Sqrt[a + b*Cot[x]^2])
```

3.50.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.50.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.68

method	result	size
derivativedivides	$-\frac{\cot(x)}{a\sqrt{a+b\cot(x)^2}}-\frac{b\cot(x)}{(a-b)a\sqrt{a+b\cot(x)^2}}+\frac{\sqrt{b^4(a-b)}\arctan\left(\frac{b^2(a-b)\cot(x)}{\sqrt{b^4(a-b)}\sqrt{a+b\cot(x)^2}}\right)}{(a-b)^2b^2}$	99
default	$-\frac{\cot(x)}{a\sqrt{a+b\cot(x)^2}}-\frac{b\cot(x)}{(a-b)a\sqrt{a+b\cot(x)^2}}+\frac{\sqrt{b^4(a-b)}\arctan\left(\frac{b^2(a-b)\cot(x)}{\sqrt{b^4(a-b)}\sqrt{a+b\cot(x)^2}}\right)}{(a-b)^2b^2}$	99

input
```
int(cot(x)^2/(a+b*cot(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-cot(x)/a/(a+b*cot(x)^2)^(1/2)-b/(a-b)*cot(x)/a/(a+b*cot(x)^2)^(1/2)+1/(a-
b)^2*(b^4*(a-b))^(1/2)/b^2*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*cot(x)^
2)^(1/2)*cot(x))
```

3.50.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. 2(51) = 102.

Time = 0.37 (sec) , antiderivative size = 388, normalized size of antiderivative = 6.58

$$\int \frac{\cot^2(x)}{(a+b\cot^2(x))^{3/2}}\,dx = \left[-\frac{((a-b)\cos(2x)-a-b)\sqrt{-a+b}\log\left(-2(a^2-2ab+b^2)\cos(2x)^2-2((a\right.}{4(a^3}\right.$$

$$-\frac{((a-b)\cos(2x)-a-b)\sqrt{a-b}\arctan\left(-\frac{\sqrt{a-b}\sqrt{\frac{(a-b)\cos(2x)-a-b}{\cos(2x)-1}}\sin(2x)}{(a-b)\cos(2x)-b}\right)+2(a-b)\sqrt{\frac{(a-b)\cos(2x)-a-b}{\cos(2x)-1}}\sin(}{2(a^3-a^2b-ab^2+b^3-(a^3-3a^2b+3ab^2-b^3)\cos(2x))}$$

input
```
integrate(cot(x)^2/(a+b*cot(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*(((a - b)*cos(2*x) - a - b)*sqrt(-a + b)*log(-2*(a^2 - 2*a*b + b^2)*
cos(2*x)^2 - 2*((a - b)*cos(2*x) - b)*sqrt(-a + b)*sqrt(((a - b)*cos(2*x)
- a - b)/(cos(2*x) - 1))*sin(2*x) + a^2 - 2*b^2 + 4*(a*b - b^2)*cos(2*x))
+ 4*(a - b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))*sin(2*x))/(a^3
 - a^2*b - a*b^2 + b^3 - (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(2*x)), -1/2*(
((a - b)*cos(2*x) - a - b)*sqrt(a - b)*arctan(-sqrt(a - b)*sqrt(((a - b)*c
os(2*x) - a - b)/(cos(2*x) - 1))*sin(2*x)/((a - b)*cos(2*x) - b)) + 2*(a -
 b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))*sin(2*x))/(a^3 - a^2*b
 - a*b^2 + b^3 - (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(2*x))]
```

3.50.6 Sympy [F]

$$\int \frac{\cot^2(x)}{(a+b\cot^2(x))^{3/2}}\,dx = \int \frac{\cot^2(x)}{(a+b\cot^2(x))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(x)**2/(a+b*cot(x)**2)**(3/2),x)
```

output
```
Integral(cot(x)**2/(a + b*cot(x)**2)**(3/2), x)
```

3.50.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cot^2(x)}{(a+b\cot^2(x))^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cot(x)^2/(a+b*cot(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

3.50.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(51) = 102$.

Time = 0.34 (sec) , antiderivative size = 159, normalized size of antiderivative = 2.69

$$\int \frac{\cot^2(x)}{(a+b\cot^2(x))^{3/2}}\,dx = \frac{\left(\sqrt{b}\log\left(\left|-\sqrt{-a+b}+\sqrt{b}\right|\right)+\sqrt{-a+b}\right)\operatorname{sgn}(\sin(x))}{a\sqrt{-a+b}\sqrt{b}-\sqrt{-a+b}b^{\frac{3}{2}}} + \frac{\frac{\sqrt{-a\cos(x)^2+b\cos(x)^2+a}\cos(x)}{(a\cos(x)^2-b\cos(x)^2-a)(a-b)} - \frac{\log\left(\left|-\sqrt{-a+b}\cos(x)+\sqrt{-a\cos(x)^2+b\cos(x)^2+a}\right|\right)}{(a-b)\sqrt{-a+b}}}{\operatorname{sgn}(\sin(x))}$$

input
```
integrate(cot(x)^2/(a+b*cot(x)^2)^(3/2),x, algorithm="giac")
```

output
```
(sqrt(b)*log(abs(-sqrt(-a + b) + sqrt(b))) + sqrt(-a + b))*sgn(sin(x))/(a*
sqrt(-a + b)*sqrt(b) - sqrt(-a + b)*b^(3/2)) + (sqrt(-a*cos(x)^2 + b*cos(x
)^2 + a)*cos(x)/((a*cos(x)^2 - b*cos(x)^2 - a)*(a - b)) - log(abs(-sqrt(-a
 + b)*cos(x) + sqrt(-a*cos(x)^2 + b*cos(x)^2 + a)))/((a - b)*sqrt(-a + b))
)/sgn(sin(x))
```

3.50.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^2(x)}{(a+b\cot^2(x))^{3/2}}\,dx = \int \frac{\cot(x)^2}{\left(b\cot(x)^2+a\right)^{3/2}}\,dx$$

input
```
int(cot(x)^2/(a + b*cot(x)^2)^(3/2),x)
```

output
```
int(cot(x)^2/(a + b*cot(x)^2)^(3/2), x)
```

3.51 $\int \frac{\cot(x)}{(a+b\cot^2(x))^{3/2}}\,dx$

3.51.1 Optimal result

Integrand size = 15, antiderivative size = 55

$$\int \frac{\cot(x)}{(a+b\cot^2(x))^{3/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}} - \frac{1}{(a-b)\sqrt{a+b\cot^2(x)}}$$

output
```
arctanh((a+b*cot(x)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(3/2)-1/(a-b)/(a+b*cot(x)^
2)^(1/2)
```

3.51.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.80

$$\int \frac{\cot(x)}{(a+b\cot^2(x))^{3/2}}\,dx = \frac{\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{a+b\cot^2(x)}{a-b}\right)}{(-a+b)\sqrt{a+b\cot^2(x)}}$$

input
```
Integrate[Cot[x]/(a + b*Cot[x]^2)^(3/2),x]
```

output
```
Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Cot[x]^2)/(a - b)]/((-a + b)*Sqrt[a
 + b*Cot[x]^2])
```

3.51.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {3042, 25, 4153, 25, 353, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(x)}{\left(a+b\cot^2(x)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\tan\left(x+\frac{\pi}{2}\right)}{\left(a+b\tan\left(x+\frac{\pi}{2}\right)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\tan\left(x+\frac{\pi}{2}\right)}{\left(b\tan\left(x+\frac{\pi}{2}\right)^2+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4153}$$

$$\int -\frac{\cot(x)}{\left(\cot^2(x)+1\right)\left(a+b\cot^2(x)\right)^{3/2}}d\cot(x)$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\cot(x)}{\left(\cot^2(x)+1\right)\left(b\cot^2(x)+a\right)^{3/2}}d\cot(x)$$

$$\Big\downarrow \text{353}$$

$$-\frac{1}{2}\int \frac{1}{\left(\cot^2(x)+1\right)\left(b\cot^2(x)+a\right)^{3/2}}d\cot^2(x)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2}\left(-\frac{\int \frac{1}{\left(\cot^2(x)+1\right)\sqrt{b\cot^2(x)+a}}d\cot^2(x)}{a-b}-\frac{2}{(a-b)\sqrt{a+b\cot^2(x)}}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(-\frac{2\int \frac{1}{\frac{\cot^4(x)}{b}-\frac{a}{b}+1}d\sqrt{b\cot^2(x)+a}}{b(a-b)}-\frac{2}{(a-b)\sqrt{a+b\cot^2(x)}}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}}-\frac{2}{(a-b)\sqrt{a+b\cot^2(x)}}\right)$$

input
```
Int[Cot[x]/(a + b*Cot[x]^2)^(3/2),x]
```

output
```
((2*ArcTanh[Sqrt[a + b*Cot[x]^2]/Sqrt[a - b]])/(a - b)^(3/2) - 2/((a - b)*
Sqrt[a + b*Cot[x]^2]))/2
```

3.51.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.51.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

method	result	size
derivativedivides	$-\frac{1}{(a-b)\sqrt{a+b\cot(x)^2}}-\frac{\arctan\left(\frac{\sqrt{a+b\cot(x)^2}}{\sqrt{-a+b}}\right)}{(a-b)\sqrt{-a+b}}$	56
default	$-\frac{1}{(a-b)\sqrt{a+b\cot(x)^2}}-\frac{\arctan\left(\frac{\sqrt{a+b\cot(x)^2}}{\sqrt{-a+b}}\right)}{(a-b)\sqrt{-a+b}}$	56

input
```
int(cot(x)/(a+b*cot(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/(a-b)/(a+b*cot(x)^2)^(1/2)-1/(a-b)/(-a+b)^(1/2)*arctan((a+b*cot(x)^2)^(
1/2)/(-a+b)^(1/2))
```

3.51.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(47) = 94$.

Time = 0.31 (sec) , antiderivative size = 344, normalized size of antiderivative = 6.25

$$\int \frac{\cot(x)}{(a+b\cot^2(x))^{3/2}}\,dx = \left[\frac{((a-b)\cos(2x)-a-b)\sqrt{a-b}\log\left(\sqrt{a-b}\sqrt{\frac{(a-b)\cos(2x)-a-b}{\cos(2x)-1}}(\cos(2x)-1)-\right.}{2(a^3-a^2b-ab^2+b^3-(a^3-3a^2\right.$$

$$-\frac{((a-b)\cos(2x)-a-b)\sqrt{-a+b}\arctan\left(-\frac{\sqrt{-a+b}\sqrt{\frac{(a-b)\cos(2x)-a-b}{\cos(2x)-1}}}{a-b}\right)-((a-b)\cos(2x)-a+b)\sqrt{\frac{(a-b)\cos}{\cos}}}{a^3-a^2b-ab^2+b^3-(a^3-3a^2b+3ab^2-b^3)\cos(2x)}$$

input
```
integrate(cot(x)/(a+b*cot(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*(((a - b)*cos(2*x) - a - b)*sqrt(a - b)*log(sqrt(a - b)*sqrt(((a - b)
*cos(2*x) - a - b)/(cos(2*x) - 1))*(cos(2*x) - 1) - (a - b)*cos(2*x) + a)
+ 2*((a - b)*cos(2*x) - a + b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) -
 1)))/(a^3 - a^2*b - a*b^2 + b^3 - (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(2*x
)), -(((a - b)*cos(2*x) - a - b)*sqrt(-a + b)*arctan(-sqrt(-a + b)*sqrt(((
a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))/(a - b)) - ((a - b)*cos(2*x) - a
+ b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1)))/(a^3 - a^2*b - a*b^2
 + b^3 - (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(2*x))]
```

3.51.6 Sympy [A] (verification not implemented)

Time = 4.82 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.60

$$\int \frac{\cot(x)}{(a+b\cot^2(x))^{3/2}}\,dx = -\begin{cases} \frac{2\left(\frac{b}{2(a-b)\sqrt{a+b\cot^2(x)}}+\frac{b\,\mathrm{atan}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{-a+b}}\right)}{2\sqrt{-a+b}(a-b)}\right)}{b} & \text{for } b\neq 0 \\ \begin{cases} \tilde{\infty}\cot^2(x) & \text{for } a^{\frac{3}{2}}=0 \\ \frac{\log\left(2a^{\frac{3}{2}}\cot^2(x)+2a^{\frac{3}{2}}\right)}{2a^{\frac{3}{2}}} & \text{otherwise} \end{cases} & \text{otherwise} \end{cases}$$

input
```
integrate(cot(x)/(a+b*cot(x)**2)**(3/2),x)
```

output
```
-Piecewise((2*(b/(2*(a - b)*sqrt(a + b*cot(x)**2)) + b*atan(sqrt(a + b*cot
(x)**2)/sqrt(-a + b))/(2*sqrt(-a + b)*(a - b)))/b, Ne(b, 0)), (Piecewise((
zoo*cot(x)**2, Eq(a**(3/2), 0)), (log(2*a**(3/2)*cot(x)**2 + 2*a**(3/2))/(
2*a**(3/2)), True)), True))
```

3.51.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cot(x)}{\left(a+b\cot^2(x)\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cot(x)/(a+b*cot(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for m
ore detail
```

3.51.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(47) = 94$.

Time = 0.30 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.91

$$\int \frac{\cot(x)}{\left(a+b\cot^2(x)\right)^{3/2}}\,dx = \frac{\log\left(|b|\right)\operatorname{sgn}(\sin\left(x\right))}{2\left(\sqrt{a-b}a-\sqrt{a-b}b\right)} - \frac{\frac{\log\left(\left|-\sqrt{a-b}\sin(x)+\sqrt{a\sin(x)^2-b\sin(x)^2+b}\right|\right)}{(a-b)^{\frac{3}{2}}}+\frac{\sin(x)}{\sqrt{a\sin(x)^2-b\sin(x)^2+b}(a-b)}}{\operatorname{sgn}\left(\sin\left(x\right)\right)}$$

input
```
integrate(cot(x)/(a+b*cot(x)^2)^(3/2),x, algorithm="giac")
```

output
```
1/2*log(abs(b))*sgn(sin(x))/(sqrt(a - b)*a - sqrt(a - b)*b) - (log(abs(-sq
rt(a - b)*sin(x) + sqrt(a*sin(x)^2 - b*sin(x)^2 + b)))/(a - b)^(3/2) + sin
(x)/(sqrt(a*sin(x)^2 - b*sin(x)^2 + b)*(a - b)))/sgn(sin(x))
```

3.51.9 Mupad [B] (verification not implemented)

Time = 14.41 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

$$\int \frac{\cot(x)}{(a+b\cot^2(x))^{3/2}}\,dx = \frac{\operatorname{atanh}\left(\frac{\sqrt{b\cot(x)^2+a}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}} - \frac{1}{(a-b)\,\sqrt{b\cot(x)^2+a}}$$

input
```
int(cot(x)/(a + b*cot(x)^2)^(3/2),x)
```

output
```
atanh((a + b*cot(x)^2)^(1/2)/(a - b)^(1/2))/(a - b)^(3/2) - 1/((a - b)*(a
+ b*cot(x)^2)^(1/2))
```

3.52 $\int \frac{\tan(x)}{\left(a+b\cot^2(x)\right)^{3/2}} \, dx$

3.52.1 Optimal result

Integrand size = 15, antiderivative size = 84

$$\int \frac{\tan(x)}{\left(a+b\cot^2(x)\right)^{3/2}} \, dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}} + \frac{b}{a(a-b)\sqrt{a+b\cot^2(x)}}$$

output
```
arctanh((a+b*cot(x)^2)^(1/2)/a^(1/2))/a^(3/2)-arctanh((a+b*cot(x)^2)^(1/2)
/(a-b)^(1/2))/(a-b)^(3/2)+b/a/(a-b)/(a+b*cot(x)^2)^(1/2)
```

3.52.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.89

$$\int \frac{\tan(x)}{\left(a+b\cot^2(x)\right)^{3/2}} \, dx = \frac{a \operatorname{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{a+b\cot^2(x)}{a-b}\right) + (-a+b) \operatorname{Hypergeometric2F1}\left(-\frac{1}{2}\right.}{a(a-b)\sqrt{a+b\cot^2(x)}}$$

input
```
Integrate[Tan[x]/(a + b*Cot[x]^2)^(3/2),x]
```

output
```
(a*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Cot[x]^2)/(a - b)] + (-a + b)*Hy
pergeometric2F1[-1/2, 1, 1/2, 1 + (b*Cot[x]^2)/a])/(a*(a - b)*Sqrt[a + b*C
ot[x]^2])
```

3.52.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.30, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 25, 4153, 25, 354, 96, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(x)}{\left(a+b\cot^2(x)\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\tan\left(x+\frac{\pi}{2}\right)\left(a+b\tan\left(x+\frac{\pi}{2}\right)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\tan\left(x+\frac{\pi}{2}\right)\left(b\tan\left(x+\frac{\pi}{2}\right)^2+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4153}$$

$$\int -\frac{\tan(x)}{\left(\cot^2(x)+1\right)\left(a+b\cot^2(x)\right)^{3/2}}d\cot(x)$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\tan(x)}{\left(\cot^2(x)+1\right)\left(b\cot^2(x)+a\right)^{3/2}}d\cot(x)$$

$$\Big\downarrow \text{354}$$

$$-\frac{1}{2}\int \frac{\tan(x)}{\left(\cot^2(x)+1\right)\left(b\cot^2(x)+a\right)^{3/2}}d\cot^2(x)$$

$$\Big\downarrow \text{96}$$

$$\frac{1}{2}\left(\frac{2b}{a(a-b)\sqrt{a+b\cot^2(x)}}-\frac{\int \frac{\left(-b\cot^2(x)+a-b\right)\tan(x)}{\left(\cot^2(x)+1\right)\sqrt{b\cot^2(x)+a}}d\cot^2(x)}{a(a-b)}\right)$$

$$\Big\downarrow \; 174$$

$$\frac{1}{2}\left(\frac{2b}{a(a-b)\sqrt{a+b\cot^2(x)}}-\frac{(a-b)\int \frac{\tan(x)}{\sqrt{b\cot^2(x)+a}}d\cot^2(x)-a\int \frac{1}{(\cot^2(x)+1)\sqrt{b\cot^2(x)+a}}d\cot^2(x)}{a(a-b)}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{2}\left(\frac{2b}{a(a-b)\sqrt{a+b\cot^2(x)}}-\frac{\frac{2(a-b)\int \frac{1}{\frac{\cot^4(x)}{b}-\frac{a}{b}}d\sqrt{b\cot^2(x)+a}}{b}-\frac{2a\int \frac{1}{\frac{\cot^4(x)}{b}-\frac{a}{b}+1}d\sqrt{b\cot^2(x)+a}}{b}}{a(a-b)}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{2}\left(\frac{2b}{a(a-b)\sqrt{a+b\cot^2(x)}}-\frac{\frac{2a\text{arctanh}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}-\frac{2(a-b)\text{arctanh}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}}{a(a-b)}\right)$$

input
```
Int[Tan[x]/(a + b*Cot[x]^2)^(3/2),x]
```

output
```
(-(((-2*(a - b)*ArcTanh[Sqrt[a + b*Cot[x]^2/Sqrt[a]])/Sqrt[a] + (2*a*ArcT
anh[Sqrt[a + b*Cot[x]^2]/Sqrt[a - b]])/Sqrt[a - b])/(a*(a - b))) + (2*b)/(
a*(a - b)*Sqrt[a + b*Cot[x]^2]))/2
```

3.52.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 96
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[f*((e + f*x)^(p + 1)/((p + 1)*(b*e - a*f)*(d*e - c*f))), x] + S
imp[1/((b*e - a*f)*(d*e - c*f))   Int[(b*d*e - b*c*f - a*d*f - b*d*f*x)*((e
 + f*x)^(p + 1)/((a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && LtQ[p, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.52. $\int \frac{\tan(x)}{(a+b\cot^2(x))^{3/2}}\,dx$

3.52.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 516 vs. $2(70) = 140$.

Time = 1.52 (sec) , antiderivative size = 517, normalized size of antiderivative = 6.15

method	result
default	$\frac{\sqrt{4}\left(\arctan\left(\frac{\sqrt{b(1-\cos(x))^4\csc(x)^4+4a(1-\cos(x))^2\csc(x)^2-2b(1-\cos(x))^2\csc(x)^2+b}\,\sin(x)}{2(1-\cos(x))\sqrt{-a+b}}\right)a^{\frac{5}{2}}\sqrt{b(1-\cos(x))^4\csc(x)^4+4a(1-\cos(x))^2 c}}{}$

input
```
int(tan(x)/(a+b*cot(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*4^(1/2)/a^(5/2)/(a-b)/(-a+b)^(1/2)*(arctan(1/2*(b*(1-cos(x))^4*csc(x)^
4+4*a*(1-cos(x))^2*csc(x)^2-2*b*(1-cos(x))^2*csc(x)^2+b)^(1/2)/(1-cos(x))*
sin(x)/(-a+b)^(1/2))*a^(5/2)*(b*(1-cos(x))^4*csc(x)^4+4*a*(1-cos(x))^2*csc
(x)^2-2*b*(1-cos(x))^2*csc(x)^2+b)^(1/2)+2*b*(-a+b)^(1/2)*a^(3/2)*(csc(x)-
cot(x))+arctanh(1/2*(b*(1-cos(x))^4*csc(x)^4+4*a*(1-cos(x))^2*csc(x)^2-2*b
*(1-cos(x))^2*csc(x)^2+b)^(1/2)/(1-cos(x))*sin(x)/a^(1/2))*a^2*(b*(1-cos(x
))^4*csc(x)^4+4*a*(1-cos(x))^2*csc(x)^2-2*b*(1-cos(x))^2*csc(x)^2+b)^(1/2)
*(-a+b)^(1/2)-arctanh(1/2*(b*(1-cos(x))^4*csc(x)^4+4*a*(1-cos(x))^2*csc(x)
^2-2*b*(1-cos(x))^2*csc(x)^2+b)^(1/2)/(1-cos(x))*sin(x)/a^(1/2))*a*(b*(1-c
os(x))^4*csc(x)^4+4*a*(1-cos(x))^2*csc(x)^2-2*b*(1-cos(x))^2*csc(x)^2+b)^(
1/2)*(-a+b)^(1/2)*b)/(1/(1-cos(x))^2*(b*(1-cos(x))^4*csc(x)^2+4*a*(1-cos(x
))^2-2*b*(1-cos(x))^2+b*sin(x)^2))^(3/2)/(1-cos(x))^3*(b*(1-cos(x))^4*csc(
x)+4*a*(1-cos(x))^2*sin(x)-2*b*(1-cos(x))^2*sin(x)+b*sin(x)^3)
```

3.52.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(70) = 140$.

Time = 0.37 (sec) , antiderivative size = 863, normalized size of antiderivative = 10.27

$$\int \frac{\tan(x)}{(a+b\cot^2(x))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tan(x)/(a+b*cot(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*(2*(a^2*b - a*b^2)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x)^2 + (a^2*b
- 2*a*b^2 + b^3 + (a^3 - 2*a^2*b + a*b^2)*tan(x)^2)*sqrt(a)*log(2*a*tan(x)
^2 + 2*sqrt(a)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x)^2 + b) - (a^3*tan(x)
^2 + a^2*b)*sqrt(a - b)*log(((2*a - b)*tan(x)^2 + 2*sqrt(a - b)*sqrt((a*ta
n(x)^2 + b)/tan(x)^2)*tan(x)^2 + b)/(tan(x)^2 + 1)))/(a^4*b - 2*a^3*b^2 +
a^2*b^3 + (a^5 - 2*a^4*b + a^3*b^2)*tan(x)^2), 1/2*(2*(a^2*b - a*b^2)*sqrt
((a*tan(x)^2 + b)/tan(x)^2)*tan(x)^2 - 2*(a^3*tan(x)^2 + a^2*b)*sqrt(-a +
b)*arctan(-sqrt(-a + b)*sqrt((a*tan(x)^2 + b)/tan(x)^2)/(a - b)) + (a^2*b
- 2*a*b^2 + b^3 + (a^3 - 2*a^2*b + a*b^2)*tan(x)^2)*sqrt(a)*log(2*a*tan(x)
^2 + 2*sqrt(a)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x)^2 + b))/(a^4*b - 2*a
^3*b^2 + a^2*b^3 + (a^5 - 2*a^4*b + a^3*b^2)*tan(x)^2), 1/2*(2*(a^2*b - a*
b^2)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x)^2 - 2*(a^2*b - 2*a*b^2 + b^3 +
 (a^3 - 2*a^2*b + a*b^2)*tan(x)^2)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*tan(x)
^2 + b)/tan(x)^2)/a) - (a^3*tan(x)^2 + a^2*b)*sqrt(a - b)*log(((2*a - b)*t
an(x)^2 + 2*sqrt(a - b)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x)^2 + b)/(tan
(x)^2 + 1)))/(a^4*b - 2*a^3*b^2 + a^2*b^3 + (a^5 - 2*a^4*b + a^3*b^2)*tan(
x)^2), ((a^2*b - a*b^2)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x)^2 - (a^2*b
- 2*a*b^2 + b^3 + (a^3 - 2*a^2*b + a*b^2)*tan(x)^2)*sqrt(-a)*arctan(sqrt(-
a)*sqrt((a*tan(x)^2 + b)/tan(x)^2)/a) - (a^3*tan(x)^2 + a^2*b)*sqrt(-a + b
)*arctan(-sqrt(-a + b)*sqrt((a*tan(x)^2 + b)/tan(x)^2)/(a - b)))/(a^4*b...
```

3.52.6 Sympy [F]

$$\int \frac{\tan(x)}{(a+b\cot^2(x))^{3/2}}\,dx = \int \frac{\tan(x)}{(a+b\cot^2(x))^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(x)/(a+b*cot(x)**2)**(3/2),x)
```

output
```
Integral(tan(x)/(a + b*cot(x)**2)**(3/2), x)
```

3.52.7 Maxima [F]

$$\int \frac{\tan(x)}{\left(a+b\cot^2(x)\right)^{3/2}}\,dx=\int \frac{\tan\left(x\right)}{\left(b\cot\left(x\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(x)/(a+b*cot(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(tan(x)/(b*cot(x)^2 + a)^(3/2), x)
```

3.52.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 295 vs. $2(70) = 140$.

Time = 0.35 (sec) , antiderivative size = 295, normalized size of antiderivative = 3.51

$$\int \frac{\tan(x)}{\left(a+b\cot^2(x)\right)^{3/2}}\,dx=$$
$$-\frac{\left(2\,a^{2}\arctan\left(-\frac{a-b}{\sqrt{-a^{2}+ab}}\right)-4\,ab\arctan\left(-\frac{a-b}{\sqrt{-a^{2}+ab}}\right)+2\,b^{2}\arctan\left(-\frac{a-b}{\sqrt{-a^{2}+ab}}\right)+\sqrt{-a^{2}+ab}a\log\left(b\right)\right)\mathrm{sgn}}{2\left(\sqrt{-a^{2}+ab}\sqrt{a-b}a^{2}-\sqrt{-a^{2}+ab}\sqrt{a-b}ab\right)}$$
$$+\frac{\frac{2\,b\sin(x)}{\sqrt{a\sin(x)^{2}-b\sin(x)^{2}+b}(a^{2}-ab)}+\frac{2\sqrt{a-b}\arctan\left(\frac{\left(\sqrt{a-b}\sin(x)-\sqrt{a\sin(x)^{2}-b\sin(x)^{2}+b}\right)^{2}-2\,a+b}{2\sqrt{-a^{2}+ab}}\right)}{\sqrt{-a^{2}+ab}a}+\frac{\log\left(\left(\sqrt{a-b}\sin(x)-\sqrt{a\sin(x)^{2}-b\sin(}\right.\right.}{(a-b)^{\frac{3}{2}}}}{2\,\mathrm{sgn}\left(\sin\left(x\right)\right)}$$

input
```
integrate(tan(x)/(a+b*cot(x)^2)^(3/2),x, algorithm="giac")
```

output
```
-1/2*(2*a^2*arctan(-(a - b)/sqrt(-a^2 + a*b)) - 4*a*b*arctan(-(a - b)/sqrt
(-a^2 + a*b)) + 2*b^2*arctan(-(a - b)/sqrt(-a^2 + a*b)) + sqrt(-a^2 + a*b)
*a*log(b))*sgn(sin(x))/(sqrt(-a^2 + a*b)*sqrt(a - b)*a^2 - sqrt(-a^2 + a*b
)*sqrt(a - b)*a*b) + 1/2*(2*b*sin(x)/(sqrt(a*sin(x)^2 - b*sin(x)^2 + b)*(a
^2 - a*b)) + 2*sqrt(a - b)*arctan(1/2*((sqrt(a - b)*sin(x) - sqrt(a*sin(x)
^2 - b*sin(x)^2 + b))^2 - 2*a + b)/sqrt(-a^2 + a*b))/(sqrt(-a^2 + a*b)*a)
+ log((sqrt(a - b)*sin(x) - sqrt(a*sin(x)^2 - b*sin(x)^2 + b))^2)/(a - b)^
(3/2))/sgn(sin(x))
```

3.52.9 Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 1451, normalized size of antiderivative = 17.27

$$\int \frac{\tan(x)}{(a+b\cot^2(x))^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(tan(x)/(a + b*cot(x)^2)^(3/2),x)
```

output
```
atanh((2*a^2*b^8*(a + b/tan(x)^2)^(1/2))/((a^3)^(1/2)*(2*a*b^8 - 12*a^2*b^
7 + 30*a^3*b^6 - 38*a^4*b^5 + 24*a^5*b^4 - 6*a^6*b^3)) - (12*a^3*b^7*(a +
b/tan(x)^2)^(1/2))/((a^3)^(1/2)*(2*a*b^8 - 12*a^2*b^7 + 30*a^3*b^6 - 38*a^
4*b^5 + 24*a^5*b^4 - 6*a^6*b^3)) + (30*a^4*b^6*(a + b/tan(x)^2)^(1/2))/((a
^3)^(1/2)*(2*a*b^8 - 12*a^2*b^7 + 30*a^3*b^6 - 38*a^4*b^5 + 24*a^5*b^4 - 6
*a^6*b^3)) - (38*a^5*b^5*(a + b/tan(x)^2)^(1/2))/((a^3)^(1/2)*(2*a*b^8 - 1
2*a^2*b^7 + 30*a^3*b^6 - 38*a^4*b^5 + 24*a^5*b^4 - 6*a^6*b^3)) + (24*a^6*b
^4*(a + b/tan(x)^2)^(1/2))/((a^3)^(1/2)*(2*a*b^8 - 12*a^2*b^7 + 30*a^3*b^6
 - 38*a^4*b^5 + 24*a^5*b^4 - 6*a^6*b^3)) - (6*a^7*b^3*(a + b/tan(x)^2)^(1/
2))/((a^3)^(1/2)*(2*a*b^8 - 12*a^2*b^7 + 30*a^3*b^6 - 38*a^4*b^5 + 24*a^5*
b^4 - 6*a^6*b^3)))/(a^3)^(1/2) - (atan(((((a - b)^3)^(1/2)*(((a + b/tan(x)
^2)^(1/2)*(2*a^3*b^7 - 10*a^4*b^6 + 22*a^5*b^5 - 26*a^6*b^4 + 16*a^7*b^3 -
 4*a^8*b^2))/2 + (((a - b)^3)^(1/2)*(12*a^5*b^7 - 2*a^4*b^8 - 28*a^6*b^6 +
 32*a^7*b^5 - 18*a^8*b^4 + 4*a^9*b^3 + ((a + b/tan(x)^2)^(1/2)*((a - b)^3)
^(1/2)*(8*a^5*b^8 - 56*a^6*b^7 + 160*a^7*b^6 - 240*a^8*b^5 + 200*a^9*b^4 -
 88*a^10*b^3 + 16*a^11*b^2))/(4*(a - b)^3)))/(2*(a - b)^3))*1i)/(a - b)^3
+ (((a - b)^3)^(1/2)*(((a + b/tan(x)^2)^(1/2)*(2*a^3*b^7 - 10*a^4*b^6 + 22
*a^5*b^5 - 26*a^6*b^4 + 16*a^7*b^3 - 4*a^8*b^2))/2 + (((a - b)^3)^(1/2)*(2
*a^4*b^8 - 12*a^5*b^7 + 28*a^6*b^6 - 32*a^7*b^5 + 18*a^8*b^4 - 4*a^9*b^3 +
 ((a + b/tan(x)^2)^(1/2)*((a - b)^3)^(1/2)*(8*a^5*b^8 - 56*a^6*b^7 + 16...
```

3.53 $\int \frac{\tan^2(x)}{\left(a+b\cot^2(x)\right)^{3/2}}\,dx$

3.53.1 Optimal result

Integrand size = 17, antiderivative size = 92

$$\int \frac{\tan^2(x)}{\left(a+b\cot^2(x)\right)^{3/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{a-b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{(a-b)^{3/2}} + \frac{b\tan(x)}{a(a-b)\sqrt{a+b\cot^2(x)}} + \frac{(a-2b)\sqrt{a+b\cot^2(x)}\tan(x)}{a^2(a-b)}$$

output
```
arctan(cot(x)*(a-b)^(1/2)/(a+b*cot(x)^2)^(1/2))/(a-b)^(3/2)+b*tan(x)/a/(a-
b)/(a+b*cot(x)^2)^(1/2)+(a-2*b)*(a+b*cot(x)^2)^(1/2)*tan(x)/a^2/(a-b)
```

3.53.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 6.95 (sec) , antiderivative size = 674, normalized size of antiderivative = 7.33

$$\int \frac{\tan^2(x)}{\left(a+b\cot^2(x)\right)^{3/2}}\,dx = \frac{\sin^2(x)\left(\frac{12b\csc^2(x)}{a-b} + \frac{8b^2\cot^2(x)\csc^2(x)}{a(a-b)} + \frac{16(a-b)\cos^2(x)\,\text{Hypergeometric2F1}\left(2,2,\frac{7}{2},\frac{(a-b)\cos^2(x)}{a}\right)}{15a}\right.}{}$$

input
```
Integrate[Tan[x]^2/(a + b*Cot[x]^2)^(3/2),x]
```

output
```
(Sin[x]^2*((12*b*Csc[x]^2)/(a - b) + (8*b^2*Cot[x]^2*Csc[x]^2)/(a*(a - b))
 + (16*(a - b)*Cos[x]^2*Hypergeometric2F1[2, 2, 7/2, ((a - b)*Cos[x]^2)/a]
)/(15*a) + (8*(a - b)*b*Cos[x]^2*Cot[x]^2*Hypergeometric2F1[2, 2, 7/2, ((a
 - b)*Cos[x]^2)/a])/(3*a^2) + (8*(a - b)*b^2*Cos[x]^2*Cot[x]^4*Hypergeomet
ric2F1[2, 2, 7/2, ((a - b)*Cos[x]^2)/a])/(5*a^3) + (8*(a - b)*Cos[x]^2*Hyp
ergeometricPFQ[{2, 2, 2}, {1, 7/2}, ((a - b)*Cos[x]^2)/a])/(15*a) + (16*(a
 - b)*b*Cos[x]^2*Cot[x]^2*HypergeometricPFQ[{2, 2, 2}, {1, 7/2}, ((a - b)*
Cos[x]^2)/a])/(15*a^2) + (8*(a - b)*b^2*Cos[x]^2*Cot[x]^4*HypergeometricPF
Q[{2, 2, 2}, {1, 7/2}, ((a - b)*Cos[x]^2)/a])/(15*a^3) + (3*a*Sec[x]^2)/(a
 - b) - (3*ArcSin[Sqrt[((a - b)*Cos[x]^2)/a]])/((((a - b)*Cos[x]^2)/a)^(3/
2)*Sqrt[((a + b*Cot[x]^2)*Sin[x]^2)/a]) - (12*b*ArcSin[Sqrt[((a - b)*Cos[x
]^2)/a]]*Cot[x]^2)/(a*(((a - b)*Cos[x]^2)/a)^(3/2)*Sqrt[((a + b*Cot[x]^2)*
Sin[x]^2)/a]) - (8*b^2*ArcSin[Sqrt[((a - b)*Cos[x]^2)/a]]*Cot[x]^4)/(a^2*(
((a - b)*Cos[x]^2)/a)^(3/2)*Sqrt[((a + b*Cot[x]^2)*Sin[x]^2)/a]) + (3*ArcS
in[Sqrt[((a - b)*Cos[x]^2)/a]])/Sqrt[((a - b)*Cos[x]^2*(a + b*Cot[x]^2)*Si
n[x]^2)/a^2] + (12*b*ArcSin[Sqrt[((a - b)*Cos[x]^2)/a]]*Cot[x]^2)/(a*Sqrt[
((a - b)*Cos[x]^2*(a + b*Cot[x]^2)*Sin[x]^2)/a^2]) + (8*b^2*ArcSin[Sqrt[((
a - b)*Cos[x]^2)/a]]*Cot[x]^4)/(a^2*Sqrt[((a - b)*Cos[x]^2*(a + b*Cot[x]^2
)*Sin[x]^2)/a^2]))*Tan[x])/(a*Sqrt[a + b*Cot[x]^2])
```

3.53.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {3042, 4153, 374, 445, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(x)}{\left(a+b\cot^2(x)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan\left(x+\frac{\pi}{2}\right)^2\left(a+b\tan\left(x+\frac{\pi}{2}\right)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{4153}$$

$$-\int \frac{\tan^2(x)}{\left(\cot^2(x)+1\right)\left(b\cot^2(x)+a\right)^{3/2}}d\cot(x)$$

$$\Big\downarrow \text{374}$$

$$\frac{b\tan(x)}{a(a-b)\sqrt{a+b\cot^2(x)}}-\frac{\int\frac{(-2b\cot^2(x)+a-2b)\tan^2(x)}{(\cot^2(x)+1)\sqrt{b\cot^2(x)+a}}d\cot(x)}{a(a-b)}$$

$$\Big\downarrow \text{445}$$

$$\frac{b\tan(x)}{a(a-b)\sqrt{a+b\cot^2(x)}}-\frac{-\frac{\int\frac{a^2}{(\cot^2(x)+1)\sqrt{b\cot^2(x)+a}}d\cot(x)}{a}-\frac{(a-2b)\tan(x)\sqrt{a+b\cot^2(x)}}{a}}{a(a-b)}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\tan(x)}{a(a-b)\sqrt{a+b\cot^2(x)}}-\frac{-a\int\frac{1}{(\cot^2(x)+1)\sqrt{b\cot^2(x)+a}}d\cot(x)-\frac{(a-2b)\tan(x)\sqrt{a+b\cot^2(x)}}{a}}{a(a-b)}$$

$$\Big\downarrow \text{291}$$

$$\frac{b\tan(x)}{a(a-b)\sqrt{a+b\cot^2(x)}}-\frac{-a\int\frac{1}{1-\frac{(b-a)\cot^2(x)}{b\cot^2(x)+a}}d\frac{\cot(x)}{\sqrt{b\cot^2(x)+a}}-\frac{(a-2b)\tan(x)\sqrt{a+b\cot^2(x)}}{a}}{a(a-b)}$$

$$\Big\downarrow \text{216}$$

$$\frac{b\tan(x)}{a(a-b)\sqrt{a+b\cot^2(x)}}-\frac{-\frac{a\arctan\left(\frac{\sqrt{a-b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{\sqrt{a-b}}-\frac{(a-2b)\tan(x)\sqrt{a+b\cot^2(x)}}{a}}{a(a-b)}$$

input
```
Int[Tan[x]^2/(a + b*Cot[x]^2)^(3/2),x]
```

output
```
(b*Tan[x])/(a*(a - b)*Sqrt[a + b*Cot[x]^2]) - (-((a*ArcTan[(Sqrt[a - b]*Co
t[x])/Sqrt[a + b*Cot[x]^2]])/Sqrt[a - b]) - ((a - 2*b)*Sqrt[a + b*Cot[x]^2
]*Tan[x])/a)/(a*(a - b))
```

3.53.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.53.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 630 vs. $2(82) = 164$.

Time = 2.93 (sec) , antiderivative size = 631, normalized size of antiderivative = 6.86

method	result
default	$\frac{\sqrt{4}\left(-\sqrt{-a+b}\,ab(1-\cos(x))^4\csc(x)^4+2\sqrt{-a+b}\,b^2(1-\cos(x))^4\csc(x)^4+\sqrt{b(1-\cos(x))^4\csc(x)^4+4a(1-\cos(x))^2\csc(x)^2-2b(1-\cos}\right.}{}$

input
```
int(tan(x)^2/(a+b*cot(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*4^(1/2)/(-a+b)^(1/2)/a^2/(a-b)*(-(-a+b)^(1/2)*a*b*(1-cos(x))^4*csc(x)^
4+2*(-a+b)^(1/2)*b^2*(1-cos(x))^4*csc(x)^4+(b*(1-cos(x))^4*csc(x)^4+4*a*(1
-cos(x))^2*csc(x)^2-2*b*(1-cos(x))^2*csc(x)^2+b)^(1/2)*ln(4*(a*(1-cos(x))^
2*csc(x)^2-b*(1-cos(x))^2*csc(x)^2+(-a+b)^(1/2)*(b*(1-cos(x))^4*csc(x)^4+4
*a*(1-cos(x))^2*csc(x)^2-2*b*(1-cos(x))^2*csc(x)^2+b)^(1/2)-a+b)/((1-cos(x
))^2*csc(x)^2+1))*a^2*(1-cos(x))^2*csc(x)^2-4*(-a+b)^(1/2)*a^2*(1-cos(x))^
2*csc(x)^2+6*(-a+b)^(1/2)*a*b*(1-cos(x))^2*csc(x)^2-4*(-a+b)^(1/2)*b^2*(1-
cos(x))^2*csc(x)^2-ln(4*(a*(1-cos(x))^2*csc(x)^2-b*(1-cos(x))^2*csc(x)^2+(
-a+b)^(1/2)*(b*(1-cos(x))^4*csc(x)^4+4*a*(1-cos(x))^2*csc(x)^2-2*b*(1-cos(
x))^2*csc(x)^2+b)^(1/2)-a+b)/((1-cos(x))^2*csc(x)^2+1))*(b*(1-cos(x))^4*cs
c(x)^4+4*a*(1-cos(x))^2*csc(x)^2-2*b*(1-cos(x))^2*csc(x)^2+b)^(1/2)*a^2-a*
(-a+b)^(1/2)*b+2*b^2*(-a+b)^(1/2))*(b*(1-cos(x))^4*csc(x)^4+4*a*(1-cos(x))
^2*csc(x)^2-2*b*(1-cos(x))^2*csc(x)^2+b)/((1-cos(x))^2*csc(x)^2-1)/(1-cos(
x))^3*sin(x)^3/(1/(1-cos(x))^2*(b*(1-cos(x))^4*csc(x)^2+4*a*(1-cos(x))^2-2
*b*(1-cos(x))^2+b*sin(x)^2))^(3/2)
```

3.53.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(82) = 164$.

Time = 0.35 (sec) , antiderivative size = 393, normalized size of antiderivative = 4.27

$$\int \frac{\tan^2(x)}{(a+b\cot^2(x))^{3/2}}\,dx = \left[\frac{\left(a^3\tan(x)^2+a^2b\right)\sqrt{-a+b}\log\left(-\frac{a^2\tan(x)^4-2\left(3a^2-4ab\right)\tan(x)^2+a^2-8ab+8b^2-4\left(a\tan(\right.}{\tan(x)^4+2\tan(x)^2\cdots}\right.}{4\left(a^4b-2a^3\cdots\right.}\right.$$

input
```
integrate(tan(x)^2/(a+b*cot(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*((a^3*tan(x)^2 + a^2*b)*sqrt(-a + b)*log(-(a^2*tan(x)^4 - 2*(3*a^2 -
4*a*b)*tan(x)^2 + a^2 - 8*a*b + 8*b^2 - 4*(a*tan(x)^3 - (a - 2*b)*tan(x))*
sqrt(-a + b)*sqrt((a*tan(x)^2 + b)/tan(x)^2))/(tan(x)^4 + 2*tan(x)^2 + 1))
 + 4*((a^3 - 2*a^2*b + a*b^2)*tan(x)^3 + (a^2*b - 3*a*b^2 + 2*b^3)*tan(x))
*sqrt((a*tan(x)^2 + b)/tan(x)^2))/(a^4*b - 2*a^3*b^2 + a^2*b^3 + (a^5 - 2*
a^4*b + a^3*b^2)*tan(x)^2), 1/2*((a^3*tan(x)^2 + a^2*b)*sqrt(a - b)*arctan
(2*sqrt(a - b)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x)/(a*tan(x)^2 - a + 2*
b)) + 2*((a^3 - 2*a^2*b + a*b^2)*tan(x)^3 + (a^2*b - 3*a*b^2 + 2*b^3)*tan(
x))*sqrt((a*tan(x)^2 + b)/tan(x)^2))/(a^4*b - 2*a^3*b^2 + a^2*b^3 + (a^5 -
 2*a^4*b + a^3*b^2)*tan(x)^2)]
```

3.53.6 Sympy [F]

$$\int \frac{\tan^2(x)}{(a+b\cot^2(x))^{3/2}}\,dx = \int \frac{\tan^2(x)}{(a+b\cot^2(x))^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(x)**2/(a+b*cot(x)**2)**(3/2),x)
```

output
```
Integral(tan(x)**2/(a + b*cot(x)**2)**(3/2), x)
```

3.53.7 Maxima [F]

$$\int \frac{\tan^2(x)}{(a+b\cot^2(x))^{3/2}}\,dx = \int \frac{\tan(x)^2}{(b\cot(x)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(x)^2/(a+b*cot(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(tan(x)^2/(b*cot(x)^2 + a)^(3/2), x)
```

3.53.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 359 vs. 2(82) = 164.

Time = 0.33 (sec) , antiderivative size = 359, normalized size of antiderivative = 3.90

$$\int \frac{\tan^2(x)}{\left(a+b\cot^2(x)\right)^{3/2}}\,dx = \frac{\left(a^3\log\left(-a-2\sqrt{-a+b}\sqrt{b}+2\,b\right)+a^2\sqrt{-a+b}\sqrt{b}\log\left(-a-2\sqrt{-a+b}\sqrt{b}+\right.\right.}{2\left(a^4\sqrt{-a+b}-a^4\sqrt{b}\right.}$$
$$+\frac{\frac{2\sqrt{-a\cos(x)^2+b\cos(x)^2+a}b^2\cos(x)}{(a^3-a^2b)\left(a\cos(x)^2-b\cos(x)^2-a\right)}-\frac{\log\left(\left(\sqrt{-a+b}\cos(x)-\sqrt{-a\cos(x)^2+b\cos(x)^2+a}\right)^2\right)}{(a-b)\sqrt{-a+b}}-\frac{4\sqrt{-a+b}}{\left(\left(\sqrt{-a+b}\cos(x)-\sqrt{-a\cos(x)^2+b\cos(x)^2}\right.\right.}}{2\,\mathrm{sgn}\left(\sin\left(x\right)\right)}$$

input
```
integrate(tan(x)^2/(a+b*cot(x)^2)^(3/2),x, algorithm="giac")
```

output
```
1/2*(a^3*log(-a - 2*sqrt(-a + b)*sqrt(b) + 2*b) + a^2*sqrt(-a + b)*sqrt(b)
*log(-a - 2*sqrt(-a + b)*sqrt(b) + 2*b) - a^2*b*log(-a - 2*sqrt(-a + b)*sq
rt(b) + 2*b) + 2*a^3 - 4*a^2*b + 2*a*sqrt(-a + b)*b^(3/2) - 2*sqrt(-a + b)
*b^(5/2) + 2*b^3)*sgn(sin(x))/(a^4*sqrt(-a + b) - a^4*sqrt(b) - 2*a^3*sqrt
(-a + b)*b + 2*a^3*b^(3/2) + a^2*sqrt(-a + b)*b^2 - a^2*b^(5/2)) + 1/2*(2*
sqrt(-a*cos(x)^2 + b*cos(x)^2 + a)*b^2*cos(x)/((a^3 - a^2*b)*(a*cos(x)^2 -
 b*cos(x)^2 - a)) - log((sqrt(-a + b)*cos(x) - sqrt(-a*cos(x)^2 + b*cos(x)
^2 + a))^2)/((a - b)*sqrt(-a + b)) - 4*sqrt(-a + b)/(((sqrt(-a + b)*cos(x)
 - sqrt(-a*cos(x)^2 + b*cos(x)^2 + a))^2 - a)*a))/sgn(sin(x))
```

3.53.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^2(x)}{\left(a+b\cot^2(x)\right)^{3/2}}\,dx = \int \frac{\tan(x)^2}{\left(b\cot\left(x\right)^2+a\right)^{3/2}}\,dx$$

input
```
int(tan(x)^2/(a + b*cot(x)^2)^(3/2),x)
```

output
```
int(tan(x)^2/(a + b*cot(x)^2)^(3/2), x)
```

3.54 $\int \frac{\cot^3(x)}{(a+b\cot^2(x))^{5/2}}\,dx$

3.54.1 Optimal result

Integrand size = 17, antiderivative size = 82

$$\int \frac{\cot^3(x)}{(a+b\cot^2(x))^{5/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{(a-b)^{5/2}} + \frac{a}{3(a-b)b\,(a+b\cot^2(x))^{3/2}} + \frac{1}{(a-b)^2\sqrt{a+b\cot^2(x)}}$$

output
```
-arctanh((a+b*cot(x)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(5/2)+1/3*a/(a-b)/b/(a+b*
cot(x)^2)^(3/2)+1/(a-b)^2/(a+b*cot(x)^2)^(1/2)
```

3.54.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.84

$$\int \frac{\cot^3(x)}{(a+b\cot^2(x))^{5/2}}\,dx = \frac{a(a-b)+3b(a+b\cot^2(x))\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{a+b\cot^2(x)}{a-b}\right)}{3(a-b)^2b\,(a+b\cot^2(x))^{3/2}}$$

input
```
Integrate[Cot[x]^3/(a + b*Cot[x]^2)^(5/2),x]
```

output
```
(a*(a - b) + 3*b*(a + b*Cot[x]^2)*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*C
ot[x]^2)/(a - b)])/(3*(a - b)^2*b*(a + b*Cot[x]^2)^(3/2))
```

3.54.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.17, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {3042, 25, 4153, 25, 354, 87, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(x)}{\left(a+b\cot^2(x)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\tan\left(x+\frac{\pi}{2}\right)^3}{\left(a+b\tan\left(x+\frac{\pi}{2}\right)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\tan\left(x+\frac{\pi}{2}\right)^3}{\left(b\tan\left(x+\frac{\pi}{2}\right)^2+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4153}$$

$$\int -\frac{\cot^3(x)}{\left(\cot^2(x)+1\right)\left(a+b\cot^2(x)\right)^{5/2}}d\cot(x)$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\cot^3(x)}{\left(\cot^2(x)+1\right)\left(b\cot^2(x)+a\right)^{5/2}}d\cot(x)$$

$$\Big\downarrow \text{354}$$

$$-\frac{1}{2}\int \frac{\cot^2(x)}{\left(\cot^2(x)+1\right)\left(b\cot^2(x)+a\right)^{5/2}}d\cot^2(x)$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{2}\left(\frac{\int \frac{1}{\left(\cot^2(x)+1\right)\left(b\cot^2(x)+a\right)^{3/2}}d\cot^2(x)}{a-b}+\frac{2a}{3b(a-b)\left(a+b\cot^2(x)\right)^{3/2}}\right)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2}\left(\frac{\frac{\int \frac{1}{\left(\cot^2(x)+1\right)\sqrt{b\cot^2(x)+a}}d\cot^2(x)}{a-b}+\frac{2}{(a-b)\sqrt{a+b\cot^2(x)}}}{a-b}+\frac{2a}{3b(a-b)\left(a+b\cot^2(x)\right)^{3/2}}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{\frac{2\int \frac{1}{\frac{\cot^4(x)}{b}-\frac{a}{b}+1}d\sqrt{b\cot^2(x)+a}}{b(a-b)}+\frac{2}{(a-b)\sqrt{a+b\cot^2(x)}}}{a-b}+\frac{2a}{3b(a-b)\left(a+b\cot^2(x)\right)^{3/2}}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{\frac{2}{(a-b)\sqrt{a+b\cot^2(x)}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}}}{a-b}+\frac{2a}{3b(a-b)\left(a+b\cot^2(x)\right)^{3/2}}\right)$$

input
```
Int[Cot[x]^3/(a + b*Cot[x]^2)^(5/2),x]
```

output
```
((2*a)/(3*(a - b)*b*(a + b*Cot[x]^2)^(3/2)) + ((-2*ArcTanh[Sqrt[a + b*Cot[
x]^2]/Sqrt[a - b]])/(a - b)^(3/2) + 2/((a - b)*Sqrt[a + b*Cot[x]^2]))/(a -
 b))/2
```

3.54.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.54. $\int \frac{\cot^3(x)}{(a+b\cot^2(x))^{5/2}}\,dx$

3.54.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.07

method	result	size
derivativedivides	$\frac{1}{3b\left(a+b\cot(x)^2\right)^{\frac{3}{2}}}+\frac{1}{3(a-b)\left(a+b\cot(x)^2\right)^{\frac{3}{2}}}+\frac{1}{(a-b)^2\sqrt{a+b\cot(x)^2}}+\frac{\arctan\left(\frac{\sqrt{a+b\cot(x)^2}}{\sqrt{-a+b}}\right)}{(a-b)^2\sqrt{-a+b}}$	88
default	$\frac{1}{3b\left(a+b\cot(x)^2\right)^{\frac{3}{2}}}+\frac{1}{3(a-b)\left(a+b\cot(x)^2\right)^{\frac{3}{2}}}+\frac{1}{(a-b)^2\sqrt{a+b\cot(x)^2}}+\frac{\arctan\left(\frac{\sqrt{a+b\cot(x)^2}}{\sqrt{-a+b}}\right)}{(a-b)^2\sqrt{-a+b}}$	88

input
```
int(cot(x)^3/(a+b*cot(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/b/(a+b*cot(x)^2)^(3/2)+1/3/(a-b)/(a+b*cot(x)^2)^(3/2)+1/(a-b)^2/(a+b*c
ot(x)^2)^(1/2)+1/(a-b)^2/(-a+b)^(1/2)*arctan((a+b*cot(x)^2)^(1/2)/(-a+b)^(
1/2))
```

3.54.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. $2(70) = 140$.

Time = 0.31 (sec) , antiderivative size = 698, normalized size of antiderivative = 8.51

$$\int \frac{\cot^3(x)}{(a+b\cot^2(x))^{5/2}}\,dx = \left[\frac{3\left(a^2b+2\,ab^2+b^3+\left(a^2b-2\,ab^2+b^3\right)\cos\left(2\,x\right)^2-2\left(a^2b-b^3\right)\cos\left(2\,x\right)\right)\sqrt{a-}}{6\left(a^5b-a^4b^2-2\,a^3b^3+2\,a\right.}\right.$$

$$-\frac{3\left(a^2b+2\,ab^2+b^3+\left(a^2b-2\,ab^2+b^3\right)\cos\left(2\,x\right)^2-2\left(a^2b-b^3\right)\cos\left(2\,x\right)\right)\sqrt{-a+b}\arctan\left(-\frac{\sqrt{-a+b}\sqrt{\frac{(a-b)c}{co}}}{a-b}\right.}{3\left(a^5b-a^4b^2-2\,a^3b^3+2\,a^2b^4+ab^5-b^6+\left(a^5b-5\,a^4b^2+10\,a^3b^3-1\right.\right.}$$

input
```
integrate(cot(x)^3/(a+b*cot(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/6*(3*(a^2*b + 2*a*b^2 + b^3 + (a^2*b - 2*a*b^2 + b^3)*cos(2*x)^2 - 2*(a
^2*b - b^3)*cos(2*x))*sqrt(a - b)*log(sqrt(a - b)*sqrt(((a - b)*cos(2*x) -
 a - b)/(cos(2*x) - 1))*(cos(2*x) - 1) - (a - b)*cos(2*x) + a) + 2*(a^3 +
a^2*b + a*b^2 - 3*b^3 + (a^3 + a^2*b - 5*a*b^2 + 3*b^3)*cos(2*x)^2 - 2*(a^
3 + a^2*b - 2*a*b^2)*cos(2*x))*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) -
 1)))/(a^5*b - a^4*b^2 - 2*a^3*b^3 + 2*a^2*b^4 + a*b^5 - b^6 + (a^5*b - 5*
a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*cos(2*x)^2 - 2*(a^5*b -
 3*a^4*b^2 + 2*a^3*b^3 + 2*a^2*b^4 - 3*a*b^5 + b^6)*cos(2*x)), -1/3*(3*(a^
2*b + 2*a*b^2 + b^3 + (a^2*b - 2*a*b^2 + b^3)*cos(2*x)^2 - 2*(a^2*b - b^3)
*cos(2*x))*sqrt(-a + b)*arctan(-sqrt(-a + b)*sqrt(((a - b)*cos(2*x) - a -
b)/(cos(2*x) - 1))/(a - b)) - (a^3 + a^2*b + a*b^2 - 3*b^3 + (a^3 + a^2*b
- 5*a*b^2 + 3*b^3)*cos(2*x)^2 - 2*(a^3 + a^2*b - 2*a*b^2)*cos(2*x))*sqrt((
(a - b)*cos(2*x) - a - b)/(cos(2*x) - 1)))/(a^5*b - a^4*b^2 - 2*a^3*b^3 +
2*a^2*b^4 + a*b^5 - b^6 + (a^5*b - 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5
*a*b^5 - b^6)*cos(2*x)^2 - 2*(a^5*b - 3*a^4*b^2 + 2*a^3*b^3 + 2*a^2*b^4 -
3*a*b^5 + b^6)*cos(2*x))]
```

3.54.6 Sympy [F]

$$\int \frac{\cot^3(x)}{(a+b\cot^2(x))^{5/2}}\,dx = \int \frac{\cot^3(x)}{(a+b\cot^2(x))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(x)**3/(a+b*cot(x)**2)**(5/2),x)
```

output
```
Integral(cot(x)**3/(a + b*cot(x)**2)**(5/2), x)
```

3.54.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cot^3(x)}{(a+b\cot^2(x))^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cot(x)^3/(a+b*cot(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for m
ore detail
```

3.54.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(70) = 140$.

Time = 0.29 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.67

$$\int \frac{\cot^3(x)}{\left(a+b\cot^2(x)\right)^{5/2}}\,dx = -\frac{\log(|b|)\,\mathrm{sgn}(\sin(x))}{2\left(\sqrt{a-b}a^2-2\sqrt{a-b}ab+\sqrt{a-b}b^2\right)} + \frac{\frac{\left(\frac{(a^3+a^2b-5\,ab^2+3\,b^3)\sin(x)^2}{a^3b-3\,a^2b^2+3\,ab^3-b^4}+\frac{3\,(ab^2-b^3)}{a^3b-3\,a^2b^2+3\,ab^3-b^4}\right)\sin(x)}{\left(a\sin(x)^2-b\sin(x)^2+b\right)^{\frac{3}{2}}}+\frac{3\,\log\left(\left|-\sqrt{a-b}\sin(x)+\sqrt{a\sin(x)^2-b\sin(x)^2+b}\right|\right)}{(a^2-2\,ab+b^2)\sqrt{a-b}}}{3\,\mathrm{sgn}(\sin(x))}$$

input
```
integrate(cot(x)^3/(a+b*cot(x)^2)^(5/2),x, algorithm="giac")
```

output
```
-1/2*log(abs(b))*sgn(sin(x))/(sqrt(a - b)*a^2 - 2*sqrt(a - b)*a*b + sqrt(a
 - b)*b^2) + 1/3*(((a^3 + a^2*b - 5*a*b^2 + 3*b^3)*sin(x)^2/(a^3*b - 3*a^2
*b^2 + 3*a*b^3 - b^4) + 3*(a*b^2 - b^3)/(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4
))*sin(x)/(a*sin(x)^2 - b*sin(x)^2 + b)^(3/2) + 3*log(abs(-sqrt(a - b)*sin
(x) + sqrt(a*sin(x)^2 - b*sin(x)^2 + b)))/((a^2 - 2*a*b + b^2)*sqrt(a - b)
))/sgn(sin(x))
```

3.54.9 Mupad [B] (verification not implemented)

Time = 16.21 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.07

$$\int \frac{\cot^3(x)}{\left(a+b\cot^2(x)\right)^{5/2}}\,dx = \frac{\frac{a}{3\,(a-b)}+\frac{b\,\left(b\cot(x)^2+a\right)}{(a-b)^2}}{b\,\left(b\cot(x)^2+a\right)^{3/2}} - \frac{\mathrm{atanh}\left(\frac{\sqrt{b\cot(x)^2+a}\,(2\,a^2-4\,a\,b+2\,b^2)}{2\,(a-b)^{5/2}}\right)}{(a-b)^{5/2}}$$

input
```
int(cot(x)^3/(a + b*cot(x)^2)^(5/2),x)
```

output

```
(a/(3*(a - b)) + (b*(a + b*cot(x)^2))/(a - b)^2)/(b*(a + b*cot(x)^2)^(3/2)
) - atanh(((a + b*cot(x)^2)^(1/2)*(2*a^2 - 4*a*b + 2*b^2))/(2*(a - b)^(5/2
)))/(a - b)^(5/2)
```

3.54. $\int \frac{\cot^3(x)}{(a+b\cot^2(x))^{5/2}}\,dx$

3.55 $\int \frac{\cot^2(x)}{(a+b\cot^2(x))^{5/2}}\,dx$

3.55.1 Optimal result

Integrand size = 17, antiderivative size = 94

$$\int \frac{\cot^2(x)}{(a+b\cot^2(x))^{5/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{a-b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{(a-b)^{5/2}} - \frac{\cot(x)}{3(a-b)\left(a+b\cot^2(x)\right)^{3/2}} - \frac{(2a+b)\cot(x)}{3a(a-b)^2\sqrt{a+b\cot^2(x)}}$$

output
```
arctan(cot(x)*(a-b)^(1/2)/(a+b*cot(x)^2)^(1/2))/(a-b)^(5/2)-1/3*cot(x)/(a-
b)/(a+b*cot(x)^2)^(3/2)-1/3*(2*a+b)*cot(x)/a/(a-b)^2/(a+b*cot(x)^2)^(1/2)
```

3.55.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 6.34 (sec) , antiderivative size = 197, normalized size of antiderivative = 2.10

$$\int \frac{\cot^2(x)}{(a+b\cot^2(x))^{5/2}}\,dx = \frac{\cos(x)\left(-12(a-b)^3\cos^3(x)\cot(x)\left(a+b\cot^2(x)\right)\text{Hypergeometric2F1}\left(2,2,\frac{9}{2},\right.\right.}{}$$

input
```
Integrate[Cot[x]^2/(a + b*Cot[x]^2)^(5/2),x]
```

output
```
(Cos[x]*(-12*(a - b)^3*Cos[x]^3*Cot[x]*(a + b*Cot[x]^2)*Hypergeometric2F1[
2, 2, 9/2, ((a - b)*Cos[x]^2)/a] - (35*a*(5*a + 2*b*Cot[x]^2)*Sin[x]*(a*((
a - 4*b)*Csc[x]^2 - 3*a*Sec[x]^2)*Sqrt[((a - b)*Cos[x]^2*(a + b*Cot[x]^2)*
Sin[x]^2)/a^2] + 3*ArcSin[Sqrt[((a - b)*Cos[x]^2)/a]]*(b*Cot[x] + a*Tan[x]
)^2))/Sqrt[((a - b)*Cos[x]^2*(a + b*Cot[x]^2)*Sin[x]^2)/a^2]))/(315*a^3*(a
 - b)^2*(a + b*Cot[x]^2)^(3/2))
```

3.55.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {3042, 4153, 373, 402, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(x)}{\left(a+b\cot^2(x)\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\tan\left(x+\frac{\pi}{2}\right)^2}{\left(a+b\tan\left(x+\frac{\pi}{2}\right)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 4153}$$

$$-\int \frac{\cot^2(x)}{\left(\cot^2(x)+1\right)\left(b\cot^2(x)+a\right)^{5/2}}d\cot(x)$$

$$\Big\downarrow \text{ 373}$$

$$\frac{\int \frac{1-2\cot^2(x)}{\left(\cot^2(x)+1\right)\left(b\cot^2(x)+a\right)^{3/2}}d\cot(x)}{3(a-b)}-\frac{\cot(x)}{3(a-b)\left(a+b\cot^2(x)\right)^{3/2}}$$

$$\Big\downarrow \text{ 402}$$

$$\frac{\frac{\int \frac{3a}{\left(\cot^2(x)+1\right)\sqrt{b\cot^2(x)+a}}d\cot(x)}{a(a-b)}-\frac{(2a+b)\cot(x)}{a(a-b)\sqrt{a+b\cot^2(x)}}}{3(a-b)}-\frac{\cot(x)}{3(a-b)\left(a+b\cot^2(x)\right)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{3\int \frac{1}{\left(\cot^2(x)+1\right)\sqrt{b\cot^2(x)+a}}d\cot(x)}{a-b}-\frac{(2a+b)\cot(x)}{a(a-b)\sqrt{a+b\cot^2(x)}}}{3(a-b)}-\frac{\cot(x)}{3(a-b)\left(a+b\cot^2(x)\right)^{3/2}}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{3\int \frac{1}{1-\frac{(b-a)\cot^2(x)}{b\cot^2(x)+a}}d\frac{\cot(x)}{\sqrt{b\cot^2(x)+a}}}{a-b}-\frac{(2a+b)\cot(x)}{a(a-b)\sqrt{a+b\cot^2(x)}}}{3(a-b)}-\frac{\cot(x)}{3(a-b)\left(a+b\cot^2(x)\right)^{3/2}}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{3\arctan\left(\frac{\sqrt{a-b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{(a-b)^{3/2}}-\frac{(2a+b)\cot(x)}{a(a-b)\sqrt{a+b\cot^2(x)}}}{3(a-b)}-\frac{\cot(x)}{3(a-b)\left(a+b\cot^2(x)\right)^{3/2}}$$

input
```
Int[Cot[x]^2/(a + b*Cot[x]^2)^(5/2),x]
```

output
```
-1/3*Cot[x]/((a - b)*(a + b*Cot[x]^2)^(3/2)) + ((3*ArcTan[(Sqrt[a - b]*Cot
[x])/Sqrt[a + b*Cot[x]^2]])/(a - b)^(3/2) - ((2*a + b)*Cot[x])/(a*(a - b)*
Sqrt[a + b*Cot[x]^2]))/(3*(a - b))
```

3.55.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

3.55. $\int \frac{\cot^2(x)}{\left(a+b\cot^2(x)\right)^{5/2}}\,dx$

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.55.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.71

method	result
derivativedivides	$-\frac{\cot(x)}{3a\left(a+b\cot(x)^2\right)^{\frac{3}{2}}}-\frac{2\cot(x)}{3a^2\sqrt{a+b\cot(x)^2}}-\frac{b\left(\frac{\cot(x)}{3a\left(a+b\cot(x)^2\right)^{\frac{3}{2}}}+\frac{2\cot(x)}{3a^2\sqrt{a+b\cot(x)^2}}\right)}{a-b}-\frac{b\cot(x)}{(a-b)^2a\sqrt{a+b\cot(x)^2}}$
default	$-\frac{\cot(x)}{3a\left(a+b\cot(x)^2\right)^{\frac{3}{2}}}-\frac{2\cot(x)}{3a^2\sqrt{a+b\cot(x)^2}}-\frac{b\left(\frac{\cot(x)}{3a\left(a+b\cot(x)^2\right)^{\frac{3}{2}}}+\frac{2\cot(x)}{3a^2\sqrt{a+b\cot(x)^2}}\right)}{a-b}-\frac{b\cot(x)}{(a-b)^2a\sqrt{a+b\cot(x)^2}}$

input
```
int(cot(x)^2/(a+b*cot(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*cot(x)/a/(a+b*cot(x)^2)^(3/2)-2/3/a^2*cot(x)/(a+b*cot(x)^2)^(1/2)-1/(
a-b)*b*(1/3*cot(x)/a/(a+b*cot(x)^2)^(3/2)+2/3/a^2*cot(x)/(a+b*cot(x)^2)^(1
/2))-1/(a-b)^2*b*cot(x)/a/(a+b*cot(x)^2)^(1/2)+1/(a-b)^3*(b^4*(a-b))^(1/2)
/b^2*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*cot(x)^2)^(1/2)*cot(x))
```

3.55.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 339 vs. $2(80) = 160$.

Time = 0.39 (sec) , antiderivative size = 720, normalized size of antiderivative = 7.66

$$\int \frac{\cot^2(x)}{(a+b\cot^2(x))^{5/2}}\,dx = \left[-\frac{3\left(a^3+2\,a^2b+ab^2+(a^3-2\,a^2b+ab^2)\cos{(2\,x)}^2-2\,(a^3-ab^2)\cos{(2\,x)}\right)\sqrt{-}}{12\,(a^6}\right.$$

input
```
integrate(cot(x)^2/(a+b*cot(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
[-1/12*(3*(a^3 + 2*a^2*b + a*b^2 + (a^3 - 2*a^2*b + a*b^2)*cos(2*x)^2 - 2*
(a^3 - a*b^2)*cos(2*x))*sqrt(-a + b)*log(-2*(a^2 - 2*a*b + b^2)*cos(2*x)^2
 + 2*((a - b)*cos(2*x) - b)*sqrt(-a + b)*sqrt(((a - b)*cos(2*x) - a - b)/(
cos(2*x) - 1))*sin(2*x) + a^2 - 2*b^2 + 4*(a*b - b^2)*cos(2*x)) + 4*(3*a^3
 - a^2*b - a*b^2 - b^3 - (3*a^3 - 5*a^2*b + a*b^2 + b^3)*cos(2*x))*sqrt(((
a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))*sin(2*x))/(a^6 - a^5*b - 2*a^4*b^
2 + 2*a^3*b^3 + a^2*b^4 - a*b^5 + (a^6 - 5*a^5*b + 10*a^4*b^2 - 10*a^3*b^3
 + 5*a^2*b^4 - a*b^5)*cos(2*x)^2 - 2*(a^6 - 3*a^5*b + 2*a^4*b^2 + 2*a^3*b^
3 - 3*a^2*b^4 + a*b^5)*cos(2*x)), 1/6*(3*(a^3 + 2*a^2*b + a*b^2 + (a^3 - 2
*a^2*b + a*b^2)*cos(2*x)^2 - 2*(a^3 - a*b^2)*cos(2*x))*sqrt(a - b)*arctan(
-sqrt(a - b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))*sin(2*x)/((a
- b)*cos(2*x) - b)) - 2*(3*a^3 - a^2*b - a*b^2 - b^3 - (3*a^3 - 5*a^2*b +
a*b^2 + b^3)*cos(2*x))*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))*sin
(2*x))/(a^6 - a^5*b - 2*a^4*b^2 + 2*a^3*b^3 + a^2*b^4 - a*b^5 + (a^6 - 5*a
^5*b + 10*a^4*b^2 - 10*a^3*b^3 + 5*a^2*b^4 - a*b^5)*cos(2*x)^2 - 2*(a^6 -
3*a^5*b + 2*a^4*b^2 + 2*a^3*b^3 - 3*a^2*b^4 + a*b^5)*cos(2*x))]
```

3.55.6 Sympy [F]

$$\int \frac{\cot^2(x)}{(a+b\cot^2(x))^{5/2}}\,dx = \int \frac{\cot^2(x)}{(a+b\cot^2(x))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(x)**2/(a+b*cot(x)**2)**(5/2),x)
```

output
```
Integral(cot(x)**2/(a + b*cot(x)**2)**(5/2), x)
```

3.55.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cot^2(x)}{(a+b\cot^2(x))^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cot(x)^2/(a+b*cot(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

3.55.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 281 vs. $2(80) = 160$.

Time = 0.34 (sec) , antiderivative size = 281, normalized size of antiderivative = 2.99

$$\int \frac{\cot^2(x)}{(a+b\cot^2(x))^{5/2}}\,dx = \frac{\left(3\,a\sqrt{b}\log\left(\left|-\sqrt{-a+b}+\sqrt{b}\right|\right)+2\,a\sqrt{-a+b}+\sqrt{-a+b}b\right)\mathrm{sgn}\left(\sin\left(x\right)\right)}{3\left(a^3\sqrt{-a+b}\sqrt{b}-2\,a^2\sqrt{-a+b}b^{\frac{3}{2}}+a\sqrt{-a+b}b^{\frac{5}{2}}\right)} - \frac{\frac{\left(\frac{(3\,a^3-5\,a^2b+ab^2+b^3)\cos(x)^2}{a^4-3\,a^3b+3\,a^2b^2-ab^3}-\frac{3\,(a^3-a^2b)}{a^4-3\,a^3b+3\,a^2b^2-ab^3}\right)\cos(x)}{(a\cos(x)^2-b\cos(x)^2-a)\sqrt{-a\cos(x)^2+b\cos(x)^2+a}}+\frac{3\,\log\left(\left|-\sqrt{-a+b}\cos(x)+\sqrt{-a\cos(x)^2+b\cos(x)^2+a}\right|\right)}{(a^2-2\,ab+b^2)\sqrt{-a+b}}}{3\,\mathrm{sgn}\left(\sin\left(x\right)\right)}$$

input
```
integrate(cot(x)^2/(a+b*cot(x)^2)^(5/2),x, algorithm="giac")
```

output
```
1/3*(3*a*sqrt(b)*log(abs(-sqrt(-a + b) + sqrt(b))) + 2*a*sqrt(-a + b) + sq
rt(-a + b)*b)*sgn(sin(x))/(a^3*sqrt(-a + b)*sqrt(b) - 2*a^2*sqrt(-a + b)*b
^(3/2) + a*sqrt(-a + b)*b^(5/2)) - 1/3*(((3*a^3 - 5*a^2*b + a*b^2 + b^3)*c
os(x)^2/(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3) - 3*(a^3 - a^2*b)/(a^4 - 3*a^3
*b + 3*a^2*b^2 - a*b^3))*cos(x)/((a*cos(x)^2 - b*cos(x)^2 - a)*sqrt(-a*cos
(x)^2 + b*cos(x)^2 + a)) + 3*log(abs(-sqrt(-a + b)*cos(x) + sqrt(-a*cos(x)
^2 + b*cos(x)^2 + a)))/((a^2 - 2*a*b + b^2)*sqrt(-a + b)))/sgn(sin(x))
```

3.55.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^2(x)}{(a+b\cot^2(x))^{5/2}}\,dx = \int \frac{\cot(x)^2}{\left(b\cot(x)^2+a\right)^{5/2}}\,dx$$

input
```
int(cot(x)^2/(a + b*cot(x)^2)^(5/2),x)
```

output
```
int(cot(x)^2/(a + b*cot(x)^2)^(5/2), x)
```

3.56 $\int \frac{\cot(x)}{\left(a+b\cot^2(x)\right)^{5/2}}\,dx$

3.56.1 Optimal result

Integrand size = 15, antiderivative size = 78

$$\int \frac{\cot(x)}{\left(a+b\cot^2(x)\right)^{5/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{(a-b)^{5/2}} - \frac{1}{3(a-b)\left(a+b\cot^2(x)\right)^{3/2}} - \frac{1}{(a-b)^2\sqrt{a+b\cot^2(x)}}$$

output
```
arctanh((a+b*cot(x)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(5/2)-1/3/(a-b)/(a+b*cot(x
)^2)^(3/2)-1/(a-b)^2/(a+b*cot(x)^2)^(1/2)
```

3.56.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.60

$$\int \frac{\cot(x)}{\left(a+b\cot^2(x)\right)^{5/2}}\,dx = -\frac{\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\frac{a+b\cot^2(x)}{a-b}\right)}{3(a-b)\left(a+b\cot^2(x)\right)^{3/2}}$$

input
```
Integrate[Cot[x]/(a + b*Cot[x]^2)^(5/2),x]
```

output
```
-1/3*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Cot[x]^2)/(a - b)]/((a - b)*(
a + b*Cot[x]^2)^(3/2))
```

3.56.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.19, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 25, 4153, 25, 353, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(x)}{\left(a+b\cot^2(x)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\tan\left(x+\frac{\pi}{2}\right)}{\left(a+b\tan\left(x+\frac{\pi}{2}\right)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\tan\left(x+\frac{\pi}{2}\right)}{\left(b\tan\left(x+\frac{\pi}{2}\right)^2+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4153}$$

$$\int -\frac{\cot(x)}{\left(\cot^2(x)+1\right)\left(a+b\cot^2(x)\right)^{5/2}}d\cot(x)$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\cot(x)}{\left(\cot^2(x)+1\right)\left(b\cot^2(x)+a\right)^{5/2}}d\cot(x)$$

$$\Big\downarrow \text{353}$$

$$-\frac{1}{2}\int \frac{1}{\left(\cot^2(x)+1\right)\left(b\cot^2(x)+a\right)^{5/2}}d\cot^2(x)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2}\left(-\frac{\int \frac{1}{\left(\cot^2(x)+1\right)\left(b\cot^2(x)+a\right)^{3/2}}d\cot^2(x)}{a-b}-\frac{2}{3(a-b)\left(a+b\cot^2(x)\right)^{3/2}}\right)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2}\left(-\frac{\frac{\int \frac{1}{\left(\cot^2(x)+1\right)\sqrt{b\cot^2(x)+a}}d\cot^2(x)}{a-b}+\frac{2}{(a-b)\sqrt{a+b\cot^2(x)}}}{a-b}-\frac{2}{3(a-b)\left(a+b\cot^2(x)\right)^{3/2}}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(-\frac{\frac{2\int \frac{1}{\frac{\cot^4(x)}{b}-\frac{a}{b}+1}d\sqrt{b\cot^2(x)+a}}{b(a-b)}+\frac{2}{(a-b)\sqrt{a+b\cot^2(x)}}}{a-b}-\frac{2}{3(a-b)\left(a+b\cot^2(x)\right)^{3/2}}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(-\frac{\frac{2}{(a-b)\sqrt{a+b\cot^2(x)}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}}}{a-b}-\frac{2}{3(a-b)\left(a+b\cot^2(x)\right)^{3/2}}\right)$$

input
```
Int[Cot[x]/(a + b*Cot[x]^2)^(5/2),x]
```

output
```
(-2/(3*(a - b)*(a + b*Cot[x]^2)^(3/2)) - ((-2*ArcTanh[Sqrt[a + b*Cot[x]^2]
/Sqrt[a - b]])/(a - b)^(3/2) + 2/((a - b)*Sqrt[a + b*Cot[x]^2]))/(a - b))/
2
```

3.56.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.56.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$-\frac{1}{3(a-b)\left(a+b\cot(x)^2\right)^{\frac{3}{2}}}-\frac{1}{(a-b)^2\sqrt{a+b\cot(x)^2}}-\frac{\arctan\left(\frac{\sqrt{a+b\cot(x)^2}}{\sqrt{-a+b}}\right)}{(a-b)^2\sqrt{-a+b}}$	75
default	$-\frac{1}{3(a-b)\left(a+b\cot(x)^2\right)^{\frac{3}{2}}}-\frac{1}{(a-b)^2\sqrt{a+b\cot(x)^2}}-\frac{\arctan\left(\frac{\sqrt{a+b\cot(x)^2}}{\sqrt{-a+b}}\right)}{(a-b)^2\sqrt{-a+b}}$	75

input
```
int(cot(x)/(a+b*cot(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/(a-b)/(a+b*cot(x)^2)^(3/2)-1/(a-b)^2/(a+b*cot(x)^2)^(1/2)-1/(a-b)^2/(
-a+b)^(1/2)*arctan((a+b*cot(x)^2)^(1/2)/(-a+b)^(1/2))
```

3.56.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 307 vs. $2(66) = 132$.

Time = 0.34 (sec) , antiderivative size = 627, normalized size of antiderivative = 8.04

$$\int \frac{\cot(x)}{\left(a+b\cot^2(x)\right)^{5/2}}\,dx = \left[\frac{3\left(\left(a^2-2\,ab+b^2\right)\cos\left(2\,x\right)^2+a^2+2\,ab+b^2-2\left(a^2-b^2\right)\cos\left(2\,x\right)\right)\sqrt{a-b}\log}{6\left(a^5-a^4b-2\,a^3b^2+2\,a^2b^3+ab^4\right.}\right.$$

input
```
integrate(cot(x)/(a+b*cot(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/6*(3*((a^2 - 2*a*b + b^2)*cos(2*x)^2 + a^2 + 2*a*b + b^2 - 2*(a^2 - b^2
)*cos(2*x))*sqrt(a - b)*log(-sqrt(a - b)*sqrt(((a - b)*cos(2*x) - a - b)/(
cos(2*x) - 1))*(cos(2*x) - 1) - (a - b)*cos(2*x) + a) - 4*(2*(a^2 - 2*a*b
+ b^2)*cos(2*x)^2 + 2*a^2 - a*b - b^2 - (4*a^2 - 5*a*b + b^2)*cos(2*x))*sq
rt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1)))/(a^5 - a^4*b - 2*a^3*b^2 +
2*a^2*b^3 + a*b^4 - b^5 + (a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b
^4 - b^5)*cos(2*x)^2 - 2*(a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4
+ b^5)*cos(2*x)), 1/3*(3*((a^2 - 2*a*b + b^2)*cos(2*x)^2 + a^2 + 2*a*b + b
^2 - 2*(a^2 - b^2)*cos(2*x))*sqrt(-a + b)*arctan(-sqrt(-a + b)*sqrt(((a -
b)*cos(2*x) - a - b)/(cos(2*x) - 1))/(a - b)) - 2*(2*(a^2 - 2*a*b + b^2)*c
os(2*x)^2 + 2*a^2 - a*b - b^2 - (4*a^2 - 5*a*b + b^2)*cos(2*x))*sqrt(((a -
 b)*cos(2*x) - a - b)/(cos(2*x) - 1)))/(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^
3 + a*b^4 - b^5 + (a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5
)*cos(2*x)^2 - 2*(a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^5)*c
os(2*x))]
```

3.56.6 Sympy [A] (verification not implemented)

Time = 7.39 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.41

$$\int \frac{\cot(x)}{(a+b\cot^2(x))^{5/2}}\,dx = -\begin{cases} \frac{2\left(\frac{b}{6(a-b)\left(a+b\cot^2(x)\right)^{\frac{3}{2}}}+\frac{b}{2(a-b)^2\sqrt{a+b\cot^2(x)}}+\frac{b\,\mathrm{atan}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{-a+b}}\right)}{2\sqrt{-a+b}(a-b)^2}\right)}{b} & \text{for } b\neq 0 \\ \begin{cases} \tilde{\infty}\cot^2(x) & \text{for } a^{\frac{5}{2}}=0 \\ \frac{\log\left(2a^{\frac{5}{2}}\cot^2(x)+2a^{\frac{5}{2}}\right)}{2a^{\frac{5}{2}}} & \text{otherwise} \end{cases} & \text{otherwise} \end{cases}$$

input
```
integrate(cot(x)/(a+b*cot(x)**2)**(5/2),x)
```

output
```
-Piecewise((2*(b/(6*(a - b)*(a + b*cot(x)**2)**(3/2)) + b/(2*(a - b)**2*sq
rt(a + b*cot(x)**2)) + b*atan(sqrt(a + b*cot(x)**2)/sqrt(-a + b))/(2*sqrt(
-a + b)*(a - b)**2))/b, Ne(b, 0)), (Piecewise((zoo*cot(x)**2, Eq(a**(5/2),
 0)), (log(2*a**(5/2)*cot(x)**2 + 2*a**(5/2))/(2*a**(5/2)), True)), True))
```

3.56.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cot(x)}{(a+b\cot^2(x))^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cot(x)/(a+b*cot(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for m
ore detail
```

3.56.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(66) = 132$.

Time = 0.32 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.76

$$\int \frac{\cot(x)}{(a+b\cot^2(x))^{5/2}}\,dx = \frac{\log(|b|)\operatorname{sgn}(\sin(x))}{2\left(\sqrt{a-b}a^2-2\sqrt{a-b}ab+\sqrt{a-b}b^2\right)} - \frac{\frac{\left(\frac{4\left(a^2b-2ab^2+b^3\right)\sin(x)^2}{a^3b-3a^2b^2+3ab^3-b^4}+\frac{3\left(ab^2-b^3\right)}{a^3b-3a^2b^2+3ab^3-b^4}\right)\sin(x)}{\left(a\sin(x)^2-b\sin(x)^2+b\right)^{\frac{3}{2}}}+\frac{3\log\left(\left|-\sqrt{a-b}\sin(x)+\sqrt{a\sin(x)^2-b\sin(x)^2+b}\right|\right)}{(a^2-2ab+b^2)\sqrt{a-b}}}{3\operatorname{sgn}(\sin(x))}$$

input
```
integrate(cot(x)/(a+b*cot(x)^2)^(5/2),x, algorithm="giac")
```

output
```
1/2*log(abs(b))*sgn(sin(x))/(sqrt(a - b)*a^2 - 2*sqrt(a - b)*a*b + sqrt(a
- b)*b^2) - 1/3*((4*(a^2*b - 2*a*b^2 + b^3)*sin(x)^2/(a^3*b - 3*a^2*b^2 +
3*a*b^3 - b^4) + 3*(a*b^2 - b^3)/(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4))*sin(
x)/(a*sin(x)^2 - b*sin(x)^2 + b)^(3/2) + 3*log(abs(-sqrt(a - b)*sin(x) + s
qrt(a*sin(x)^2 - b*sin(x)^2 + b)))/((a^2 - 2*a*b + b^2)*sqrt(a - b)))/sgn(
sin(x))
```

3.56.9 Mupad [B] (verification not implemented)

Time = 17.27 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.05

$$\int \frac{\cot(x)}{(a+b\cot^2(x))^{5/2}}\,dx = \frac{\operatorname{atanh}\left(\frac{\sqrt{b\cot(x)^2+a}\,(2a^2-4ab+2b^2)}{2(a-b)^{5/2}}\right)}{(a-b)^{5/2}} - \frac{\frac{1}{3(a-b)}+\frac{b\cot(x)^2+a}{(a-b)^2}}{\left(b\cot(x)^2+a\right)^{3/2}}$$

input
```
int(cot(x)/(a + b*cot(x)^2)^(5/2),x)
```

output
```
atanh(((a + b*cot(x)^2)^(1/2)*(2*a^2 - 4*a*b + 2*b^2))/(2*(a - b)^(5/2)))/
(a - b)^(5/2) - (1/(3*(a - b)) + (a + b*cot(x)^2)/(a - b)^2)/(a + b*cot(x)
^2)^(3/2)
```

3.57 $\int \frac{\tan(x)}{\left(a+b\cot^2(x)\right)^{5/2}}\,dx$

3.57.1 Optimal result

Integrand size = 15, antiderivative size = 118

$$\int \frac{\tan(x)}{\left(a+b\cot^2(x)\right)^{5/2}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{(a-b)^{5/2}}+\frac{b}{3a(a-b)\left(a+b\cot^2(x)\right)^{3/2}}+\frac{(2a-b)b}{a^2(a-b)^2\sqrt{a+b\cot^2(x)}}$$

output
```
arctanh((a+b*cot(x)^2)^(1/2)/a^(1/2))/a^(5/2)-arctanh((a+b*cot(x)^2)^(1/2)
/(a-b)^(1/2))/(a-b)^(5/2)+1/3*b/a/(a-b)/(a+b*cot(x)^2)^(3/2)+(2*a-b)*b/a^2
/(a-b)^2/(a+b*cot(x)^2)^(1/2)
```

3.57.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.66

$$\int \frac{\tan(x)}{\left(a+b\cot^2(x)\right)^{5/2}}\,dx=\frac{a\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\frac{a+b\cot^2(x)}{a-b}\right)+(-a+b)\,\text{Hypergeometric2F1}\left(-\right.}{3a(a-b)\left(a+b\cot^2(x)\right)^{3/2}}$$

input
```
Integrate[Tan[x]/(a + b*Cot[x]^2)^(5/2),x]
```

output
```
(a*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Cot[x]^2)/(a - b)] + (-a + b)*H
ypergeometric2F1[-3/2, 1, -1/2, 1 + (b*Cot[x]^2)/a])/(3*a*(a - b)*(a + b*C
ot[x]^2)^(3/2))
```

3.57.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.35, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.733$, Rules used = {3042, 25, 4153, 25, 354, 96, 169, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(x)}{\left(a+b\cot^2(x)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\tan\left(x+\frac{\pi}{2}\right)\left(a+b\tan\left(x+\frac{\pi}{2}\right)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\tan\left(x+\frac{\pi}{2}\right)\left(b\tan\left(x+\frac{\pi}{2}\right)^2+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4153}$$

$$\int -\frac{\tan(x)}{\left(\cot^2(x)+1\right)\left(a+b\cot^2(x)\right)^{5/2}}d\cot(x)$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\tan(x)}{\left(\cot^2(x)+1\right)\left(b\cot^2(x)+a\right)^{5/2}}d\cot(x)$$

$$\Big\downarrow \text{354}$$

$$-\frac{1}{2}\int \frac{\tan(x)}{\left(\cot^2(x)+1\right)\left(b\cot^2(x)+a\right)^{5/2}}d\cot^2(x)$$

$$\Big\downarrow \text{96}$$

$$\frac{1}{2}\left(\frac{2b}{3a(a-b)\left(a+b\cot^2(x)\right)^{3/2}}-\frac{\int \frac{(-b\cot^2(x)+a-b)\tan(x)}{(\cot^2(x)+1)(b\cot^2(x)+a)^{3/2}}d\cot^2(x)}{a(a-b)}\right)$$

$$\Big\downarrow \text{169}$$

$$\frac{1}{2}\left(\frac{2b}{3a(a-b)\left(a+b\cot^2(x)\right)^{3/2}}-\frac{-\frac{2\int -\frac{\left((a-b)^2-(2a-b)b\cot^2(x)\right)\tan(x)}{2\left(\cot^2(x)+1\right)\sqrt{b\cot^2(x)+a}}d\cot^2(x)}{a(a-b)}-\frac{2b(2a-b)}{a(a-b)\sqrt{a+b\cot^2(x)}}}{a(a-b)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{2b}{3a(a-b)\left(a+b\cot^2(x)\right)^{3/2}}-\frac{\frac{\int \frac{\left((a-b)^2-(2a-b)b\cot^2(x)\right)\tan(x)}{\left(\cot^2(x)+1\right)\sqrt{b\cot^2(x)+a}}d\cot^2(x)}{a(a-b)}-\frac{2b(2a-b)}{a(a-b)\sqrt{a+b\cot^2(x)}}}{a(a-b)}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{2}\left(\frac{2b}{3a(a-b)\left(a+b\cot^2(x)\right)^{3/2}}-\frac{\frac{(a-b)^2\int \frac{\tan(x)}{\sqrt{b\cot^2(x)+a}}d\cot^2(x)-a^2\int \frac{1}{\left(\cot^2(x)+1\right)\sqrt{b\cot^2(x)+a}}d\cot^2(x)}{a(a-b)}-\frac{2b(2a-b)}{a(a-b)\sqrt{a+b\cot^2(x)}}}{a(a-b)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{2b}{3a(a-b)\left(a+b\cot^2(x)\right)^{3/2}}-\frac{\frac{\frac{2(a-b)^2\int \frac{1}{\frac{\cot^4(x)}{b}-\frac{a}{b}}d\sqrt{b\cot^2(x)+a}}{b}-\frac{2a^2\int \frac{1}{\frac{\cot^4(x)}{b}-\frac{a}{b}+1}d\sqrt{b\cot^2(x)+a}}{b}}{a(a-b)}-\frac{2b(2a-b)}{a(a-b)\sqrt{a+b\cot^2(x)}}}{a(a-b)}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{2b}{3a(a-b)\left(a+b\cot^2(x)\right)^{3/2}}-\frac{\frac{\frac{2a^2\text{arctanh}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}-\frac{2(a-b)^2\text{arctanh}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}}{a(a-b)}-\frac{2b(2a-b)}{a(a-b)\sqrt{a+b\cot^2(x)}}}{a(a-b)}\right)$$

input
```
Int[Tan[x]/(a + b*Cot[x]^2)^(5/2),x]
```

3.57. $\int \frac{\tan(x)}{(a+b\cot^2(x))^{5/2}}\,dx$

output
```
((2*b)/(3*a*(a - b)*(a + b*Cot[x]^2)^(3/2)) - (((-2*(a - b)^2*ArcTanh[Sqrt
[a + b*Cot[x]^2]/Sqrt[a]])/Sqrt[a] + (2*a^2*ArcTanh[Sqrt[a + b*Cot[x]^2]/S
qrt[a - b]])/Sqrt[a - b])/(a*(a - b)) - (2*(2*a - b)*b)/(a*(a - b)*Sqrt[a
+ b*Cot[x]^2]))/(a*(a - b)))/2
```

3.57.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 96
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[f*((e + f*x)^(p + 1)/((p + 1)*(b*e - a*f)*(d*e - c*f))), x] + S
imp[1/((b*e - a*f)*(d*e - c*f))   Int[(b*d*e - b*c*f - a*d*f - b*d*f*x)*((e
 + f*x)^(p + 1)/((a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && LtQ[p, -1]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.57.4 Maple [F]

$$\int \frac{\tan(x)}{\left(a+b\cot(x)^2\right)^{\frac{5}{2}}}dx$$

input
```
int(tan(x)/(a+b*cot(x)^2)^(5/2),x)
```

output
```
int(tan(x)/(a+b*cot(x)^2)^(5/2),x)
```

3.57.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 369 vs. $2(100) = 200$.

Time = 0.49 (sec) , antiderivative size = 1531, normalized size of antiderivative = 12.97

$$\int \frac{\tan(x)}{(a+b\cot^2(x))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tan(x)/(a+b*cot(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/6*(3*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5 + (a^5 - 3*a^4*b + 3*a^3*b^2
- a^2*b^3)*tan(x)^4 + 2*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*tan(x)^2)*
sqrt(a)*log(2*a*tan(x)^2 + 2*sqrt(a)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x
)^2 + b) + 3*(a^5*tan(x)^4 + 2*a^4*b*tan(x)^2 + a^3*b^2)*sqrt(a - b)*log((
(2*a - b)*tan(x)^2 - 2*sqrt(a - b)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x)^
2 + b)/(tan(x)^2 + 1)) + 2*((7*a^4*b - 11*a^3*b^2 + 4*a^2*b^3)*tan(x)^4 +
3*(2*a^3*b^2 - 3*a^2*b^3 + a*b^4)*tan(x)^2)*sqrt((a*tan(x)^2 + b)/tan(x)^2
))/(a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5 + (a^8 - 3*a^7*b + 3*a^6*b^2
 - a^5*b^3)*tan(x)^4 + 2*(a^7*b - 3*a^6*b^2 + 3*a^5*b^3 - a^4*b^4)*tan(x)^
2), -1/6*(6*(a^5*tan(x)^4 + 2*a^4*b*tan(x)^2 + a^3*b^2)*sqrt(-a + b)*arcta
n(-sqrt(-a + b)*sqrt((a*tan(x)^2 + b)/tan(x)^2)/(a - b)) - 3*(a^3*b^2 - 3*
a^2*b^3 + 3*a*b^4 - b^5 + (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*tan(x)^4 +
 2*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*tan(x)^2)*sqrt(a)*log(2*a*tan(x
)^2 + 2*sqrt(a)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x)^2 + b) - 2*((7*a^4*
b - 11*a^3*b^2 + 4*a^2*b^3)*tan(x)^4 + 3*(2*a^3*b^2 - 3*a^2*b^3 + a*b^4)*t
an(x)^2)*sqrt((a*tan(x)^2 + b)/tan(x)^2))/(a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4
 - a^3*b^5 + (a^8 - 3*a^7*b + 3*a^6*b^2 - a^5*b^3)*tan(x)^4 + 2*(a^7*b - 3
*a^6*b^2 + 3*a^5*b^3 - a^4*b^4)*tan(x)^2), -1/6*(6*(a^3*b^2 - 3*a^2*b^3 +
3*a*b^4 - b^5 + (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*tan(x)^4 + 2*(a^4*b
- 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*tan(x)^2)*sqrt(-a)*arctan(sqrt(-a)*sqr...
```

3.57.6 Sympy [F]

$$\int \frac{\tan(x)}{(a+b\cot^2(x))^{5/2}}\,dx = \int \frac{\tan{(x)}}{(a+b\cot^2{(x)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(x)/(a+b*cot(x)**2)**(5/2),x)
```

output
```
Integral(tan(x)/(a + b*cot(x)**2)**(5/2), x)
```

3.57.7 Maxima [F]

$$\int \frac{\tan(x)}{(a+b\cot^2(x))^{5/2}}\,dx = \int \frac{\tan(x)}{\left(b\cot(x)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(x)/(a+b*cot(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(tan(x)/(b*cot(x)^2 + a)^(5/2), x)
```

3.57.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 483 vs. $2(100) = 200$.

Time = 0.38 (sec) , antiderivative size = 483, normalized size of antiderivative = 4.09

$$\int \frac{\tan(x)}{(a+b\cot^2(x))^{5/2}}\,dx =$$

$$-\frac{\left(2\,a^3\arctan\left(-\frac{a-b}{\sqrt{-a^2+ab}}\right)-6\,a^2b\arctan\left(-\frac{a-b}{\sqrt{-a^2+ab}}\right)+6\,ab^2\arctan\left(-\frac{a-b}{\sqrt{-a^2+ab}}\right)-2\,b^3\arctan\left(-\frac{a-b}{\sqrt{-a^2+a}}\right.\right.}{2\left(\sqrt{-a^2+ab}\sqrt{a-b}a^4-2\sqrt{-a^2+ab}\sqrt{a-b}a^3b+\sqrt{-a^2+ab}\sqrt{a-b}\right.}$$

$$+\frac{\frac{2\left(\frac{\left(7\,a^5b^2-17\,a^4b^3+13\,a^3b^4-3\,a^2b^5\right)\sin(x)^2}{a^7b-3\,a^6b^2+3\,a^5b^3-a^4b^4}+\frac{3\left(2\,a^4b^3-3\,a^3b^4+a^2b^5\right)}{a^7b-3\,a^6b^2+3\,a^5b^3-a^4b^4}\right)\sin(x)}{\left(a\sin(x)^2-b\sin(x)^2+b\right)^{\frac{3}{2}}}+\frac{3\log\left(\left(\sqrt{a-b}\sin(x)-\sqrt{a\sin(x)^2-b\sin(x)^2+b}\right)^2\right)}{(a^2-2\,ab+b^2)\sqrt{a-b}}+\frac{6\sqrt{a-}}{}}{6\,\mathrm{sgn}\left(\sin(x)\right)}$$

input
```
integrate(tan(x)/(a+b*cot(x)^2)^(5/2),x, algorithm="giac")
```

output
```
-1/2*(2*a^3*arctan(-(a - b)/sqrt(-a^2 + a*b)) - 6*a^2*b*arctan(-(a - b)/sq
rt(-a^2 + a*b)) + 6*a*b^2*arctan(-(a - b)/sqrt(-a^2 + a*b)) - 2*b^3*arctan
(-(a - b)/sqrt(-a^2 + a*b)) + sqrt(-a^2 + a*b)*a^2*log(b))*sgn(sin(x))/(sq
rt(-a^2 + a*b)*sqrt(a - b)*a^4 - 2*sqrt(-a^2 + a*b)*sqrt(a - b)*a^3*b + sq
rt(-a^2 + a*b)*sqrt(a - b)*a^2*b^2) + 1/6*(2*((7*a^5*b^2 - 17*a^4*b^3 + 13
*a^3*b^4 - 3*a^2*b^5)*sin(x)^2/(a^7*b - 3*a^6*b^2 + 3*a^5*b^3 - a^4*b^4) +
 3*(2*a^4*b^3 - 3*a^3*b^4 + a^2*b^5)/(a^7*b - 3*a^6*b^2 + 3*a^5*b^3 - a^4*
b^4))*sin(x)/(a*sin(x)^2 - b*sin(x)^2 + b)^(3/2) + 3*log((sqrt(a - b)*sin(
x) - sqrt(a*sin(x)^2 - b*sin(x)^2 + b))^2)/((a^2 - 2*a*b + b^2)*sqrt(a - b
)) + 6*sqrt(a - b)*arctan(1/2*((sqrt(a - b)*sin(x) - sqrt(a*sin(x)^2 - b*s
in(x)^2 + b))^2 - 2*a + b)/sqrt(-a^2 + a*b))/(sqrt(-a^2 + a*b)*a^2))/sgn(s
in(x))
```

3.57.9 Mupad [B] (verification not implemented)

Time = 13.68 (sec) , antiderivative size = 2817, normalized size of antiderivative = 23.87

$$\int \frac{\tan(x)}{(a+b\cot^2(x))^{5/2}}\,dx = \text{Too large to display}$$

input
```
int(tan(x)/(a + b*cot(x)^2)^(5/2),x)
```

output
```
atanh((2*a^5*b^13*(a + b/tan(x)^2)^(1/2))/((a^5)^(1/2)*(2*a^3*b^13 - 22*a^
4*b^12 + 110*a^5*b^11 - 330*a^6*b^10 + 660*a^7*b^9 - 922*a^8*b^8 + 912*a^9
*b^7 - 630*a^10*b^6 + 290*a^11*b^5 - 80*a^12*b^4 + 10*a^13*b^3)) - (22*a^6
*b^12*(a + b/tan(x)^2)^(1/2))/((a^5)^(1/2)*(2*a^3*b^13 - 22*a^4*b^12 + 110
*a^5*b^11 - 330*a^6*b^10 + 660*a^7*b^9 - 922*a^8*b^8 + 912*a^9*b^7 - 630*a
^10*b^6 + 290*a^11*b^5 - 80*a^12*b^4 + 10*a^13*b^3)) + (110*a^7*b^11*(a +
b/tan(x)^2)^(1/2))/((a^5)^(1/2)*(2*a^3*b^13 - 22*a^4*b^12 + 110*a^5*b^11 -
 330*a^6*b^10 + 660*a^7*b^9 - 922*a^8*b^8 + 912*a^9*b^7 - 630*a^10*b^6 + 2
90*a^11*b^5 - 80*a^12*b^4 + 10*a^13*b^3)) - (330*a^8*b^10*(a + b/tan(x)^2)
^(1/2))/((a^5)^(1/2)*(2*a^3*b^13 - 22*a^4*b^12 + 110*a^5*b^11 - 330*a^6*b^
10 + 660*a^7*b^9 - 922*a^8*b^8 + 912*a^9*b^7 - 630*a^10*b^6 + 290*a^11*b^5
 - 80*a^12*b^4 + 10*a^13*b^3)) + (660*a^9*b^9*(a + b/tan(x)^2)^(1/2))/((a^
5)^(1/2)*(2*a^3*b^13 - 22*a^4*b^12 + 110*a^5*b^11 - 330*a^6*b^10 + 660*a^7
*b^9 - 922*a^8*b^8 + 912*a^9*b^7 - 630*a^10*b^6 + 290*a^11*b^5 - 80*a^12*b
^4 + 10*a^13*b^3)) - (922*a^10*b^8*(a + b/tan(x)^2)^(1/2))/((a^5)^(1/2)*(2
*a^3*b^13 - 22*a^4*b^12 + 110*a^5*b^11 - 330*a^6*b^10 + 660*a^7*b^9 - 922*
a^8*b^8 + 912*a^9*b^7 - 630*a^10*b^6 + 290*a^11*b^5 - 80*a^12*b^4 + 10*a^1
3*b^3)) + (912*a^11*b^7*(a + b/tan(x)^2)^(1/2))/((a^5)^(1/2)*(2*a^3*b^13 -
 22*a^4*b^12 + 110*a^5*b^11 - 330*a^6*b^10 + 660*a^7*b^9 - 922*a^8*b^8 + 9
12*a^9*b^7 - 630*a^10*b^6 + 290*a^11*b^5 - 80*a^12*b^4 + 10*a^13*b^3)) ...
```

3.58 $\int \frac{\tan^2(x)}{(a+b\cot^2(x))^{5/2}}\,dx$

3.58.1 Optimal result

Integrand size = 17, antiderivative size = 141

$$\int \frac{\tan^2(x)}{(a+b\cot^2(x))^{5/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{a-b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{(a-b)^{5/2}} + \frac{b\tan(x)}{3a(a-b)\left(a+b\cot^2(x)\right)^{3/2}} + \frac{(7a-4b)b\tan(x)}{3a^2(a-b)^2\sqrt{a+b\cot^2(x)}} + \frac{(a-4b)(3a-2b)\sqrt{a+b\cot^2(x)}\tan(x)}{3a^3(a-b)^2}$$

output
```
arctan(cot(x)*(a-b)^(1/2)/(a+b*cot(x)^2)^(1/2))/(a-b)^(5/2)+1/3*b*tan(x)/a
/(a-b)/(a+b*cot(x)^2)^(3/2)+1/3*(7*a-4*b)*b*tan(x)/a^2/(a-b)^2/(a+b*cot(x)
^2)^(1/2)+1/3*(a-4*b)*(3*a-2*b)*(a+b*cot(x)^2)^(1/2)*tan(x)/a^3/(a-b)^2
```

3.58.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 8.16 (sec) , antiderivative size = 1450, normalized size of antiderivative = 10.28

$$\int \frac{\tan^2(x)}{(a+b\cot^2(x))^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[Tan[x]^2/(a + b*Cot[x]^2)^(5/2),x]
```

output
```
(Sin[x]^2*((-16*b^3*(Cot[x] + Cot[x]^3)^2)/(a*(a - b)^2) + (40*b*Csc[x]^2)
/(a - b) + (160*b^2*Cot[x]^2*Csc[x]^2)/(3*a*(a - b)) + (64*b^3*Cot[x]^4*Cs
c[x]^2)/(3*a^2*(a - b)) - (40*b^2*Csc[x]^4)/(a - b)^2 + (92*(a - b)*Cos[x]
^2*Hypergeometric2F1[2, 2, 9/2, ((a - b)*Cos[x]^2)/a])/(105*a) + (124*(a -
 b)*b*Cos[x]^2*Cot[x]^2*Hypergeometric2F1[2, 2, 9/2, ((a - b)*Cos[x]^2)/a]
)/(35*a^2) + (152*(a - b)*b^2*Cos[x]^2*Cot[x]^4*Hypergeometric2F1[2, 2, 9/
2, ((a - b)*Cos[x]^2)/a])/(35*a^3) + (176*(a - b)*b^3*Cos[x]^2*Cot[x]^6*Hy
pergeometric2F1[2, 2, 9/2, ((a - b)*Cos[x]^2)/a])/(105*a^4) + (24*(a - b)*
Cos[x]^2*HypergeometricPFQ[{2, 2, 2}, {1, 9/2}, ((a - b)*Cos[x]^2)/a])/(35
*a) + (16*(a - b)*b*Cos[x]^2*Cot[x]^2*HypergeometricPFQ[{2, 2, 2}, {1, 9/2
}, ((a - b)*Cos[x]^2)/a])/(7*a^2) + (88*(a - b)*b^2*Cos[x]^2*Cot[x]^4*Hype
rgeometricPFQ[{2, 2, 2}, {1, 9/2}, ((a - b)*Cos[x]^2)/a])/(35*a^3) + (32*(
a - b)*b^3*Cos[x]^2*Cot[x]^6*HypergeometricPFQ[{2, 2, 2}, {1, 9/2}, ((a -
b)*Cos[x]^2)/a])/(35*a^4) + (16*(a - b)*Cos[x]^2*HypergeometricPFQ[{2, 2,
2, 2}, {1, 1, 9/2}, ((a - b)*Cos[x]^2)/a])/(105*a) + (16*(a - b)*b*Cos[x]^
2*Cot[x]^2*HypergeometricPFQ[{2, 2, 2, 2}, {1, 1, 9/2}, ((a - b)*Cos[x]^2)
/a])/(35*a^2) + (16*(a - b)*b^2*Cos[x]^2*Cot[x]^4*HypergeometricPFQ[{2, 2,
 2, 2}, {1, 1, 9/2}, ((a - b)*Cos[x]^2)/a])/(35*a^3) + (16*(a - b)*b^3*Cos
[x]^2*Cot[x]^6*HypergeometricPFQ[{2, 2, 2, 2}, {1, 1, 9/2}, ((a - b)*Cos[x
]^2)/a])/(105*a^4) + (20*a*Sec[x]^2)/(3*(a - b)) - (30*a*b*Csc[x]^2*Sec...
```

3.58.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {3042, 4153, 374, 441, 445, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(x)}{\left(a+b\cot^2(x)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan\left(x+\frac{\pi}{2}\right)^2\left(a+b\tan\left(x+\frac{\pi}{2}\right)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{4153}$$

$$-\int \frac{\tan^2(x)}{\left(\cot^2(x)+1\right)\left(b\cot^2(x)+a\right)^{5/2}}d\cot(x)$$

$$\Big\downarrow \; 374$$

$$\frac{b\tan(x)}{3a(a-b)\left(a+b\cot^2(x)\right)^{3/2}}-\frac{\int \frac{\left(-4b\cot^2(x)+3a-4b\right)\tan^2(x)}{\left(\cot^2(x)+1\right)\left(b\cot^2(x)+a\right)^{3/2}}d\cot(x)}{3a(a-b)}$$

$$\Big\downarrow \; 441$$

$$\frac{b\tan(x)}{3a(a-b)\left(a+b\cot^2(x)\right)^{3/2}}-\frac{\frac{\int \frac{\left((a-4b)(3a-2b)-2(7a-4b)b\cot^2(x)\right)\tan^2(x)}{\left(\cot^2(x)+1\right)\sqrt{b\cot^2(x)+a}}d\cot(x)}{a(a-b)}-\frac{b(7a-4b)\tan(x)}{a(a-b)\sqrt{a+b\cot^2(x)}}}{3a(a-b)}$$

$$\Big\downarrow \; 445$$

$$\frac{b\tan(x)}{3a(a-b)\left(a+b\cot^2(x)\right)^{3/2}}-\frac{\frac{-\frac{\int \frac{3a^3}{\left(\cot^2(x)+1\right)\sqrt{b\cot^2(x)+a}}d\cot(x)}{a}-\frac{(a-4b)(3a-2b)\tan(x)\sqrt{a+b\cot^2(x)}}{a}}{a(a-b)}-\frac{b(7a-4b)\tan(x)}{a(a-b)\sqrt{a+b\cot^2(x)}}}{3a(a-b)}$$

$$\Big\downarrow \; 27$$

$$\frac{b\tan(x)}{3a(a-b)\left(a+b\cot^2(x)\right)^{3/2}}-\frac{\frac{-3a^2\int \frac{1}{\left(\cot^2(x)+1\right)\sqrt{b\cot^2(x)+a}}d\cot(x)-\frac{(a-4b)(3a-2b)\tan(x)\sqrt{a+b\cot^2(x)}}{a}}{a(a-b)}-\frac{b(7a-4b)\tan(x)}{a(a-b)\sqrt{a+b\cot^2(x)}}}{3a(a-b)}$$

$$\Big\downarrow \; 291$$

$$\frac{b\tan(x)}{3a(a-b)\left(a+b\cot^2(x)\right)^{3/2}}-\frac{\frac{-3a^2\int \frac{1}{1-\frac{(b-a)\cot^2(x)}{b\cot^2(x)+a}}d\frac{\cot(x)}{\sqrt{b\cot^2(x)+a}}-\frac{(a-4b)(3a-2b)\tan(x)\sqrt{a+b\cot^2(x)}}{a}}{a(a-b)}-\frac{b(7a-4b)\tan(x)}{a(a-b)\sqrt{a+b\cot^2(x)}}}{3a(a-b)}$$

$$\Big\downarrow \; 216$$

$$\frac{b\tan(x)}{3a(a-b)\left(a+b\cot^2(x)\right)^{3/2}}-\frac{\frac{-\frac{3a^2\arctan\left(\frac{\sqrt{a-b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{\sqrt{a-b}}-\frac{(a-4b)(3a-2b)\tan(x)\sqrt{a+b\cot^2(x)}}{a}}{a(a-b)}-\frac{b(7a-4b)\tan(x)}{a(a-b)\sqrt{a+b\cot^2(x)}}}{3a(a-b)}$$

3.58. $\displaystyle\int \frac{\tan^2(x)}{\left(a+b\cot^2(x)\right)^{5/2}}\,dx$

input
```
Int[Tan[x]^2/(a + b*Cot[x]^2)^(5/2),x]
```

output
```
(b*Tan[x])/(3*a*(a - b)*(a + b*Cot[x]^2)^(3/2)) - (-((((7*a - 4*b)*b*Tan[x]
)/(a*(a - b)*Sqrt[a + b*Cot[x]^2])) + ((-3*a^2*ArcTan[(Sqrt[a - b]*Cot[x])
/Sqrt[a + b*Cot[x]^2]])/Sqrt[a - b] - ((a - 4*b)*(3*a - 2*b)*Sqrt[a + b*Co
t[x]^2]*Tan[x])/a)/(a*(a - b)))/(3*a*(a - b))
```

3.58.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.58.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 933 vs. $2(123) = 246$.

Time = 1.56 (sec) , antiderivative size = 934, normalized size of antiderivative = 6.62

method	result	size
default	Expression too large to display	934

input
```
int(tan(x)^2/(a+b*cot(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*4^(1/2)/(-a+b)^(1/2)/(a-b)^2/a^3*(-3*(-a+b)^(1/2)*a^2*b^2*(1-cos(x))^8
*csc(x)^8+14*(-a+b)^(1/2)*a*b^3*(1-cos(x))^8*csc(x)^8-8*(-a+b)^(1/2)*b^4*(
1-cos(x))^8*csc(x)^8-24*(-a+b)^(1/2)*a^3*b*(1-cos(x))^6*csc(x)^6+96*(-a+b)
^(1/2)*a^2*b^2*(1-cos(x))^6*csc(x)^6-104*(-a+b)^(1/2)*a*b^3*(1-cos(x))^6*c
sc(x)^6+32*(-a+b)^(1/2)*b^4*(1-cos(x))^6*csc(x)^6+3*ln(4*(a*(1-cos(x))^2*c
sc(x)^2-b*(1-cos(x))^2*csc(x)^2+(-a+b)^(1/2)*(b*(1-cos(x))^4*csc(x)^4+4*a*
(1-cos(x))^2*csc(x)^2-2*b*(1-cos(x))^2*csc(x)^2+b)^(1/2)-a+b)/((1-cos(x))^
2*csc(x)^2+1))*(b*(1-cos(x))^4*csc(x)^4+4*a*(1-cos(x))^2*csc(x)^2-2*b*(1-c
os(x))^2*csc(x)^2+b)^(3/2)*a^3*(1-cos(x))^2*csc(x)^2-48*(-a+b)^(1/2)*a^4*(
1-cos(x))^4*csc(x)^4+144*(-a+b)^(1/2)*a^3*b*(1-cos(x))^4*csc(x)^4-234*(-a+
b)^(1/2)*a^2*b^2*(1-cos(x))^4*csc(x)^4+180*(-a+b)^(1/2)*a*b^3*(1-cos(x))^4
*csc(x)^4-48*(-a+b)^(1/2)*b^4*(1-cos(x))^4*csc(x)^4-3*ln(4*(a*(1-cos(x))^2
*csc(x)^2-b*(1-cos(x))^2*csc(x)^2+(-a+b)^(1/2)*(b*(1-cos(x))^4*csc(x)^4+4*
a*(1-cos(x))^2*csc(x)^2-2*b*(1-cos(x))^2*csc(x)^2+b)^(1/2)-a+b)/((1-cos(x)
)^2*csc(x)^2+1))*(b*(1-cos(x))^4*csc(x)^4+4*a*(1-cos(x))^2*csc(x)^2-2*b*(1
-cos(x))^2*csc(x)^2+b)^(3/2)*a^3-24*(-a+b)^(1/2)*a^3*b*(1-cos(x))^2*csc(x)
^2+96*(-a+b)^(1/2)*a^2*b^2*(1-cos(x))^2*csc(x)^2-104*(-a+b)^(1/2)*a*b^3*(1
-cos(x))^2*csc(x)^2+32*(-a+b)^(1/2)*b^4*(1-cos(x))^2*csc(x)^2-3*(-a+b)^(1/
2)*a^2*b^2+14*(-a+b)^(1/2)*a*b^3-8*(-a+b)^(1/2)*b^4)*(b*(1-cos(x))^4*csc(x
)^4+4*a*(1-cos(x))^2*csc(x)^2-2*b*(1-cos(x))^2*csc(x)^2+b)/((1-cos(x))^...
```

3.58.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 297 vs. $2(123) = 246$.

Time = 0.34 (sec) , antiderivative size = 647, normalized size of antiderivative = 4.59

$$\int \frac{\tan^2(x)}{(a+b\cot^2(x))^{5/2}}\,dx = \left[-\frac{3\left(a^5\tan(x)^4+2\,a^4b\tan(x)^2+a^3b^2\right)\sqrt{-a+b}\log\left(-\frac{a^2\tan(x)^4-2\,(3\,a^2-4\,ab)\tan(x}{}\right.}{}\right.$$

input
```
integrate(tan(x)^2/(a+b*cot(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
[-1/12*(3*(a^5*tan(x)^4 + 2*a^4*b*tan(x)^2 + a^3*b^2)*sqrt(-a + b)*log(-(a
^2*tan(x)^4 - 2*(3*a^2 - 4*a*b)*tan(x)^2 + a^2 - 8*a*b + 8*b^2 + 4*(a*tan(
x)^3 - (a - 2*b)*tan(x))*sqrt(-a + b)*sqrt((a*tan(x)^2 + b)/tan(x)^2))/(ta
n(x)^4 + 2*tan(x)^2 + 1)) - 4*(3*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*tan
(x)^5 + 3*(2*a^4*b - 9*a^3*b^2 + 11*a^2*b^3 - 4*a*b^4)*tan(x)^3 + (3*a^3*b
^2 - 17*a^2*b^3 + 22*a*b^4 - 8*b^5)*tan(x))*sqrt((a*tan(x)^2 + b)/tan(x)^2
))/(a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5 + (a^8 - 3*a^7*b + 3*a^6*b^2
 - a^5*b^3)*tan(x)^4 + 2*(a^7*b - 3*a^6*b^2 + 3*a^5*b^3 - a^4*b^4)*tan(x)^
2), 1/6*(3*(a^5*tan(x)^4 + 2*a^4*b*tan(x)^2 + a^3*b^2)*sqrt(a - b)*arctan(
2*sqrt(a - b)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x)/(a*tan(x)^2 - a + 2*b
)) + 2*(3*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*tan(x)^5 + 3*(2*a^4*b - 9*
a^3*b^2 + 11*a^2*b^3 - 4*a*b^4)*tan(x)^3 + (3*a^3*b^2 - 17*a^2*b^3 + 22*a*
b^4 - 8*b^5)*tan(x))*sqrt((a*tan(x)^2 + b)/tan(x)^2))/(a^6*b^2 - 3*a^5*b^3
 + 3*a^4*b^4 - a^3*b^5 + (a^8 - 3*a^7*b + 3*a^6*b^2 - a^5*b^3)*tan(x)^4 +
2*(a^7*b - 3*a^6*b^2 + 3*a^5*b^3 - a^4*b^4)*tan(x)^2)]
```

3.58.6 Sympy [F]

$$\int \frac{\tan^2(x)}{(a+b\cot^2(x))^{5/2}}\,dx = \int \frac{\tan^2(x)}{(a+b\cot^2(x))^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(x)**2/(a+b*cot(x)**2)**(5/2),x)
```

output
```
Integral(tan(x)**2/(a + b*cot(x)**2)**(5/2), x)
```

3.58.7 Maxima [F]

$$\int \frac{\tan^2(x)}{(a+b\cot^2(x))^{5/2}}\,dx = \int \frac{\tan(x)^2}{(b\cot(x)^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(x)^2/(a+b*cot(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(tan(x)^2/(b*cot(x)^2 + a)^(5/2), x)
```

3.58.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 537 vs. $2(123) = 246$.

Time = 0.36 (sec) , antiderivative size = 537, normalized size of antiderivative = 3.81

$$\int \frac{\tan^2(x)}{(a+b\cot^2(x))^{5/2}}\,dx = \frac{\left(3\,a^4\sqrt{b}\log\left(-a-2\,\sqrt{-a+b}\sqrt{b}+2\,b\right)+3\,a^3\sqrt{-a+b}b\log\left(-a-2\,\sqrt{-a+b}\sqrt{\right.}\right.}{6\left(a^6\sqrt{-a\right.}}$$

$$-\frac{\frac{2\left(\frac{(9\,a^5b^2-23\,a^4b^3+19\,a^3b^4-5\,a^2b^5)\cos(x)^2}{a^8-3\,a^7b+3\,a^6b^2-a^5b^3}-\frac{3\,(3\,a^5b^2-5\,a^4b^3+2\,a^3b^4)}{a^8-3\,a^7b+3\,a^6b^2-a^5b^3}\right)\cos(x)}{(a\cos(x)^2-b\cos(x)^2-a)\sqrt{-a\cos(x)^2+b\cos(x)^2+a}}+\frac{3\,\log\left(\left(\sqrt{-a+b}\cos(x)-\sqrt{-a\cos(x)^2+b\cos(x)^2+a}\right)^2\right)}{(a^2-2\,ab+b^2)\sqrt{-a+b}}+\cdots}{6\,\mathrm{sgn}\left(\sin\left(x\right)\right)}$$

input
```
integrate(tan(x)^2/(a+b*cot(x)^2)^(5/2),x, algorithm="giac")
```

output
```
1/6*(3*a^4*sqrt(b)*log(-a - 2*sqrt(-a + b)*sqrt(b) + 2*b) + 3*a^3*sqrt(-a
+ b)*b*log(-a - 2*sqrt(-a + b)*sqrt(b) + 2*b) - 3*a^3*b^(3/2)*log(-a - 2*s
qrt(-a + b)*sqrt(b) + 2*b) + 6*a^4*sqrt(b) - 18*a^3*b^(3/2) + 16*a^2*sqrt(
-a + b)*b^2 + 2*a^2*b^(5/2) - 26*a*sqrt(-a + b)*b^3 + 20*a*b^(7/2) + 10*sq
rt(-a + b)*b^4 - 10*b^(9/2))*sgn(sin(x))/(a^6*sqrt(-a + b)*sqrt(b) - a^6*b
 - 3*a^5*sqrt(-a + b)*b^(3/2) + 3*a^5*b^2 + 3*a^4*sqrt(-a + b)*b^(5/2) - 3
*a^4*b^3 - a^3*sqrt(-a + b)*b^(7/2) + a^3*b^4) - 1/6*(2*((9*a^5*b^2 - 23*a
^4*b^3 + 19*a^3*b^4 - 5*a^2*b^5)*cos(x)^2/(a^8 - 3*a^7*b + 3*a^6*b^2 - a^5
*b^3) - 3*(3*a^5*b^2 - 5*a^4*b^3 + 2*a^3*b^4)/(a^8 - 3*a^7*b + 3*a^6*b^2 -
 a^5*b^3))*cos(x)/((a*cos(x)^2 - b*cos(x)^2 - a)*sqrt(-a*cos(x)^2 + b*cos(
x)^2 + a)) + 3*log((sqrt(-a + b)*cos(x) - sqrt(-a*cos(x)^2 + b*cos(x)^2 +
a))^2)/((a^2 - 2*a*b + b^2)*sqrt(-a + b)) + 12*sqrt(-a + b)/(((sqrt(-a + b
)*cos(x) - sqrt(-a*cos(x)^2 + b*cos(x)^2 + a))^2 - a)*a^2))/sgn(sin(x))
```

3.58.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^2(x)}{(a+b\cot^2(x))^{5/2}}\,dx = \int \frac{\tan(x)^2}{\left(b\cot(x)^2+a\right)^{5/2}}\,dx$$

input
```
int(tan(x)^2/(a + b*cot(x)^2)^(5/2),x)
```

output
```
int(tan(x)^2/(a + b*cot(x)^2)^(5/2), x)
```

3.59 $\int \frac{1}{1+\cot^3(x)}\,dx$

3.59.1 Optimal result

Integrand size = 8, antiderivative size = 37

$$\int \frac{1}{1+\cot^3(x)}\,dx = \frac{x}{2} - \frac{1}{6}\log(1+\cot(x)) + \frac{1}{3}\log\left(1-\cot(x)+\cot^2(x)\right) + \frac{1}{2}\log(\sin(x))$$

output
```
1/2*x-1/6*ln(1+cot(x))+1/3*ln(1-cot(x)+cot(x)^2)+1/2*ln(sin(x))
```

3.59.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.54

$$\int \frac{1}{1+\cot^3(x)}\,dx = \left(-\frac{1}{4}-\frac{i}{4}\right)\log(i-\tan(x)) - \left(\frac{1}{4}-\frac{i}{4}\right)\log(i+\tan(x)) - \frac{1}{6}\log(1+\tan(x)) + \frac{1}{3}\log\left(1-\tan(x)+\tan^2(x)\right)$$

input
```
Integrate[(1 + Cot[x]^3)^(-1),x]
```

output
```
(-1/4 - I/4)*Log[I - Tan[x]] - (1/4 - I/4)*Log[I + Tan[x]] - Log[1 + Tan[x
]]/6 + Log[1 - Tan[x] + Tan[x]^2]/3
```

3.59.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 4144, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cot^3(x)+1} \, dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{1-\tan\left(x+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \; 4144$$

$$-\int \frac{1}{\left(\cot^2(x)+1\right)\left(\cot^3(x)+1\right)}d\cot(x)$$

$$\Big\downarrow \; 7276$$

$$-\int \left(\frac{1-2\cot(x)}{3\left(\cot^2(x)-\cot(x)+1\right)}+\frac{1}{6(\cot(x)+1)}+\frac{\cot(x)+1}{2\left(\cot^2(x)+1\right)}\right)d\cot(x)$$

$$\Big\downarrow \; 2009$$

$$-\frac{1}{2}\arctan(\cot(x))-\frac{1}{4}\log\left(\cot^2(x)+1\right)+\frac{1}{3}\log\left(\cot^2(x)-\cot(x)+1\right)-\frac{1}{6}\log(\cot(x)+1)$$

input
```
Int[(1 + Cot[x]^3)^(-1),x]
```

output
```
-1/2*ArcTan[Cot[x]] - Log[1 + Cot[x]]/6 - Log[1 + Cot[x]^2]/4 + Log[1 - Co
t[x] + Cot[x]^2]/3
```

3.59.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.59.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

method	result	size
parallelrisch	$\frac{x}{2}+\ln\left(\frac{1}{\left(\tan\left(x\right)+1\right)^{\frac{1}{6}}}\right)+\ln\left(\frac{1}{\left(\sec\left(x\right)^{2}\right)^{\frac{1}{4}}}\right)+\ln\left(\left(-\tan\left(x\right)+\sec\left(x\right)^{2}\right)^{\frac{1}{3}}\right)$	31
norman	$\frac{x}{2}-\frac{\ln\left(\tan\left(x\right)+1\right)}{6}-\frac{\ln\left(\tan\left(x\right)^{2}+1\right)}{4}+\frac{\ln\left(\tan\left(x\right)^{2}-\tan\left(x\right)+1\right)}{3}$	34
risch	$\frac{x}{2}-\frac{i x}{2}-\frac{\ln\left({\mathrm e}^{2 i x}+i\right)}{6}+\frac{\ln\left({\mathrm e}^{4 i x}-4 i {\mathrm e}^{2 i x}-1\right)}{3}$	38
derivativedivides	$\frac{\ln\left(1-\cot\left(x\right)+\cot\left(x\right)^{2}\right)}{3}-\frac{\ln\left(\cot\left(x\right)^{2}+1\right)}{4}-\frac{\pi}{4}+\frac{\operatorname{arccot}\left(\cot\left(x\right)\right)}{2}-\frac{\ln\left(1+\cot\left(x\right)\right)}{6}$	39
default	$\frac{\ln\left(1-\cot\left(x\right)+\cot\left(x\right)^{2}\right)}{3}-\frac{\ln\left(\cot\left(x\right)^{2}+1\right)}{4}-\frac{\pi}{4}+\frac{\operatorname{arccot}\left(\cot\left(x\right)\right)}{2}-\frac{\ln\left(1+\cot\left(x\right)\right)}{6}$	39

input
```
int(1/(1+cot(x)^3),x,method=_RETURNVERBOSE)
```

output
```
1/2*x+ln(1/(tan(x)+1)^(1/6))+ln(1/(sec(x)^2)^(1/4))+ln((-tan(x)+sec(x)^2)^
(1/3))
```

3.59.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.65

$$\int \frac{1}{1+\cot^3(x)}\,dx = \frac{1}{2}\,x - \frac{1}{12}\,\log\left(\sin\left(2\,x\right)+1\right) + \frac{1}{3}\,\log\left(-\frac{1}{2}\,\sin\left(2\,x\right)+1\right)$$

input
```
integrate(1/(1+cot(x)^3),x, algorithm="fricas")
```

output
```
1/2*x - 1/12*log(sin(2*x) + 1) + 1/3*log(-1/2*sin(2*x) + 1)
```

3.59.6 Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int \frac{1}{1+\cot^3(x)}\,dx = \frac{x}{2} - \frac{\log\left(\tan\left(x\right)+1\right)}{6} - \frac{\log\left(\tan^2\left(x\right)+1\right)}{4} + \frac{\log\left(\tan^2\left(x\right)-\tan\left(x\right)+1\right)}{3}$$

input
```
integrate(1/(1+cot(x)**3),x)
```

output
```
x/2 - log(tan(x) + 1)/6 - log(tan(x)**2 + 1)/4 + log(tan(x)**2 - tan(x) +
1)/3
```

3.59.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int \frac{1}{1+\cot^3(x)}\,dx = \frac{1}{2}\,x + \frac{1}{3}\,\log\left(\tan\left(x\right)^2 - \tan\left(x\right)+1\right) - \frac{1}{4}\,\log\left(\tan\left(x\right)^2+1\right) - \frac{1}{6}\,\log\left(\tan\left(x\right)+1\right)$$

input
```
integrate(1/(1+cot(x)^3),x, algorithm="maxima")
```

output
```
1/2*x + 1/3*log(tan(x)^2 - tan(x) + 1) - 1/4*log(tan(x)^2 + 1) - 1/6*log(t
an(x) + 1)
```

3.59.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int \frac{1}{1+\cot^3(x)}\,dx = \frac{1}{2}\,x + \frac{1}{3}\,\log\left(\tan\left(x\right)^2 - \tan\left(x\right) + 1\right) - \frac{1}{4}\,\log\left(\tan\left(x\right)^2 + 1\right) - \frac{1}{6}\,\log\left(\left|\tan\left(x\right) + 1\right|\right)$$

input
```
integrate(1/(1+cot(x)^3),x, algorithm="giac")
```

output
```
1/2*x + 1/3*log(tan(x)^2 - tan(x) + 1) - 1/4*log(tan(x)^2 + 1) - 1/6*log(a
bs(tan(x) + 1))
```

3.59.9 Mupad [B] (verification not implemented)

Time = 13.79 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+\cot^3(x)}\,dx = x\left(\frac{1}{2} - \frac{1}{2}\mathrm{i}\right) - \frac{\ln\left(12\,\mathrm{e}^{x\,2\mathrm{i}} + 12\mathrm{i}\right)}{6} + \frac{\ln\left(\mathrm{e}^{x\,4\mathrm{i}} - 1 - \mathrm{e}^{x\,2\mathrm{i}}\,4\mathrm{i}\right)}{3}$$

input
```
int(1/(cot(x)^3 + 1),x)
```

output
```
x*(1/2 - 1i/2) - log(12*exp(x*2i) + 12i)/6 + log(exp(x*4i) - exp(x*2i)*4i
- 1)/3
```

3.60 $\int \cot(x)\sqrt{a+b\cot^4(x)}\,dx$

3.60.1 Optimal result

Integrand size = 15, antiderivative size = 90

$$\int \cot(x)\sqrt{a+b\cot^4(x)}\,dx = \frac{1}{2}\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\cot^2(x)}{\sqrt{a+b\cot^4(x)}}\right) + \frac{1}{2}\sqrt{a+b}\operatorname{arctanh}\left(\frac{a-b\cot^2(x)}{\sqrt{a+b}\sqrt{a+b\cot^4(x)}}\right) - \frac{1}{2}\sqrt{a+b\cot^4(x)}$$

output
```
1/2*arctanh(cot(x)^2*b^(1/2)/(a+b*cot(x)^4)^(1/2))*b^(1/2)+1/2*arctanh((a-
b*cot(x)^2)/(a+b)^(1/2)/(a+b*cot(x)^4)^(1/2))*(a+b)^(1/2)-1/2*(a+b*cot(x)^
4)^(1/2)
```

3.60.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.96

$$\int \cot(x)\sqrt{a+b\cot^4(x)}\,dx = \frac{1}{2}\left(\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\cot^2(x)}{\sqrt{a+b\cot^4(x)}}\right) + \sqrt{a+b}\operatorname{arctanh}\left(\frac{a-b\cot^2(x)}{\sqrt{a+b}\sqrt{a+b\cot^4(x)}}\right) - \sqrt{a+b\cot^4(x)}\right)$$

input
```
Integrate[Cot[x]*Sqrt[a + b*Cot[x]^4],x]
```

output
```
(Sqrt[b]*ArcTanh[(Sqrt[b]*Cot[x]^2)/Sqrt[a + b*Cot[x]^4]] + Sqrt[a + b]*Ar
cTanh[(a - b*Cot[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Cot[x]^4])] - Sqrt[a + b*Co
t[x]^4])/2
```

3.60.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.96, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.733, Rules used = {3042, 25, 4153, 25, 1577, 493, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(x)\sqrt{a+b\cot^4(x)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int -\tan\left(x+\frac{\pi}{2}\right)\sqrt{a+b\tan\left(x+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \; 25$$

$$-\int \tan\left(x+\frac{\pi}{2}\right)\sqrt{b\tan\left(x+\frac{\pi}{2}\right)^4+a}dx$$

$$\Big\downarrow \; 4153$$

$$\int -\frac{\cot(x)\sqrt{a+b\cot^4(x)}}{\cot^2(x)+1}d\cot(x)$$

$$\Big\downarrow \; 25$$

$$-\int \frac{\cot(x)\sqrt{b\cot^4(x)+a}}{\cot^2(x)+1}d\cot(x)$$

$$\Big\downarrow \; 1577$$

$$-\frac{1}{2}\int \frac{\sqrt{b\cot^4(x)+a}}{\cot^2(x)+1}d\cot^2(x)$$

$$\Big\downarrow \; 493$$

$$\frac{1}{2}\left(-\int \frac{a-b\cot^2(x)}{\left(\cot^2(x)+1\right)\sqrt{b\cot^4(x)+a}}d\cot^2(x)-\sqrt{a+b\cot^4(x)}\right)$$

$$\Big\downarrow \text{719}$$

$$\frac{1}{2}\left(b\int \frac{1}{\sqrt{b\cot^4(x)+a}}d\cot^2(x)-(a+b)\int \frac{1}{(\cot^2(x)+1)\sqrt{b\cot^4(x)+a}}d\cot^2(x)-\sqrt{a+b\cot^4(x)}\right)$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{2}\left(b\int \frac{1}{1-b\cot^4(x)}d\frac{\cot^2(x)}{\sqrt{b\cot^4(x)+a}}-(a+b)\int \frac{1}{(\cot^2(x)+1)\sqrt{b\cot^4(x)+a}}d\cot^2(x)-\sqrt{a+b\cot^4(x)}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(-(a+b)\int \frac{1}{(\cot^2(x)+1)\sqrt{b\cot^4(x)+a}}d\cot^2(x)+\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\cot^2(x)}{\sqrt{a+b\cot^4(x)}}\right)-\sqrt{a+b\cot^4(x)}\right)$$

$$\Big\downarrow \text{488}$$

$$\frac{1}{2}\left((a+b)\int \frac{1}{-\cot^4(x)+a+b}d\frac{a-b\cot^2(x)}{\sqrt{b\cot^4(x)+a}}+\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\cot^2(x)}{\sqrt{a+b\cot^4(x)}}\right)-\sqrt{a+b\cot^4(x)}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\cot^2(x)}{\sqrt{a+b\cot^4(x)}}\right)+\sqrt{a+b}\text{arctanh}\left(\frac{a-b\cot^2(x)}{\sqrt{a+b}\sqrt{a+b\cot^4(x)}}\right)-\sqrt{a+b\cot^4(x)}\right)$$

input
```
Int[Cot[x]*Sqrt[a + b*Cot[x]^4],x]
```

output
```
(Sqrt[b]*ArcTanh[(Sqrt[b]*Cot[x]^2)/Sqrt[a + b*Cot[x]^4]] + Sqrt[a + b]*Ar
cTanh[(a - b*Cot[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Cot[x]^4])] - Sqrt[a + b*Co
t[x]^4])/2
```

3.60.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 493
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 2*p + 1))), x] + Simp[2*(p/(d*(n +
 2*p + 1)))   Int[(c + d*x)^n*(a + b*x^2)^(p - 1)*(a*d - b*c*x), x], x] /;
FreeQ[{a, b, c, d, n}, x] && GtQ[p, 0] && NeQ[n + 2*p + 1, 0] && ( !Rationa
lQ[n] || LtQ[n, 1]) &&  !ILtQ[n + 2*p, 0] && IntQuadraticQ[a, 0, b, c, d, n
, p, x]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1577
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, c, d, e, p, q}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.60. $\int \cot(x)\sqrt{a+b\cot^4(x)}\,dx$

3.60.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.54

method	result
derivativedivides	$-\frac{\sqrt{b\left(\cot(x)^2+1\right)^2-2b\left(\cot(x)^2+1\right)+a+b}}{2}+\frac{\sqrt{b}\,\ln\left(\frac{b\left(\cot(x)^2+1\right)-b}{\sqrt{b}}+\sqrt{b\left(\cot(x)^2+1\right)^2-2b\left(\cot(x)^2+1\right)+a+b}\right)}{2}+$
default	$-\frac{\sqrt{b\left(\cot(x)^2+1\right)^2-2b\left(\cot(x)^2+1\right)+a+b}}{2}+\frac{\sqrt{b}\,\ln\left(\frac{b\left(\cot(x)^2+1\right)-b}{\sqrt{b}}+\sqrt{b\left(\cot(x)^2+1\right)^2-2b\left(\cot(x)^2+1\right)+a+b}\right)}{2}+$

input
```
int(cot(x)*(a+b*cot(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(b*(cot(x)^2+1)^2-2*b*(cot(x)^2+1)+a+b)^(1/2)+1/2*b^(1/2)*ln((b*(cot(
x)^2+1)-b)/b^(1/2)+(b*(cot(x)^2+1)^2-2*b*(cot(x)^2+1)+a+b)^(1/2))+1/2*(a+b
)^(1/2)*ln((2*a+2*b-2*b*(cot(x)^2+1)+2*(a+b)^(1/2)*(b*(cot(x)^2+1)^2-2*b*(
cot(x)^2+1)+a+b)^(1/2))/(cot(x)^2+1))
```

3.60.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 252 vs. 2(72) = 144.

Time = 0.45 (sec) , antiderivative size = 1063, normalized size of antiderivative = 11.81

$$\int \cot(x)\sqrt{a+b\cot^4(x)}\,dx = \text{Too large to display}$$

input
```
integrate(cot(x)*(a+b*cot(x)^4)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*sqrt(a + b)*log(1/2*(a^2 + 2*a*b + b^2)*cos(2*x)^2 + 1/2*a^2 + 1/2*b^
2 + 1/2*((a + b)*cos(2*x)^2 - 2*a*cos(2*x) + a - b)*sqrt(a + b)*sqrt(((a +
 b)*cos(2*x)^2 - 2*(a - b)*cos(2*x) + a + b)/(cos(2*x)^2 - 2*cos(2*x) + 1)
) - (a^2 - b^2)*cos(2*x)) + 1/4*sqrt(b)*log(-((a + 2*b)*cos(2*x)^2 - 2*(co
s(2*x)^2 - 1)*sqrt(b)*sqrt(((a + b)*cos(2*x)^2 - 2*(a - b)*cos(2*x) + a +
b)/(cos(2*x)^2 - 2*cos(2*x) + 1)) - 2*(a - 2*b)*cos(2*x) + a + 2*b)/(cos(2
*x)^2 - 2*cos(2*x) + 1)) - 1/2*sqrt(((a + b)*cos(2*x)^2 - 2*(a - b)*cos(2*
x) + a + b)/(cos(2*x)^2 - 2*cos(2*x) + 1)), 1/2*sqrt(-b)*arctan(sqrt(-b)*s
qrt(((a + b)*cos(2*x)^2 - 2*(a - b)*cos(2*x) + a + b)/(cos(2*x)^2 - 2*cos(
2*x) + 1))*(cos(2*x) - 1)/(b*cos(2*x) + b)) + 1/4*sqrt(a + b)*log(1/2*(a^2
 + 2*a*b + b^2)*cos(2*x)^2 + 1/2*a^2 + 1/2*b^2 + 1/2*((a + b)*cos(2*x)^2 -
 2*a*cos(2*x) + a - b)*sqrt(a + b)*sqrt(((a + b)*cos(2*x)^2 - 2*(a - b)*co
s(2*x) + a + b)/(cos(2*x)^2 - 2*cos(2*x) + 1)) - (a^2 - b^2)*cos(2*x)) - 1
/2*sqrt(((a + b)*cos(2*x)^2 - 2*(a - b)*cos(2*x) + a + b)/(cos(2*x)^2 - 2*
cos(2*x) + 1)), -1/2*sqrt(-a - b)*arctan(((a + b)*cos(2*x)^2 - 2*a*cos(2*x
) + a - b)*sqrt(-a - b)*sqrt(((a + b)*cos(2*x)^2 - 2*(a - b)*cos(2*x) + a
+ b)/(cos(2*x)^2 - 2*cos(2*x) + 1))/((a^2 + 2*a*b + b^2)*cos(2*x)^2 + a^2
+ 2*a*b + b^2 - 2*(a^2 - b^2)*cos(2*x))) + 1/4*sqrt(b)*log(-((a + 2*b)*cos
(2*x)^2 - 2*(cos(2*x)^2 - 1)*sqrt(b)*sqrt(((a + b)*cos(2*x)^2 - 2*(a - b)*
cos(2*x) + a + b)/(cos(2*x)^2 - 2*cos(2*x) + 1)) - 2*(a - 2*b)*cos(2*x)...
```

3.60.6 Sympy [F]

$$\int \cot(x)\sqrt{a+b\cot^4(x)}\,dx = \int \sqrt{a+b\cot^4{(x)}}\cot{(x)}\,dx$$

input
```
integrate(cot(x)*(a+b*cot(x)**4)**(1/2),x)
```

output
```
Integral(sqrt(a + b*cot(x)**4)*cot(x), x)
```

3.60.7 Maxima [F]

$$\int \cot(x)\sqrt{a+b\cot^4(x)}\,dx = \int \sqrt{b\cot(x)^4+a}\cot(x)\,dx$$

input
```
integrate(cot(x)*(a+b*cot(x)^4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*cot(x)^4 + a)*cot(x), x)
```

3.60.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. 2(72) = 144.

Time = 0.30 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.27

$$\int \cot(x)\sqrt{a+b\cot^4(x)}\,dx = -\frac{b\arctan\left(-\frac{\sqrt{a+b}\sin(x)^2-\sqrt{a\sin(x)^4+b\sin(x)^4-2b\sin(x)^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}} - \frac{1}{2}\sqrt{a+b}\log\left(\left|-\left(\sqrt{a+b}\sin(x)^2-\sqrt{a\sin(x)^4+b\sin(x)^4-2b\sin(x)^2+b}\right)(a+b)+\sqrt{a+b}b\right|\right) - \frac{\left(\sqrt{a+b}\sin(x)^2-\sqrt{a\sin(x)^4+b\sin(x)^4-2b\sin(x)^2+b}\right)b-\sqrt{a+b}b}{\left(\sqrt{a+b}\sin(x)^2-\sqrt{a\sin(x)^4+b\sin(x)^4-2b\sin(x)^2+b}\right)^2-b}$$

input
```
integrate(cot(x)*(a+b*cot(x)^4)^(1/2),x, algorithm="giac")
```

output
```
-b*arctan(-(sqrt(a + b)*sin(x)^2 - sqrt(a*sin(x)^4 + b*sin(x)^4 - 2*b*sin(
x)^2 + b))/sqrt(-b))/sqrt(-b) - 1/2*sqrt(a + b)*log(abs(-(sqrt(a + b)*sin(
x)^2 - sqrt(a*sin(x)^4 + b*sin(x)^4 - 2*b*sin(x)^2 + b))*(a + b) + sqrt(a
+ b)*b)) - ((sqrt(a + b)*sin(x)^2 - sqrt(a*sin(x)^4 + b*sin(x)^4 - 2*b*sin
(x)^2 + b))*b - sqrt(a + b)*b)/((sqrt(a + b)*sin(x)^2 - sqrt(a*sin(x)^4 +
b*sin(x)^4 - 2*b*sin(x)^2 + b))^2 - b)
```

3.60.9 Mupad [F(-1)]

Timed out.

$$\int \cot(x)\sqrt{a+b\cot^4(x)}\,dx = \int \cot(x)\ \sqrt{b\cot{(x)}^4+a}\,dx$$

input
```
int(cot(x)*(a + b*cot(x)^4)^(1/2),x)
```

output
```
int(cot(x)*(a + b*cot(x)^4)^(1/2), x)
```

3.61 $\int \cot(x)\left(a+b\cot^4(x)\right)^{3/2}\,dx$

3.61.1 Optimal result

Integrand size = 15, antiderivative size = 126

$$\int \cot(x)\left(a+b\cot^4(x)\right)^{3/2}\,dx=\frac{1}{4}\sqrt{b}(3a+2b)\text{arctanh}\left(\frac{\sqrt{b}\cot^2(x)}{\sqrt{a+b\cot^4(x)}}\right)+\frac{1}{2}(a+b)^{3/2}\text{arctanh}\left(\frac{a-b\cot^2(x)}{\sqrt{a+b}\sqrt{a+b\cot^4(x)}}\right)-\frac{1}{4}\left(2(a+b)-b\cot^2(x)\right)\sqrt{a+b\cot^4(x)}-\frac{1}{6}\left(a+b\cot^4(x)\right)^{3/2}$$

output
```
1/2*(a+b)^(3/2)*arctanh((a-b*cot(x)^2)/(a+b)^(1/2)/(a+b*cot(x)^4)^(1/2))-1
/6*(a+b*cot(x)^4)^(3/2)+1/4*(3*a+2*b)*arctanh(cot(x)^2*b^(1/2)/(a+b*cot(x)
^4)^(1/2))*b^(1/2)-1/4*(2*a+2*b-b*cot(x)^2)*(a+b*cot(x)^4)^(1/2)
```

3.61.2 Mathematica [A] (verified)

Time = 4.81 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.33

$$\int \cot(x)\left(a+b\cot^4(x)\right)^{3/2}\,dx=\frac{1}{12}\left(6\sqrt{b}(a+b)\text{arctanh}\left(\frac{\sqrt{b}\cot^2(x)}{\sqrt{a+b\cot^4(x)}}\right)+6(a+b)^{3/2}\text{arctanh}\left(\frac{a-b\cot^2(x)}{\sqrt{a+b}\sqrt{a+b\cot^4(x)}}\right)-\sqrt{a+b\cot^4(x)}\left(8a+6b-3b\cot^2(x)+2b\cot^4(x)\right)+\frac{3\sqrt{a}\sqrt{b}\text{ar}}{}\right.$$

input
```
Integrate[Cot[x]*(a + b*Cot[x]^4)^(3/2),x]
```

output
```
(6*Sqrt[b]*(a + b)*ArcTanh[(Sqrt[b]*Cot[x]^2)/Sqrt[a + b*Cot[x]^4]] + 6*(a
 + b)^(3/2)*ArcTanh[(a - b*Cot[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Cot[x]^4])] -
 Sqrt[a + b*Cot[x]^4]*(8*a + 6*b - 3*b*Cot[x]^2 + 2*b*Cot[x]^4) + (3*Sqrt[
a]*Sqrt[b]*ArcSinh[(Sqrt[b]*Cot[x]^2)/Sqrt[a]]*Sqrt[a + b*Cot[x]^4])/Sqrt[
1 + (b*Cot[x]^4)/a])/12
```

3.61.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.03, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.867$, Rules used = {3042, 25, 4153, 25, 1577, 493, 682, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(x) \left(a+b\cot^4(x)\right)^{3/2} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\tan\left(x+\frac{\pi}{2}\right)\left(a+b\tan\left(x+\frac{\pi}{2}\right)^4\right)^{3/2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \tan\left(x+\frac{\pi}{2}\right)\left(b\tan\left(x+\frac{\pi}{2}\right)^4+a\right)^{3/2}dx$$

$$\Big\downarrow \text{4153}$$

$$\int -\frac{\cot(x)\left(a+b\cot^4(x)\right)^{3/2}}{\cot^2(x)+1}d\cot(x)$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\cot(x)\left(b\cot^4(x)+a\right)^{3/2}}{\cot^2(x)+1}d\cot(x)$$

$$\Big\downarrow \text{1577}$$

$$-\frac{1}{2}\int \frac{\left(b\cot^4(x)+a\right)^{3/2}}{\cot^2(x)+1}d\cot^2(x)$$

$$\Big\downarrow \text{493}$$

$$\frac{1}{2}\left(-\int \frac{\left(a-b\cot^2(x)\right)\sqrt{b\cot^4(x)+a}}{\cot^2(x)+1}d\cot^2(x)-\frac{1}{3}\left(a+b\cot^4(x)\right)^{3/2}\right)$$

$\downarrow$ 682

$$\frac{1}{2}\left(-\frac{\int \frac{b\left(a(2a+b)-b(3a+2b)\cot^2(x)\right)}{\left(\cot^2(x)+1\right)\sqrt{b\cot^4(x)+a}}d\cot^2(x)}{2b}-\frac{1}{3}\left(a+b\cot^4(x)\right)^{3/2}-\frac{1}{2}\left(2(a+b)-b\cot^2(x)\right)\sqrt{a+b\cot^4(x)}\right)$$

$\downarrow$ 27

$$\frac{1}{2}\left(-\frac{1}{2}\int \frac{a(2a+b)-b(3a+2b)\cot^2(x)}{\left(\cot^2(x)+1\right)\sqrt{b\cot^4(x)+a}}d\cot^2(x)-\frac{1}{3}\left(a+b\cot^4(x)\right)^{3/2}-\frac{1}{2}\left(2(a+b)-b\cot^2(x)\right)\sqrt{a+b\cot^4(x}\right.$$

$\downarrow$ 719

$$\frac{1}{2}\left(\frac{1}{2}\left(b(3a+2b)\int \frac{1}{\sqrt{b\cot^4(x)+a}}d\cot^2(x)-2(a+b)^2\int \frac{1}{\left(\cot^2(x)+1\right)\sqrt{b\cot^4(x)+a}}d\cot^2(x)\right)-\frac{1}{3}\left(a+b\,\mathrm{co}\right.\right.$$

$\downarrow$ 224

$$\frac{1}{2}\left(\frac{1}{2}\left(b(3a+2b)\int \frac{1}{1-b\cot^4(x)}d\frac{\cot^2(x)}{\sqrt{b\cot^4(x)+a}}-2(a+b)^2\int \frac{1}{\left(\cot^2(x)+1\right)\sqrt{b\cot^4(x)+a}}d\cot^2(x)\right)-\frac{1}{3}\left(a\right.\right.$$

$\downarrow$ 219

$$\frac{1}{2}\left(\frac{1}{2}\left(\sqrt{b}(3a+2b)\text{arctanh}\left(\frac{\sqrt{b}\cot^2(x)}{\sqrt{a+b\cot^4(x)}}\right)-2(a+b)^2\int \frac{1}{\left(\cot^2(x)+1\right)\sqrt{b\cot^4(x)+a}}d\cot^2(x)\right)-\frac{1}{3}\left(a+b\right.\right.$$

$\downarrow$ 488

$$\frac{1}{2}\left(\frac{1}{2}\left(2(a+b)^2\int \frac{1}{-\cot^4(x)+a+b}d\frac{a-b\cot^2(x)}{\sqrt{b\cot^4(x)+a}}+\sqrt{b}(3a+2b)\text{arctanh}\left(\frac{\sqrt{b}\cot^2(x)}{\sqrt{a+b\cot^4(x)}}\right)\right)-\frac{1}{3}\left(a+b\cot^4\right.\right.$$

$\downarrow$ 219

$$\frac{1}{2}\left(\frac{1}{2}\left(2(a+b)^{3/2}\text{arctanh}\left(\frac{a-b\cot^2(x)}{\sqrt{a+b}\sqrt{a+b\cot^4(x)}}\right)+\sqrt{b}(3a+2b)\text{arctanh}\left(\frac{\sqrt{b}\cot^2(x)}{\sqrt{a+b\cot^4(x)}}\right)\right)-\frac{1}{3}\left(a+b\cot^4(x\right.\right.$$

input
```
Int[Cot[x]*(a + b*Cot[x]^4)^(3/2),x]
```

3.61. $\int \cot(x)\left(a+b\cot^4(x)\right)^{3/2}\,dx$

output
```
((Sqrt[b]*(3*a + 2*b)*ArcTanh[(Sqrt[b]*Cot[x]^2)/Sqrt[a + b*Cot[x]^4]] + 2
*(a + b)^(3/2)*ArcTanh[(a - b*Cot[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Cot[x]^4])
])/2 - ((2*(a + b) - b*Cot[x]^2)*Sqrt[a + b*Cot[x]^4])/2 - (a + b*Cot[x]^4
)^(3/2)/3)/2
```

3.61.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 493
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 2*p + 1))), x] + Simp[2*(p/(d*(n +
 2*p + 1)))   Int[(c + d*x)^n*(a + b*x^2)^(p - 1)*(a*d - b*c*x), x], x] /;
FreeQ[{a, b, c, d, n}, x] && GtQ[p, 0] && NeQ[n + 2*p + 1, 0] && ( !Rationa
lQ[n] || LtQ[n, 1]) &&  !ILtQ[n + 2*p, 0] && IntQuadraticQ[a, 0, b, c, d, n
, p, x]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1577
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, c, d, e, p, q}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.61.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(103) = 206$.

Time = 0.07 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.94

method	result
derivativedivides	$\frac{b^{\frac{3}{2}} \ln\left(\sqrt{b}\,\cot(x)^2+\sqrt{a+b\cot(x)^4}\right)}{2}-\frac{b^2\left(\frac{\cot(x)^4\sqrt{a+b\cot(x)^4}}{3b}-\frac{2a\sqrt{a+b\cot(x)^4}}{3b^2}\right)}{2}-\frac{b\sqrt{a+b\cot(x)^4}}{2}+\sqrt{b}\,a\ln\big($
default	$\frac{b^{\frac{3}{2}} \ln\left(\sqrt{b}\,\cot(x)^2+\sqrt{a+b\cot(x)^4}\right)}{2}-\frac{b^2\left(\frac{\cot(x)^4\sqrt{a+b\cot(x)^4}}{3b}-\frac{2a\sqrt{a+b\cot(x)^4}}{3b^2}\right)}{2}-\frac{b\sqrt{a+b\cot(x)^4}}{2}+\sqrt{b}\,a\ln\big($

input
```
int(cot(x)*(a+b*cot(x)^4)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*b^(3/2)*ln(b^(1/2)*cot(x)^2+(a+b*cot(x)^4)^(1/2))-1/2*b^2*(1/3*cot(x)^
4/b*(a+b*cot(x)^4)^(1/2)-2/3*a/b^2*(a+b*cot(x)^4)^(1/2))-1/2*b*(a+b*cot(x)
^4)^(1/2)+b^(1/2)*a*ln(b^(1/2)*cot(x)^2+(a+b*cot(x)^4)^(1/2))+1/2*b^2*(1/2
*cot(x)^2/b*(a+b*cot(x)^4)^(1/2)-1/2*a/b^(3/2)*ln(b^(1/2)*cot(x)^2+(a+b*co
t(x)^4)^(1/2)))-a*(a+b*cot(x)^4)^(1/2)+1/2*(a^2+2*a*b+b^2)/(a+b)^(1/2)*ln(
(2*a+2*b-2*b*(cot(x)^2+1)+2*(a+b)^(1/2)*(b*(cot(x)^2+1)^2-2*b*(cot(x)^2+1)
+a+b)^(1/2))/(cot(x)^2+1))
```

3.61.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 357 vs. 2(104) = 208.

Time = 0.48 (sec) , antiderivative size = 1486, normalized size of antiderivative = 11.79

$$\int \cot(x)\left(a+b\cot^4(x)\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(cot(x)*(a+b*cot(x)^4)^(3/2),x, algorithm="fricas")
```

output
```
[1/24*(6*((a + b)*cos(2*x)^2 - 2*(a + b)*cos(2*x) + a + b)*sqrt(a + b)*log
(1/2*(a^2 + 2*a*b + b^2)*cos(2*x)^2 + 1/2*a^2 + 1/2*b^2 + 1/2*((a + b)*cos
(2*x)^2 - 2*a*cos(2*x) + a - b)*sqrt(a + b)*sqrt(((a + b)*cos(2*x)^2 - 2*(
a - b)*cos(2*x) + a + b)/(cos(2*x)^2 - 2*cos(2*x) + 1)) - (a^2 - b^2)*cos(
2*x)) + 3*((3*a + 2*b)*cos(2*x)^2 - 2*(3*a + 2*b)*cos(2*x) + 3*a + 2*b)*sq
rt(b)*log(-((a + 2*b)*cos(2*x)^2 - 2*(cos(2*x)^2 - 1)*sqrt(b)*sqrt(((a + b
)*cos(2*x)^2 - 2*(a - b)*cos(2*x) + a + b)/(cos(2*x)^2 - 2*cos(2*x) + 1))
- 2*(a - 2*b)*cos(2*x) + a + 2*b)/(cos(2*x)^2 - 2*cos(2*x) + 1)) - 2*((8*a
 + 11*b)*cos(2*x)^2 - 8*(2*a + b)*cos(2*x) + 8*a + 5*b)*sqrt(((a + b)*cos(
2*x)^2 - 2*(a - b)*cos(2*x) + a + b)/(cos(2*x)^2 - 2*cos(2*x) + 1)))/(cos(
2*x)^2 - 2*cos(2*x) + 1), 1/12*(3*((3*a + 2*b)*cos(2*x)^2 - 2*(3*a + 2*b)*
cos(2*x) + 3*a + 2*b)*sqrt(-b)*arctan(sqrt(-b)*sqrt(((a + b)*cos(2*x)^2 -
2*(a - b)*cos(2*x) + a + b)/(cos(2*x)^2 - 2*cos(2*x) + 1))*(cos(2*x) - 1)/
(b*cos(2*x) + b)) + 3*((a + b)*cos(2*x)^2 - 2*(a + b)*cos(2*x) + a + b)*sq
rt(a + b)*log(1/2*(a^2 + 2*a*b + b^2)*cos(2*x)^2 + 1/2*a^2 + 1/2*b^2 + 1/2
*((a + b)*cos(2*x)^2 - 2*a*cos(2*x) + a - b)*sqrt(a + b)*sqrt(((a + b)*cos
(2*x)^2 - 2*(a - b)*cos(2*x) + a + b)/(cos(2*x)^2 - 2*cos(2*x) + 1)) - (a^
2 - b^2)*cos(2*x)) - ((8*a + 11*b)*cos(2*x)^2 - 8*(2*a + b)*cos(2*x) + 8*a
 + 5*b)*sqrt(((a + b)*cos(2*x)^2 - 2*(a - b)*cos(2*x) + a + b)/(cos(2*x)^2
 - 2*cos(2*x) + 1)))/(cos(2*x)^2 - 2*cos(2*x) + 1), -1/24*(12*((a + b)*...
```

3.61.6 Sympy [F]

$$\int \cot(x)\left(a+b\cot^4(x)\right)^{3/2}\,dx=\int \left(a+b\cot^4\left(x\right)\right)^{\frac{3}{2}}\cot\left(x\right)dx$$

input
```
integrate(cot(x)*(a+b*cot(x)**4)**(3/2),x)
```

output
```
Integral((a + b*cot(x)**4)**(3/2)*cot(x), x)
```

3.61.7 Maxima [F]

$$\int \cot(x)\left(a+b\cot^4(x)\right)^{3/2}\,dx=\int \left(b\cot(x)^4+a\right)^{\frac{3}{2}}\cot(x)\,dx$$

input
```
integrate(cot(x)*(a+b*cot(x)^4)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*cot(x)^4 + a)^(3/2)*cot(x), x)
```

3.61.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 445 vs. 2(104) = 208.

Time = 0.54 (sec) , antiderivative size = 445, normalized size of antiderivative = 3.53

$$\int \cot(x)\left(a+b\cot^4(x)\right)^{3/2}\,dx=$$

$$-\frac{(3\,ab+2\,b^2)\arctan\left(-\frac{\sqrt{a+b}\sin(x)^2-\sqrt{a\sin(x)^4+b\sin(x)^4-2\,b\sin(x)^2+b}}{\sqrt{-b}}\right)}{2\sqrt{-b}}$$

$$-\frac{(a^2+2\,ab+b^2)\log\left(\left|-\left(\sqrt{a+b}\sin(x)^2-\sqrt{a\sin(x)^4+b\sin(x)^4-2\,b\sin(x)^2+b}\right)(a+b)+\sqrt{a+b}b\right|\right.}{2\sqrt{a+b}}$$

$$-\frac{3\left(\sqrt{a+b}\sin(x)^2-\sqrt{a\sin(x)^4+b\sin(x)^4-2\,b\sin(x)^2+b}\right)^5(5\,ab+6\,b^2)+8\left(\sqrt{a+b}\sin(x)^2-\sqrt{a\sin\ldots}\right.}{}$$

input
```
integrate(cot(x)*(a+b*cot(x)^4)^(3/2),x, algorithm="giac")
```

output
```
-1/2*(3*a*b + 2*b^2)*arctan(-(sqrt(a + b)*sin(x)^2 - sqrt(a*sin(x)^4 + b*s
in(x)^4 - 2*b*sin(x)^2 + b))/sqrt(-b))/sqrt(-b) - 1/2*(a^2 + 2*a*b + b^2)*
log(abs(-(sqrt(a + b)*sin(x)^2 - sqrt(a*sin(x)^4 + b*sin(x)^4 - 2*b*sin(x)
^2 + b))*(a + b) + sqrt(a + b)*b))/sqrt(a + b) - 1/6*(3*(sqrt(a + b)*sin(x
)^2 - sqrt(a*sin(x)^4 + b*sin(x)^4 - 2*b*sin(x)^2 + b))^5*(5*a*b + 6*b^2)
+ 8*(sqrt(a + b)*sin(x)^2 - sqrt(a*sin(x)^4 + b*sin(x)^4 - 2*b*sin(x)^2 +
b))^3*b^3 - 12*(sqrt(a + b)*sin(x)^2 - sqrt(a*sin(x)^4 + b*sin(x)^4 - 2*b*
sin(x)^2 + b))^4*(a*b + 3*b^2)*sqrt(a + b) + 12*(a*b^2 + b^3)*(sqrt(a + b)
*sin(x)^2 - sqrt(a*sin(x)^4 + b*sin(x)^4 - 2*b*sin(x)^2 + b))^2*sqrt(a + b
) + 3*(3*a*b^3 + 2*b^4)*(sqrt(a + b)*sin(x)^2 - sqrt(a*sin(x)^4 + b*sin(x)
^4 - 2*b*sin(x)^2 + b)) - 8*(a*b^3 + b^4)*sqrt(a + b))/((sqrt(a + b)*sin(x
)^2 - sqrt(a*sin(x)^4 + b*sin(x)^4 - 2*b*sin(x)^2 + b))^2 - b)^3
```

3.61.9 Mupad [F(-1)]

Timed out.

$$\int \cot(x) \left(a + b\cot^4(x)\right)^{3/2} \, dx = \int \cot(x) \, \left(b\cot(x)^4 + a\right)^{3/2} \, dx$$

input
```
int(cot(x)*(a + b*cot(x)^4)^(3/2),x)
```

output
```
int(cot(x)*(a + b*cot(x)^4)^(3/2), x)
```

3.62 $\int \frac{\cot(x)}{\sqrt{a+b\cot^4(x)}}\,dx$

3.62.1 Optimal result

Integrand size = 15, antiderivative size = 41

$$\int \frac{\cot(x)}{\sqrt{a+b\cot^4(x)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{a-b\cot^2(x)}{\sqrt{a+b}\sqrt{a+b\cot^4(x)}}\right)}{2\sqrt{a+b}}$$

output
```
1/2*arctanh((a-b*cot(x)^2)/(a+b)^(1/2)/(a+b*cot(x)^4)^(1/2))/(a+b)^(1/2)
```

3.62.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{\cot(x)}{\sqrt{a+b\cot^4(x)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{a-b\cot^2(x)}{\sqrt{a+b}\sqrt{a+b\cot^4(x)}}\right)}{2\sqrt{a+b}}$$

input
```
Integrate[Cot[x]/Sqrt[a + b*Cot[x]^4],x]
```

output
```
ArcTanh[(a - b*Cot[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Cot[x]^4])]/(2*Sqrt[a + b
])
```

3.62.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {3042, 25, 4153, 25, 1577, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(x)}{\sqrt{a+b\cot^4(x)}}dx$$

$$\Big\downarrow \ 3042$$

$$\int -\frac{\tan\left(x+\frac{\pi}{2}\right)}{\sqrt{a+b\tan\left(x+\frac{\pi}{2}\right)^4}}dx$$

$$\Big\downarrow \ 25$$

$$-\int \frac{\tan\left(x+\frac{\pi}{2}\right)}{\sqrt{b\tan\left(x+\frac{\pi}{2}\right)^4+a}}dx$$

$$\Big\downarrow \ 4153$$

$$\int -\frac{\cot(x)}{(\cot^2(x)+1)\sqrt{a+b\cot^4(x)}}d\cot(x)$$

$$\Big\downarrow \ 25$$

$$-\int \frac{\cot(x)}{(\cot^2(x)+1)\sqrt{b\cot^4(x)+a}}d\cot(x)$$

$$\Big\downarrow \ 1577$$

$$-\frac{1}{2}\int \frac{1}{(\cot^2(x)+1)\sqrt{b\cot^4(x)+a}}d\cot^2(x)$$

$$\Big\downarrow \ 488$$

$$\frac{1}{2}\int \frac{1}{-\cot^4(x)+a+b}d\frac{a-b\cot^2(x)}{\sqrt{b\cot^4(x)+a}}$$

$$\Big\downarrow \ 219$$

$$\frac{\text{arctanh}\left(\frac{a-b\cot^2(x)}{\sqrt{a+b}\sqrt{a+b\cot^4(x)}}\right)}{2\sqrt{a+b}}$$

input
```
Int[Cot[x]/Sqrt[a + b*Cot[x]^4],x]
```

output
```
ArcTanh[(a - b*Cot[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Cot[x]^4])]/(2*Sqrt[a + b
])
```

3.62.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 1577
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, c, d, e, p, q}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.62. $\int \frac{\cot(x)}{\sqrt{a+b\cot^4(x)}}\,dx$

3.62.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.59

method	result	size
derivativedivides	$\frac{\ln\left(\frac{2a+2b-2b\left(\cot(x)^{2}+1\right)+2\sqrt{a+b}\,\sqrt{b\left(\cot(x)^{2}+1\right)^{2}-2b\left(\cot(x)^{2}+1\right)+a+b}}{\cot(x)^{2}+1}\right)}{2\sqrt{a+b}}$	65
default	$\frac{\ln\left(\frac{2a+2b-2b\left(\cot(x)^{2}+1\right)+2\sqrt{a+b}\,\sqrt{b\left(\cot(x)^{2}+1\right)^{2}-2b\left(\cot(x)^{2}+1\right)+a+b}}{\cot(x)^{2}+1}\right)}{2\sqrt{a+b}}$	65

input
```
int(cot(x)/(a+b*cot(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/(a+b)^(1/2)*ln((2*a+2*b-2*b*(cot(x)^2+1)+2*(a+b)^(1/2)*(b*(cot(x)^2+1)
^2-2*b*(cot(x)^2+1)+a+b)^(1/2))/(cot(x)^2+1))
```

3.62.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(35) = 70.

Time = 0.39 (sec) , antiderivative size = 264, normalized size of antiderivative = 6.44

$$\int \frac{\cot(x)}{\sqrt{a+b\cot^4(x)}}\,dx$$

$$= \left[\frac{\log\left(\frac{1}{2}\left(a^2+2\,ab+b^2\right)\cos\left(2\,x\right)^2+\frac{1}{2}\,a^2+\frac{1}{2}\,b^2+\frac{1}{2}\left(\left(a+b\right)\cos\left(2\,x\right)^2-2\,a\cos\left(2\,x\right)+a-b\right)\sqrt{a+b}\sqrt{\frac{(a-}{}}\right.}{4\sqrt{a+b}}\right.$$

$$\left.-\frac{\sqrt{-a-b}\arctan\left(\frac{\left(\left(a+b\right)\cos(2\,x)^2-2\,a\cos(2\,x)+a-b\right)\sqrt{-a-b}\sqrt{\frac{(a+b)\cos(2\,x)^2-2\,(a-b)\cos(2\,x)+a+b}{\cos(2\,x)^2-2\,\cos(2\,x)+1}}}{\left(a^2+2\,ab+b^2\right)\cos(2\,x)^2+a^2+2\,ab+b^2-2\,\left(a^2-b^2\right)\cos(2\,x)}\right)}{2\,(a+b)}\right]$$

input
```
integrate(cot(x)/(a+b*cot(x)^4)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*log(1/2*(a^2 + 2*a*b + b^2)*cos(2*x)^2 + 1/2*a^2 + 1/2*b^2 + 1/2*((a
+ b)*cos(2*x)^2 - 2*a*cos(2*x) + a - b)*sqrt(a + b)*sqrt(((a + b)*cos(2*x)
^2 - 2*(a - b)*cos(2*x) + a + b)/(cos(2*x)^2 - 2*cos(2*x) + 1)) - (a^2 - b
^2)*cos(2*x))/sqrt(a + b), -1/2*sqrt(-a - b)*arctan(((a + b)*cos(2*x)^2 -
2*a*cos(2*x) + a - b)*sqrt(-a - b)*sqrt(((a + b)*cos(2*x)^2 - 2*(a - b)*co
s(2*x) + a + b)/(cos(2*x)^2 - 2*cos(2*x) + 1))/((a^2 + 2*a*b + b^2)*cos(2*
x)^2 + a^2 + 2*a*b + b^2 - 2*(a^2 - b^2)*cos(2*x)))/(a + b)]
```

3.62.6 Sympy [F]

$$\int \frac{\cot(x)}{\sqrt{a+b\cot^4(x)}}\,dx=\int \frac{\cot{(x)}}{\sqrt{a+b\cot^4{(x)}}}\,dx$$

input
```
integrate(cot(x)/(a+b*cot(x)**4)**(1/2),x)
```

output
```
Integral(cot(x)/sqrt(a + b*cot(x)**4), x)
```

3.62.7 Maxima [F]

$$\int \frac{\cot(x)}{\sqrt{a+b\cot^4(x)}}\,dx=\int \frac{\cot{(x)}}{\sqrt{b\cot{(x)}^4+a}}\,dx$$

input
```
integrate(cot(x)/(a+b*cot(x)^4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(cot(x)/sqrt(b*cot(x)^4 + a), x)
```

3.62.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.41

$$\int \frac{\cot(x)}{\sqrt{a+b\cot^4(x)}}\,dx = -\frac{\log\left(\left|-\left(\sqrt{a+b}\sin{(x)}^2-\sqrt{a\sin{(x)}^4+b\sin{(x)}^4-2\,b\sin{(x)}^2+b}\right)(a+b)+\sqrt{a+b}b\right|\right)}{2\sqrt{a+b}}$$

input
```
integrate(cot(x)/(a+b*cot(x)^4)^(1/2),x, algorithm="giac")
```

output
```
-1/2*log(abs(-(sqrt(a + b)*sin(x)^2 - sqrt(a*sin(x)^4 + b*sin(x)^4 - 2*b*s
in(x)^2 + b))*(a + b) + sqrt(a + b)*b))/sqrt(a + b)
```

3.62.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot(x)}{\sqrt{a+b\cot^4(x)}}\,dx = \int \frac{\cot(x)}{\sqrt{b\cot{(x)}^4+a}}\,dx$$

input
```
int(cot(x)/(a + b*cot(x)^4)^(1/2),x)
```

output
```
int(cot(x)/(a + b*cot(x)^4)^(1/2), x)
```

3.63 $\int \frac{\cot(x)}{(a+b\cot^4(x))^{3/2}}\,dx$

3.63.1 Optimal result

Integrand size = 15, antiderivative size = 74

$$\int \frac{\cot(x)}{(a+b\cot^4(x))^{3/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{a-b\cot^2(x)}{\sqrt{a+b}\sqrt{a+b\cot^4(x)}}\right)}{2(a+b)^{3/2}} - \frac{a+b\cot^2(x)}{2a(a+b)\sqrt{a+b\cot^4(x)}}$$

output
```
1/2*arctanh((a-b*cot(x)^2)/(a+b)^(1/2)/(a+b*cot(x)^4)^(1/2))/(a+b)^(3/2)+1
/2*(-a-b*cot(x)^2)/a/(a+b)/(a+b*cot(x)^4)^(1/2)
```

3.63.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.99

$$\int \frac{\cot(x)}{(a+b\cot^4(x))^{3/2}}\,dx = \frac{1}{2}\left(\frac{\operatorname{arctanh}\left(\frac{a-b\cot^2(x)}{\sqrt{a+b}\sqrt{a+b\cot^4(x)}}\right)}{(a+b)^{3/2}} - \frac{a+b\cot^2(x)}{a(a+b)\sqrt{a+b\cot^4(x)}}\right)$$

input
```
Integrate[Cot[x]/(a + b*Cot[x]^4)^(3/2),x]
```

output
```
(ArcTanh[(a - b*Cot[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Cot[x]^4])]/(a + b)^(3/2
) - (a + b*Cot[x]^2)/(a*(a + b)*Sqrt[a + b*Cot[x]^4]))/2
```

3.63.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3042, 25, 4153, 25, 1577, 496, 25, 27, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(x)}{\left(a+b\cot^4(x)\right)^{3/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{\tan\left(x+\frac{\pi}{2}\right)}{\left(a+b\tan\left(x+\frac{\pi}{2}\right)^4\right)^{3/2}}dx$$

$$\Big\downarrow \; 25$$

$$-\int \frac{\tan\left(x+\frac{\pi}{2}\right)}{\left(b\tan\left(x+\frac{\pi}{2}\right)^4+a\right)^{3/2}}dx$$

$$\Big\downarrow \; 4153$$

$$\int -\frac{\cot(x)}{\left(\cot^2(x)+1\right)\left(a+b\cot^4(x)\right)^{3/2}}d\cot(x)$$

$$\Big\downarrow \; 25$$

$$-\int \frac{\cot(x)}{\left(\cot^2(x)+1\right)\left(b\cot^4(x)+a\right)^{3/2}}d\cot(x)$$

$$\Big\downarrow \; 1577$$

$$-\frac{1}{2}\int \frac{1}{\left(\cot^2(x)+1\right)\left(b\cot^4(x)+a\right)^{3/2}}d\cot^2(x)$$

$$\Big\downarrow \; 496$$

$$\frac{1}{2}\left(\frac{\int -\frac{a}{\left(\cot^2(x)+1\right)\sqrt{b\cot^4(x)+a}}d\cot^2(x)}{a(a+b)}-\frac{a+b\cot^2(x)}{a(a+b)\sqrt{a+b\cot^4(x)}}\right)$$

$$\Big\downarrow \; 25$$

$$\frac{1}{2}\left(-\frac{\int \frac{a}{\left(\cot^2(x)+1\right)\sqrt{b\cot^4(x)+a}}d\cot^2(x)}{a(a+b)}-\frac{a+b\cot^2(x)}{a(a+b)\sqrt{a+b\cot^4(x)}}\right)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{2}\left(-\frac{\int \frac{1}{(\cot^2(x)+1)\sqrt{b\cot^4(x)+a}}d\cot^2(x)}{a+b}-\frac{a+b\cot^2(x)}{a(a+b)\sqrt{a+b\cot^4(x)}}\right)$$

$$\Big\downarrow \; 488$$

$$\frac{1}{2}\left(\frac{\int \frac{1}{-\cot^4(x)+a+b}d\frac{a-b\cot^2(x)}{\sqrt{b\cot^4(x)+a}}}{a+b}-\frac{a+b\cot^2(x)}{a(a+b)\sqrt{a+b\cot^4(x)}}\right)$$

$$\Big\downarrow \; 219$$

$$\frac{1}{2}\left(\frac{\text{arctanh}\left(\frac{a-b\cot^2(x)}{\sqrt{a+b}\sqrt{a+b\cot^4(x)}}\right)}{(a+b)^{3/2}}-\frac{a+b\cot^2(x)}{a(a+b)\sqrt{a+b\cot^4(x)}}\right)$$

input
```
Int[Cot[x]/(a + b*Cot[x]^4)^(3/2),x]
```

output
```
(ArcTanh[(a - b*Cot[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Cot[x]^4])]/(a + b)^(3/2
) - (a + b*Cot[x]^2)/(a*(a + b)*Sqrt[a + b*Cot[x]^4]))/2
```

3.63.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 1577
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, c, d, e, p, q}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.63.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(65) = 130$.

Time = 1.02 (sec) , antiderivative size = 248, normalized size of antiderivative = 3.35

method	result
derivativedivides	$-\frac{b\ln\left(\frac{2a+2b-2b\left(\cot(x)^2+1\right)+2\sqrt{a+b}\,\sqrt{b\left(\cot(x)^2+1\right)^2-2b\left(\cot(x)^2+1\right)+a+b}}{\cot(x)^2+1}\right)}{2\left(b+\sqrt{-ab}\right)\left(-b+\sqrt{-ab}\right)\sqrt{a+b}}+\frac{\sqrt{\left(\cot(x)^2+\frac{\sqrt{-ab}}{b}\right)^2 b-2\sqrt{-ab}\left(\cot(x)\cdots\right.}}{4a\left(-b+\sqrt{-ab}\right)\left(\cot(x)^2+\frac{\sqrt{-a\cdots}}{b}\right.}$
default	$-\frac{b\ln\left(\frac{2a+2b-2b\left(\cot(x)^2+1\right)+2\sqrt{a+b}\,\sqrt{b\left(\cot(x)^2+1\right)^2-2b\left(\cot(x)^2+1\right)+a+b}}{\cot(x)^2+1}\right)}{2\left(b+\sqrt{-ab}\right)\left(-b+\sqrt{-ab}\right)\sqrt{a+b}}+\frac{\sqrt{\left(\cot(x)^2+\frac{\sqrt{-ab}}{b}\right)^2 b-2\sqrt{-ab}\left(\cot(x)\cdots\right.}}{4a\left(-b+\sqrt{-ab}\right)\left(\cot(x)^2+\frac{\sqrt{-a\cdots}}{b}\right.}$

input
```
int(cot(x)/(a+b*cot(x)^4)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*b/(b+(-a*b)^(1/2))/(-b+(-a*b)^(1/2))/(a+b)^(1/2)*ln((2*a+2*b-2*b*(cot
(x)^2+1)+2*(a+b)^(1/2)*(b*(cot(x)^2+1)^2-2*b*(cot(x)^2+1)+a+b)^(1/2))/(cot
(x)^2+1))+1/4/a/(-b+(-a*b)^(1/2))/(cot(x)^2+(-a*b)^(1/2)/b)*((cot(x)^2+(-a
*b)^(1/2)/b)^2*b-2*(-a*b)^(1/2)*(cot(x)^2+(-a*b)^(1/2)/b))^(1/2)-1/4/a/(b+
(-a*b)^(1/2))/(cot(x)^2-(-a*b)^(1/2)/b)*((cot(x)^2-(-a*b)^(1/2)/b)^2*b+2*(
-a*b)^(1/2)*(cot(x)^2-(-a*b)^(1/2)/b))^(1/2)
```

3.63.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(64) = 128$.

Time = 0.45 (sec) , antiderivative size = 670, normalized size of antiderivative = 9.05

$$\int \frac{\cot(x)}{(a+b\cot^4(x))^{3/2}}\,dx = \left[\frac{\left((a^2+ab)\cos(2x)^2+a^2+ab-2(a^2-ab)\cos(2x)\right)\sqrt{a+b}\log\left(\frac{1}{2}(a^2+2ab+\right.}{}\right.$$

$$-\frac{\left((a^2+ab)\cos(2x)^2+a^2+ab-2(a^2-ab)\cos(2x)\right)\sqrt{-a-b}\arctan\left(\frac{\left((a+b)\cos(2x)^2-2a\cos(2x)+a-b\right)\sqrt{-a-b}}{(a^2+2ab+b^2)\cos(2x)^2+a^2+2ab}\right.}{2\left(a^4+3a^3b+3a^2b^2+ab^3+(a^4+3a^3b+\right.}$$

input
```
integrate(cot(x)/(a+b*cot(x)^4)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(((a^2 + a*b)*cos(2*x)^2 + a^2 + a*b - 2*(a^2 - a*b)*cos(2*x))*sqrt(a
 + b)*log(1/2*(a^2 + 2*a*b + b^2)*cos(2*x)^2 + 1/2*a^2 + 1/2*b^2 + 1/2*((a
 + b)*cos(2*x)^2 - 2*a*cos(2*x) + a - b)*sqrt(a + b)*sqrt(((a + b)*cos(2*x
)^2 - 2*(a - b)*cos(2*x) + a + b)/(cos(2*x)^2 - 2*cos(2*x) + 1)) - (a^2 -
b^2)*cos(2*x)) - 2*((a^2 - b^2)*cos(2*x)^2 + a^2 + 2*a*b + b^2 - 2*(a^2 +
a*b)*cos(2*x))*sqrt(((a + b)*cos(2*x)^2 - 2*(a - b)*cos(2*x) + a + b)/(cos
(2*x)^2 - 2*cos(2*x) + 1)))/(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3 + (a^4 + 3*
a^3*b + 3*a^2*b^2 + a*b^3)*cos(2*x)^2 - 2*(a^4 + a^3*b - a^2*b^2 - a*b^3)*
cos(2*x)), -1/2*(((a^2 + a*b)*cos(2*x)^2 + a^2 + a*b - 2*(a^2 - a*b)*cos(2
*x))*sqrt(-a - b)*arctan(((a + b)*cos(2*x)^2 - 2*a*cos(2*x) + a - b)*sqrt(
-a - b)*sqrt(((a + b)*cos(2*x)^2 - 2*(a - b)*cos(2*x) + a + b)/(cos(2*x)^2
 - 2*cos(2*x) + 1))/((a^2 + 2*a*b + b^2)*cos(2*x)^2 + a^2 + 2*a*b + b^2 -
2*(a^2 - b^2)*cos(2*x))) + ((a^2 - b^2)*cos(2*x)^2 + a^2 + 2*a*b + b^2 - 2
*(a^2 + a*b)*cos(2*x))*sqrt(((a + b)*cos(2*x)^2 - 2*(a - b)*cos(2*x) + a +
 b)/(cos(2*x)^2 - 2*cos(2*x) + 1)))/(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3 + (
a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cos(2*x)^2 - 2*(a^4 + a^3*b - a^2*b^2 -
 a*b^3)*cos(2*x))]
```

3.63.6 Sympy [F]

$$\int \frac{\cot(x)}{(a+b\cot^4(x))^{3/2}}\,dx = \int \frac{\cot{(x)}}{(a+b\cot^4{(x)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(x)/(a+b*cot(x)**4)**(3/2),x)
```

output
```
Integral(cot(x)/(a + b*cot(x)**4)**(3/2), x)
```

3.63.7 Maxima [F]

$$\int \frac{\cot(x)}{(a+b\cot^4(x))^{3/2}}\,dx = \int \frac{\cot{(x)}}{(b\cot{(x)}^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(x)/(a+b*cot(x)^4)^(3/2),x, algorithm="maxima")
```

output
```
integrate(cot(x)/(b*cot(x)^4 + a)^(3/2), x)
```

3.63.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.50

$$\int \frac{\cot(x)}{(a+b\cot^4(x))^{3/2}}\,dx = -\frac{\frac{(a-b)\sin(x)^2}{a^2+ab}+\frac{b}{a^2+ab}}{2\sqrt{a\sin(x)^4+b\sin(x)^4-2b\sin(x)^2+b}} - \frac{\log\left(\left|-\left(\sqrt{a+b}\sin(x)^2-\sqrt{a\sin(x)^4+b\sin(x)^4-2b\sin(x)^2+b}\right)\sqrt{a+b}+b\right|\right)}{2(a+b)^{\frac{3}{2}}}$$

input
```
integrate(cot(x)/(a+b*cot(x)^4)^(3/2),x, algorithm="giac")
```

output
```
-1/2*((a - b)*sin(x)^2/(a^2 + a*b) + b/(a^2 + a*b))/sqrt(a*sin(x)^4 + b*si
n(x)^4 - 2*b*sin(x)^2 + b) - 1/2*log(abs(-(sqrt(a + b)*sin(x)^2 - sqrt(a*s
in(x)^4 + b*sin(x)^4 - 2*b*sin(x)^2 + b))*sqrt(a + b) + b))/(a + b)^(3/2)
```

3.63.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot(x)}{(a+b\cot^4(x))^{3/2}}\,dx = \int \frac{\cot(x)}{\left(b\cot(x)^4+a\right)^{3/2}}\,dx$$

input
```
int(cot(x)/(a + b*cot(x)^4)^(3/2),x)
```

output
```
int(cot(x)/(a + b*cot(x)^4)^(3/2), x)
```

3.64 $\int \frac{\cot(x)}{\left(a+b\cot^4(x)\right)^{5/2}}\,dx$

3.64.1 Optimal result

Integrand size = 15, antiderivative size = 117

$$\int \frac{\cot(x)}{\left(a+b\cot^4(x)\right)^{5/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{a-b\cot^2(x)}{\sqrt{a+b}\sqrt{a+b\cot^4(x)}}\right)}{2(a+b)^{5/2}} - \frac{a+b\cot^2(x)}{6a(a+b)\left(a+b\cot^4(x)\right)^{3/2}} - \frac{3a^2+b(5a+2b)\cot^2(x)}{6a^2(a+b)^2\sqrt{a+b\cot^4(x)}}$$

output
```
1/2*arctanh((a-b*cot(x)^2)/(a+b)^(1/2)/(a+b*cot(x)^4)^(1/2))/(a+b)^(5/2)+1
/6*(-a-b*cot(x)^2)/a/(a+b)/(a+b*cot(x)^4)^(3/2)+1/6*(-3*a^2-b*(5*a+2*b)*co
t(x)^2)/a^2/(a+b)^2/(a+b*cot(x)^4)^(1/2)
```

3.64.2 Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.97

$$\int \frac{\cot(x)}{\left(a+b\cot^4(x)\right)^{5/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{a-b\cot^2(x)}{\sqrt{a+b}\sqrt{a+b\cot^4(x)}}\right)}{2(a+b)^{5/2}} - \frac{a^2(4a+b)+3ab(2a+b)\cot^2(x)+3a^2b\cot^4(x)+b^2(5a+2b)\cot^6(x)}{6a^2(a+b)^2\left(a+b\cot^4(x)\right)^{3/2}}$$

input
```
Integrate[Cot[x]/(a + b*Cot[x]^4)^(5/2),x]
```

output
```
ArcTanh[(a - b*Cot[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Cot[x]^4])]/(2*(a + b)^(5
/2)) - (a^2*(4*a + b) + 3*a*b*(2*a + b)*Cot[x]^2 + 3*a^2*b*Cot[x]^4 + b^2*
(5*a + 2*b)*Cot[x]^6)/(6*a^2*(a + b)^2*(a + b*Cot[x]^4)^(3/2))
```

3.64.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.11, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.733$, Rules used = {3042, 25, 4153, 25, 1577, 496, 25, 686, 27, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(x)}{(a+b\cot^4(x))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\tan\left(x+\frac{\pi}{2}\right)}{\left(a+b\tan\left(x+\frac{\pi}{2}\right)^4\right)^{5/2}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\tan\left(x+\frac{\pi}{2}\right)}{\left(b\tan\left(x+\frac{\pi}{2}\right)^4+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4153}$$

$$\int -\frac{\cot(x)}{(\cot^2(x)+1)\,(a+b\cot^4(x))^{5/2}}d\cot(x)$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\cot(x)}{(\cot^2(x)+1)\,(b\cot^4(x)+a)^{5/2}}d\cot(x)$$

$$\Big\downarrow \text{1577}$$

$$-\frac{1}{2}\int \frac{1}{(\cot^2(x)+1)\,(b\cot^4(x)+a)^{5/2}}d\cot^2(x)$$

$$\Big\downarrow \text{496}$$

$$\frac{1}{2}\left(\frac{\int -\frac{2b\cot^2(x)+3a+2b}{(\cot^2(x)+1)(b\cot^4(x)+a)^{3/2}}d\cot^2(x)}{3a(a+b)}-\frac{a+b\cot^2(x)}{3a(a+b)\,(a+b\cot^4(x))^{3/2}}\right)$$

$$\Big\downarrow \; 25$$

$$\frac{1}{2}\left(-\frac{\int \frac{2b\cot^2(x)+3a+2b}{(\cot^2(x)+1)(b\cot^4(x)+a)^{3/2}}d\cot^2(x)}{3a(a+b)}-\frac{a+b\cot^2(x)}{3a(a+b)\left(a+b\cot^4(x)\right)^{3/2}}\right)$$

$$\Big\downarrow \; 686$$

$$\frac{1}{2}\left(-\frac{\frac{3a^2+b(5a+2b)\cot^2(x)}{a(a+b)\sqrt{a+b\cot^4(x)}}-\frac{\int -\frac{3a^2b}{\left(\cot^2(x)+1\right)\sqrt{b\cot^4(x)+a}}d\cot^2(x)}{ab(a+b)}}{3a(a+b)}-\frac{a+b\cot^2(x)}{3a(a+b)\left(a+b\cot^4(x)\right)^{3/2}}\right)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{2}\left(-\frac{\frac{3a\int \frac{1}{\left(\cot^2(x)+1\right)\sqrt{b\cot^4(x)+a}}d\cot^2(x)}{a+b}+\frac{3a^2+b(5a+2b)\cot^2(x)}{a(a+b)\sqrt{a+b\cot^4(x)}}}{3a(a+b)}-\frac{a+b\cot^2(x)}{3a(a+b)\left(a+b\cot^4(x)\right)^{3/2}}\right)$$

$$\Big\downarrow \; 488$$

$$\frac{1}{2}\left(-\frac{\frac{3a^2+b(5a+2b)\cot^2(x)}{a(a+b)\sqrt{a+b\cot^4(x)}}-\frac{3a\int \frac{1}{-\cot^4(x)+a+b}d\frac{a-b\cot^2(x)}{\sqrt{b\cot^4(x)+a}}}{a+b}}{3a(a+b)}-\frac{a+b\cot^2(x)}{3a(a+b)\left(a+b\cot^4(x)\right)^{3/2}}\right)$$

$$\Big\downarrow \; 219$$

$$\frac{1}{2}\left(-\frac{\frac{3a^2+b(5a+2b)\cot^2(x)}{a(a+b)\sqrt{a+b\cot^4(x)}}-\frac{3a\,\text{arctanh}\left(\frac{a-b\cot^2(x)}{\sqrt{a+b}\sqrt{a+b\cot^4(x)}}\right)}{(a+b)^{3/2}}}{3a(a+b)}-\frac{a+b\cot^2(x)}{3a(a+b)\left(a+b\cot^4(x)\right)^{3/2}}\right)$$

input
```
Int[Cot[x]/(a + b*Cot[x]^4)^(5/2),x]
```

output
```
(-1/3*(a + b*Cot[x]^2)/(a*(a + b)*(a + b*Cot[x]^4)^(3/2)) - ((-3*a*ArcTanh
[(a - b*Cot[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Cot[x]^4])])/(a + b)^(3/2) + (3*
a^2 + b*(5*a + 2*b)*Cot[x]^2)/(a*(a + b)*Sqrt[a + b*Cot[x]^4]))/(3*a*(a +
b)))/2
```

3.64. $\int \frac{\cot(x)}{(a+b\cot^4(x))^{5/2}}\,dx$

3.64.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1577
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, c, d, e, p, q}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.64. $\int \frac{\cot(x)}{(a+b\cot^4(x))^{5/2}}\,dx$

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.64.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 585 vs. $2(105) = 210$.

Time = 0.14 (sec) , antiderivative size = 586, normalized size of antiderivative = 5.01

method	result
derivativedivides	$\frac{b^2 \ln\left(\frac{2a+2b-2b\left(\cot(x)^2+1\right)+2\sqrt{a+b}\,\sqrt{b\left(\cot(x)^2+1\right)^2-2b\left(\cot(x)^2+1\right)+a+b}}{\cot(x)^2+1}\right)}{2\left(b+\sqrt{-ab}\right)^2\left(-b+\sqrt{-ab}\right)^2\sqrt{a+b}} - \frac{\frac{\sqrt{\left(\cot(x)^2+\frac{\sqrt{-ab}}{b}\right)^2 b-2\sqrt{-ab}\left(\cot(x)^2+\cdots\right.}}{3\sqrt{-ab}\left(\cot(x)^2+\frac{\sqrt{-ab}}{b}\right)^2}\cdots}{8\cdots}$
default	$\frac{b^2 \ln\left(\frac{2a+2b-2b\left(\cot(x)^2+1\right)+2\sqrt{a+b}\,\sqrt{b\left(\cot(x)^2+1\right)^2-2b\left(\cot(x)^2+1\right)+a+b}}{\cot(x)^2+1}\right)}{2\left(b+\sqrt{-ab}\right)^2\left(-b+\sqrt{-ab}\right)^2\sqrt{a+b}} - \frac{\frac{\sqrt{\left(\cot(x)^2+\frac{\sqrt{-ab}}{b}\right)^2 b-2\sqrt{-ab}\left(\cot(x)^2+\cdots\right.}}{3\sqrt{-ab}\left(\cot(x)^2+\frac{\sqrt{-ab}}{b}\right)^2}\cdots}{8\cdots}$

input
```
int(cot(x)/(a+b*cot(x)^4)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*b^2/(b+(-a*b)^(1/2))^2/(-b+(-a*b)^(1/2))^2/(a+b)^(1/2)*ln((2*a+2*b-2*b
*(cot(x)^2+1)+2*(a+b)^(1/2)*(b*(cot(x)^2+1)^2-2*b*(cot(x)^2+1)+a+b)^(1/2))
/(cot(x)^2+1))-1/8/(-b+(-a*b)^(1/2))/a*(1/3/(-a*b)^(1/2)/(cot(x)^2+(-a*b)^
(1/2)/b)^2*((cot(x)^2+(-a*b)^(1/2)/b)^2*b-2*(-a*b)^(1/2)*(cot(x)^2+(-a*b)^
(1/2)/b))^(1/2)-1/3/a/(cot(x)^2+(-a*b)^(1/2)/b)*((cot(x)^2+(-a*b)^(1/2)/b)
^2*b-2*(-a*b)^(1/2)*(cot(x)^2+(-a*b)^(1/2)/b))^(1/2))+1/8/(b+(-a*b)^(1/2))
/a*(-1/3/(-a*b)^(1/2)/(cot(x)^2-(-a*b)^(1/2)/b)^2*((cot(x)^2-(-a*b)^(1/2)/
b)^2*b+2*(-a*b)^(1/2)*(cot(x)^2-(-a*b)^(1/2)/b))^(1/2)-1/3/a/(cot(x)^2-(-a
*b)^(1/2)/b)*((cot(x)^2-(-a*b)^(1/2)/b)^2*b+2*(-a*b)^(1/2)*(cot(x)^2-(-a*b
)^(1/2)/b))^(1/2))+1/8*(2*(-a*b)^(1/2)-b)/(-b+(-a*b)^(1/2))^2/a^2/(cot(x)^
2+(-a*b)^(1/2)/b)*((cot(x)^2+(-a*b)^(1/2)/b)^2*b-2*(-a*b)^(1/2)*(cot(x)^2+
(-a*b)^(1/2)/b))^(1/2)-1/8*(2*(-a*b)^(1/2)+b)/(b+(-a*b)^(1/2))^2/a^2/(cot(
x)^2-(-a*b)^(1/2)/b)*((cot(x)^2-(-a*b)^(1/2)/b)^2*b+2*(-a*b)^(1/2)*(cot(x)
^2-(-a*b)^(1/2)/b))^(1/2)
```

3.64.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 686 vs. $2(103) = 206$.

Time = 0.52 (sec) , antiderivative size = 1365, normalized size of antiderivative = 11.67

$$\int \frac{\cot(x)}{(a+b\cot^4(x))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(x)/(a+b*cot(x)^4)^(5/2),x, algorithm="fricas")
```

output
```
[1/12*(3*((a^4 + 2*a^3*b + a^2*b^2)*cos(2*x)^4 + a^4 + 2*a^3*b + a^2*b^2 -
 4*(a^4 - a^2*b^2)*cos(2*x)^3 + 2*(3*a^4 - 2*a^3*b + 3*a^2*b^2)*cos(2*x)^2
 - 4*(a^4 - a^2*b^2)*cos(2*x))*sqrt(a + b)*log(1/2*(a^2 + 2*a*b + b^2)*cos
(2*x)^2 + 1/2*a^2 + 1/2*b^2 + 1/2*((a + b)*cos(2*x)^2 - 2*a*cos(2*x) + a -
 b)*sqrt(a + b)*sqrt(((a + b)*cos(2*x)^2 - 2*(a - b)*cos(2*x) + a + b)/(co
s(2*x)^2 - 2*cos(2*x) + 1)) - (a^2 - b^2)*cos(2*x)) - 4*((2*a^4 + a^3*b -
5*a^2*b^2 - 5*a*b^3 - b^4)*cos(2*x)^4 + 2*a^4 + 7*a^3*b + 9*a^2*b^2 + 5*a*
b^3 + b^4 - 2*(4*a^4 + 2*a^3*b - a^2*b^2 + 2*a*b^3 + b^4)*cos(2*x)^3 + 12*
(a^4 + a^3*b)*cos(2*x)^2 - 2*(4*a^4 + 8*a^3*b + 3*a^2*b^2 - 2*a*b^3 - b^4)
*cos(2*x))*sqrt(((a + b)*cos(2*x)^2 - 2*(a - b)*cos(2*x) + a + b)/(cos(2*x
)^2 - 2*cos(2*x) + 1)))/(a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b
^4 + a^2*b^5 + (a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*
b^5)*cos(2*x)^4 - 4*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a
^2*b^5)*cos(2*x)^3 + 2*(3*a^7 + 7*a^6*b + 6*a^5*b^2 + 6*a^4*b^3 + 7*a^3*b^
4 + 3*a^2*b^5)*cos(2*x)^2 - 4*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a
^3*b^4 - a^2*b^5)*cos(2*x)), -1/6*(3*((a^4 + 2*a^3*b + a^2*b^2)*cos(2*x)^4
 + a^4 + 2*a^3*b + a^2*b^2 - 4*(a^4 - a^2*b^2)*cos(2*x)^3 + 2*(3*a^4 - 2*a
^3*b + 3*a^2*b^2)*cos(2*x)^2 - 4*(a^4 - a^2*b^2)*cos(2*x))*sqrt(-a - b)*ar
ctan(((a + b)*cos(2*x)^2 - 2*a*cos(2*x) + a - b)*sqrt(-a - b)*sqrt(((a + b
)*cos(2*x)^2 - 2*(a - b)*cos(2*x) + a + b)/(cos(2*x)^2 - 2*cos(2*x) + 1...
```

3.64.6 Sympy [F]

$$\int \frac{\cot(x)}{(a+b\cot^4(x))^{5/2}}\,dx = \int \frac{\cot{(x)}}{(a+b\cot^4{(x)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(x)/(a+b*cot(x)**4)**(5/2),x)
```

output
```
Integral(cot(x)/(a + b*cot(x)**4)**(5/2), x)
```

3.64.7 Maxima [F]

$$\int \frac{\cot(x)}{(a+b\cot^4(x))^{5/2}}\,dx = \int \frac{\cot(x)}{\left(b\cot(x)^4+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(x)/(a+b*cot(x)^4)^(5/2),x, algorithm="maxima")
```

output
```
integrate(cot(x)/(b*cot(x)^4 + a)^(5/2), x)
```

3.64.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 276 vs. $2(103) = 206$.

Time = 0.31 (sec) , antiderivative size = 276, normalized size of antiderivative = 2.36

$$\int \frac{\cot(x)}{(a+b\cot^4(x))^{5/2}}\,dx = -\frac{\left(2\left(\frac{(2a^3b-a^2b^2-4ab^3-b^4)\sin(x)^2}{a^4b+2a^3b^2+a^2b^3}+\frac{3(3ab^3+b^4)}{a^4b+2a^3b^2+a^2b^3}\right)\sin(x)^2+\frac{3(a^2b^2-5ab^3-2b^4)}{a^4b+2a^3b^2+a^2b^3}\right)\sin(x)^2+\frac{5ab^3+2b^4}{a^4b+2a^3b^2+a^2b^3}}{6\left(a\sin(x)^4+b\sin(x)^4-2b\sin(x)^2+b\right)^{\frac{3}{2}}} - \frac{\log\left(\left|-\left(\sqrt{a+b}\sin(x)^2-\sqrt{a\sin(x)^4+b\sin(x)^4-2b\sin(x)^2+b}\right)\sqrt{a+b}+b\right|\right)}{2(a^2+2ab+b^2)\sqrt{a+b}}$$

input
```
integrate(cot(x)/(a+b*cot(x)^4)^(5/2),x, algorithm="giac")
```

output
```
-1/6*((2*((2*a^3*b - a^2*b^2 - 4*a*b^3 - b^4)*sin(x)^2/(a^4*b + 2*a^3*b^2
+ a^2*b^3) + 3*(3*a*b^3 + b^4)/(a^4*b + 2*a^3*b^2 + a^2*b^3))*sin(x)^2 + 3
*(a^2*b^2 - 5*a*b^3 - 2*b^4)/(a^4*b + 2*a^3*b^2 + a^2*b^3))*sin(x)^2 + (5*
a*b^3 + 2*b^4)/(a^4*b + 2*a^3*b^2 + a^2*b^3))/(a*sin(x)^4 + b*sin(x)^4 - 2
*b*sin(x)^2 + b)^(3/2) - 1/2*log(abs(-(sqrt(a + b)*sin(x)^2 - sqrt(a*sin(x
)^4 + b*sin(x)^4 - 2*b*sin(x)^2 + b))*sqrt(a + b) + b))/((a^2 + 2*a*b + b^
2)*sqrt(a + b))
```

3.64.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot(x)}{(a+b\cot^4(x))^{5/2}}\,dx = \int \frac{\cot(x)}{\left(b\cot\left(x\right)^4+a\right)^{5/2}}\,dx$$

input
```
int(cot(x)/(a + b*cot(x)^4)^(5/2),x)
```

output
```
int(cot(x)/(a + b*cot(x)^4)^(5/2), x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)



(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
            'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
            'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
            'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
            'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))

leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation =" "
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
return grade, grade_annotation
```